Numerical Investigation of DC Flow Loss in Thermoacoustic Systems

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ABSTRACT

Gedeon DC flows impose a serious influence on the performance of thermoacoustic systems with topological loops. This article gives a quantitative analysis of DC flow influenced in a normal double inlet pulse tube cooler and a feedback pulse tube cooler with a numerical model based on linear thermoacoustic theory. Different configuration characteristics and cold end temperatures are evaluated. The results help to establish a reasonable evaluation of DC flow influence on the system performance.

INTRODUCTION

Topological loops exist in many different high efficiency thermoacoustic systems due to the requirement of phase relationship between pressure and volume flow rate[1] or feedback of acoustic work[2]. Two typical cooler systems, the double-inlet pulse tube cooler (DPTC) and feedback pulse tube cooler (FPTC), are shown in Fig. 1. While it has already been known that such loops could give rise to Gedeon DC flow[3], the correct method to estimate the loss caused by such DC flows is still ambiguous. It has been pointed out[4] that DC flow loss cannot be simply estimated with the equation:

$$Q_{dc,loss} \approx \dot{m}_{dc} C_p \left(T_0 - T_c\right) \qquad (1)$$

where, $\dot{m}_{dc}$, C_p, T_0 and T_c, are the DC mass flow rate, isobaric specific heat, ambient heat exchanger temperature and cold end temperature, respectively. The equation is unreasonable from a physical viewpoint and the DC flow actually influences the cooling power in an indirect way. The authors[5] give a numerical analysis of a miniature pulse tube cooler and have shown that Eq. (1) overestimates the loss when the DC flow amount is within a certain range. However, the discussion has been limited to the miniature pulse tube cooler and the conclusion may be different on a different geometric configuration, e.g. a feedback pulse tube cooler, or with a different operating temperature. This article further extends the discussion to a double-inlet pulse tube cooler aimed at 80 K operation and a feedback pulse tube cooler aimed at domestic refrigeration. The next section gives a brief introduction of the numeric model. The calculation results are introduced in the third section. Finally, conclusions are made.

NUMERICAL MODEL

The numerical model is based on linear thermoacoustic theory.[6,7] The main control equations in complex form are:

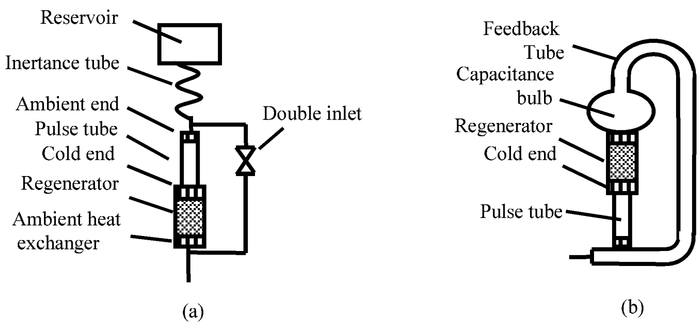

Figure 1. Two types of pulse tube cooler with topological loop: (a) double-inlet pulse tube cooler; (b) feedback pulse tube cooler

$$\frac{d\hat{U}}{dx} + R_1\hat{p} - R_3\hat{U} = 0 \tag{2}$$

$$\frac{d\hat{p}}{dx} + R_2\hat{U} = 0 \tag{3}$$

$$\frac{dH}{dx} = Q \quad H = c_1 + c_2\frac{dT_x}{dx} \tag{4}$$

$\hat{p}, \hat{U}, and\ T_x$ are complex pressure wave amplitude, complex volume flow rate amplitude and mean temperature of the gas, respectively. H is the total energy including enthalpy flow integrated over the fluid passage cross section and static heat conduction. Q is the heat input to the gas from the outside of the system. R_1, R_2, R_3, c_1, c_2 are functions of parameters such as the fluid passage geometry, frequency, and average pressure.

The working gas, helium, is taken as an ideal gas with temperature-dependent heat conductivity and viscosity. The regenerator material also has temperature-dependent heat conductivity and specific heat. With these equations and boundary conditions, the calculation[8] can start. Furthermore, there is no attempt in the program to find the source of DC flow. The magnitude of DC flow is set manually to investigate the influence.

CALCULATION RESULTS AND DISCUSSIONS

Before introducing the calculation results, two normalized parameters are defined. The first is the relative DC flow magnitude N:

$$N = \frac{\dot{m}_{dc}}{\dot{m}_{regen,Th}} \times 100\% \tag{5}$$

which is the ratio between DC mass flow rate $\dot{m}_{dc}$ and amplitude of oscillating mass flow rate at the hot end of the regenerator $\dot{m}_{regen,Th}$. By changing this value, its influence is seen on the cooling power and temperature distributions in the following cooler systems. For both systems investigated, the positive value of N means the DC flow is from the regenerator hot end to the regenerator cold end through the regenerator, while a negative value means the opposite direction.

