A Model for Parametric Analysis of Pulse Tube Losses in Pulse Tube Refrigerators

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ABSTRACT

A loss parameter is introduced into the first order thermodynamic model of a pulse tube consisting of a hot, a cold and an adiabatic section to quantify the pulse tube irreversibility. The entire pulse tube is assumed to be adiabatic while the loss parameter simulates its irreversibility. It is shown that the primary effect of the parameter is to modify the temperature as a function of time in the cold and hot sections of the pulse tube as well as have a secondary effect on the mass flow rate as a function of time at both ends of the pulse tube. The model is incorporated into a first order model of an Inertance Tube Pulse Tube Refrigerator (ITPTR). A distributed parameter model of the inertance tube is developed to simulate the fluid flow in the tube and to quantify the phase shift between the mass flow rate and the pressure at the tube's entrance. The effect of the loss parameter in the pulse tube on the energy and exergy flow in the ITPTR is investigated. The irreversibility distribution of the ITPTR, including the pulse tube, is quantified and the effect of the loss parameter on its cooling capacity is presented.

INTRODUCTION

Pulse Tube Refrigerators (PTRs) play an important role in satisfying the need for cryogenic cooling of space-based infrared detectors as well as electronics requiring coolers with high reliability, low vibration, and high efficiency. Usually, three types of phase-shifting processes exist on PTRs that control the phase shift between the mass flow rate and pressure.[1] The more conventional are Orifice Pulse Tube Refrigerators (OPTRs) where the mass flow rate and pressure are in phase at the orifice. In Double Inlet Pulse Tube Refrigerators (DIPTRs), a bypass valve between the warm end of pulse tube and warm end of the regenerator is used to provide a proper phase-shifting mechanism. In Inertance Tube Pulse Tube Refrigerators (ITPTRs), which are the focus of this study, the phase shifting is provided by an inertance tube replacing the orifice.[2,3] Figure 1 shows the important components of ITPTRs. A review of these phase-shifting mechanisms is given by Radebaugh.[1] The thermodynamics of PTRs have been under study by several investigators.[4,5,6,7]

In first-order models, usually used in design analysis and parametric studies of ITPTRs, a lumped parameter approximation is used to take into account the inertance, compliance, and the

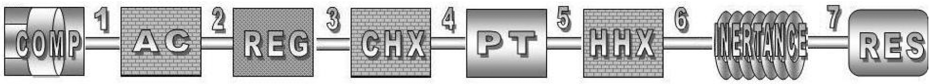

Figure 1. Inertance Tube Pulse Tube Refrigerator (ITPTR)

fluid flow resistance associated with oscillating flow in the inertance tube. In the present study, a previously developed and convenient correlation for entire ranges of laminar and turbulent flow is used with a modified distributed component model to integrate the inertance tube with other components of ITPTRs.[8] The distributed component method used in this study divides the inertance tube into several sections and applies the lumped parameter model to each section. In addition, the pulse tube is modeled using a parameter to quantify the losses in the pulse tube itself. A system of nonlinear ODEs is developed for analysis and optimization of the ITPTRs including the new models for the inertance tube and the pulse tube. The emphasis in this first order model is to study the effect of the loss parameter in the pulse tube, as it is integrated with the effect of the inertance tube in the theoretical analysis and optimization of ITPTRs.

MODELING THE PULSE TUBE AND INERTANCE TUBE

One first order method applied to the modeling of ideal pulse tubes has been based on dividing the pulse tube into three sections.[9,10] The first section adjacent to the Cold Heat Exchanger (CHX) consists of an open system exchanging mass and energy with the CHX. The second section is adjacent to the Hot Heat Exchanger (HHX) modeled as an open system exchanging mass and energy with the HHX. The third section is a closed adiabatic system acting as a buffer between the hot and cold sections and exchanging energy only with the two sections. An adiabatic efficiency parameter η is introduced for the hot and cold sections representing the irreversibility and losses in the pulse tube. The efficiency parameter in the hot and cold sections reduces the magnitude of work transfer during compression and expansion similar to the adiabatic efficiency used in modeling of the compressor and the turbine in thermodynamic analysis of gas cycles. The conservation of mass and energy for these three sections in the pulse tube results in 6 simultaneous Ordinary Differential Equations (ODEs) for the time dependent parameters V_c, V_h, T_4, T_5, $\dot{m}_4$ and $\dot{m}_5$.

$$\dot{m}_4 - \frac{P}{RT_4}\dot{V}_4 + \frac{PV_c}{RT_4^2}\dot{T}_4 = \frac{V_c}{RT_4}\frac{dP}{dt} \tag{1}$$

$$\dot{m}_4 - Z\frac{P}{RT_4}\dot{V}_c = \frac{V_c}{\gamma RT_4}\frac{dP}{dt} \tag{2}$$

$$ZP\dot{V}_c + ZP\dot{V}_h = \frac{V_t - V_h - V_c}{\gamma}\frac{dP}{dt} \tag{3}$$

$$\dot{m}_5 + \frac{P}{RT_5}\dot{V}_h - \frac{PV_h}{RT_5^2}\dot{T}_5 = -\frac{V_h}{RT_5}\frac{dP}{dt} \tag{4}$$

$$\dot{m}_5 + Z\frac{P}{RT_5}\dot{V}_h = -\frac{V_h}{\gamma RT_5}\frac{dP}{dt} \tag{5}$$

where $Z = [1+\eta(\gamma-1)]/\gamma$, R is the gas constant, γ is the specific heat ratio and P is the pressure in the pulse tube. No pressure drop in the pulse tube is considered.

The inertance tube is modeled by $2n + 1$ ODEs using the distributed model of the inertance tube and is given as follows:

$$\dot{m}_j = \dot{m}_{j+1} + C_j\frac{dP_j}{dt} \tag{6}$$

$$P_j - P_{j+1} = R_j\dot{m}_{j+1} + L_j\frac{d\dot{m}_{j+1}}{dt} \quad , j = 1 \ldots n \tag{7}$$

$$\dot{m}_{n+1} = V_{ir}\frac{dP_{n+1}}{dt} \tag{8}$$

where the following variables are given as:

$$P_1 = P, A = \frac{\pi}{4}d^2, C_1 = \frac{A\frac{l}{n}}{2\gamma RT_h