The second parameter is normalized cooling power loss, which is defined as

$$\eta = \frac{Q_c - Q_{c,dc}}{\dot{m}_{dc} C_p \left(T_h - T_c\right)} \tag{6}$$

Q_c is the cooling power with N equal to zero at the specified cold end temperature which will be intentionally selected to ensure cooling power is bigger than zero even when DC flow magnitude is largest. $Q_{c,dc}$ is the cooling power with a certain DC flow, T_h, T_c are the temperatures of the hot and cold end heat exchangers. This parameter actually shows the extent Eq. (1) overestimates the real cooling power loss.

Double Inlet Pulse Tube Cooler

Main components of the pulse tube cooler according to Fig.1(a) and the operating conditions are listed in Table 1. From the calculation, the cooler can reach a temperature of about 30 K. By setting the cold end temperature to 80 K, the magnitude of the DC flow influence on the system performance is seen when it is varied by up to ±0.3%.

One issue should be addressed before this discussion. With a given pressure wave input, the DC flow amount will change the input power of the system. To put the discussion in a more common background, Figure 2 shows how the DC flow magnitude changes the input power. As can be seen, the input power only varies within a small range, which will not affect the validity of the following discussion. Meanwhile, Figure 2 also shows that cooling power at 80 K first decreases slowly from 5.8 W with zero DC flow to around 5 W with 0.1% DC flow magnitude, then it quickly drops with increased DC flow magnitude.

Figure 3 shows how the DC flow affects the temperature distribution along the regenerator and the pulse tube. It can be seen that *N* larger than 0.1% already seriously changes the temperature gradient, which can be easily detected in experiments. Figure 4 shows how the normalized cooling

Table 1. Details of the normal double inlet pulse tube cooler

Regenerator	Length 50mm, I.D. 25mm, filled with 400# stainless steel mesh
Pulse tube	Length 60mm, I.D.12mm
Operating conditions	3.0MPa, 30Hz, Inlet pressure amplitude 0.3MPa, Ambient heat exchanger temperature is fixed at 299K

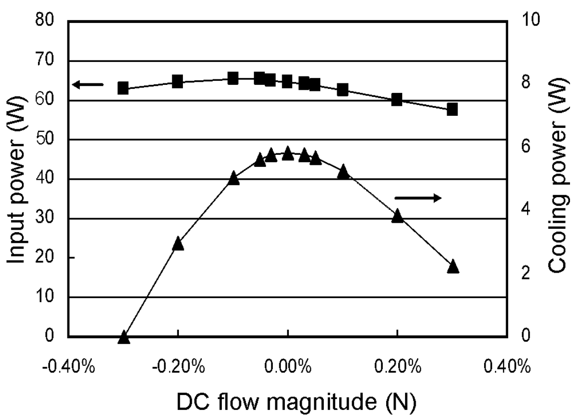

Figure 2. Influence of DC flow magnitude on input power and cooling power, DPTC

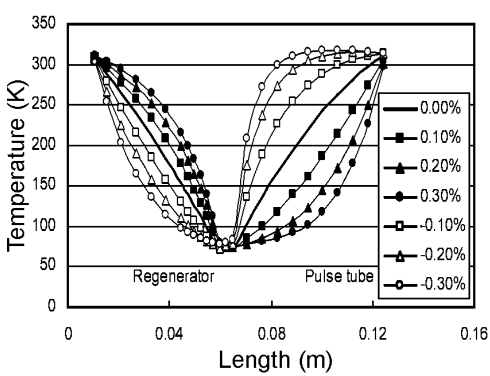

Figure 3. Influence of DC flow magnitude on temperature distribution, DPTC

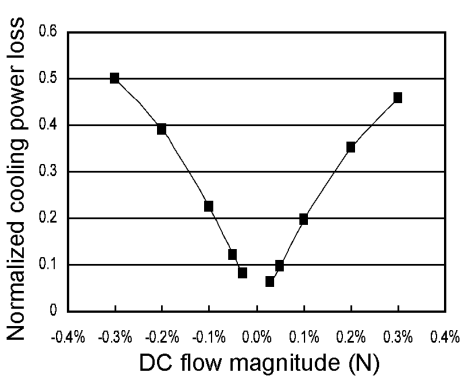

Figure 4. Normalized cooling power loss vs. DC flow magnitude, DPTC

power loss changes with *N*. As the results clearly indicate, Eq. (1) does overestimate the real cooling power loss.

Feedback Pulse Tube Cooler

The main components of the feedback pulse tube cooler and the operating conditions are listed in Table 2. By setting the cold end temperature at 240 K, we will see how the magnitude of the DC flow, which varies up to ±2.0% , changes the system performance. Figure 5 shows the influence of the DC flow magnitude on the input power and cooling power. The calculation indicates that the system input power changes within 1%. This may be related to the small temperature span of the regenerator. With zero DC flow, the system can provide 117 W @ 240 K with 77.6 W of acoustic power input.

Figure 6 shows how the DC flow magnitude changes the temperature distribution. Compared with the results on the double-inlet pulse tube cooler, the abnormality of temperature gradient becomes most apparent when DC flow magnitude N is over 1.0% which is several times larger than the case with a double-inlet pulse tube cooler. Figure 7 also shows Eq. (1) overestimates the real cooling power loss.

CONCLUSION

Influences of Gedeon DC flow on the performance of two typical thermoacoustic coolers, which have different configuration characteristics and cold end temperatures, has been investigated through a numerical model based on linear thermoacoustic theory.

Both results indicate that the traditional Eq. (1) has overestimated the cooling power loss due to DC flow. Meanwhile, it has also been shown that the magnitude of the influence depends on the specific cooler type. In the case of double-inlet pulse tube cooler for 80 K operation, the normalized

Table 2. Details of the feedback pulse tube cooler

Regenerator	Length 35mm, I.D.50mm, filled with 120# stainless steel mesh
Pulse tube	Length 50mm, I.D.50mm
Other components	Refer to [8] for details
Operating conditions	2.5MPa, 60Hz, Inlet pressure amplitude 0.12MPa, Ambient heat exchanger temperature is fixed at 288K

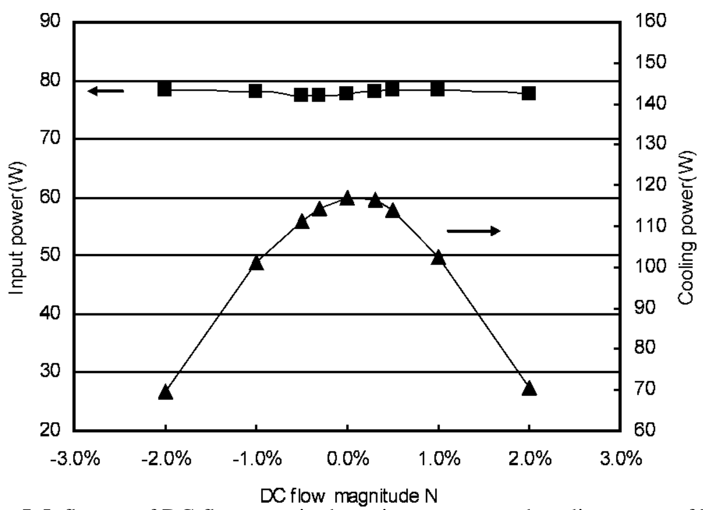

Figure 5. Influence of DC flow magnitude on input power and cooling power of FPTC

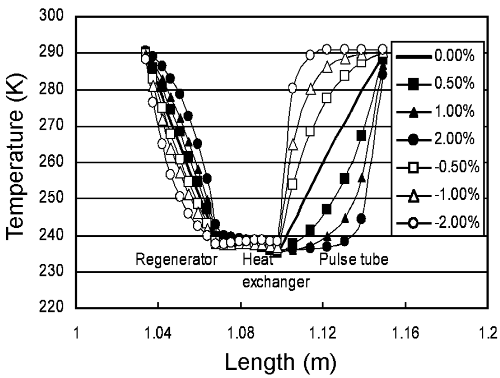

Figure 6. Influence of DC flow magnitude on temperature distribution, FPTC

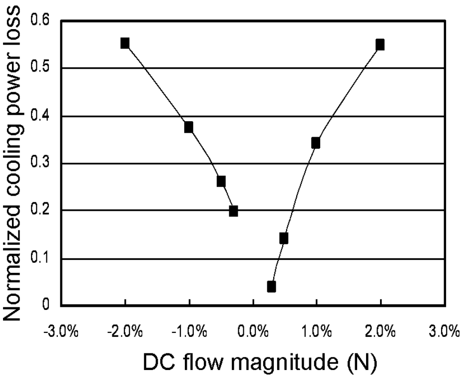

Figure 7. Normalized cooling power loss vs. DC flow magnitude, FPTC

DC flow magnitude over 0.2% could already seriously change the system performance and temperature distribution along the regenerator and the pulse tube axis. However, in the case of a feedback pulse tube cooler for 240 K operation, the magnitude can reach as high as 1.0% before it shows a serious influence. These results imply that DC flow loss should be carefully evaluated according to system characteristics and may be different from case to case.

ACKNOWLEDGMENT

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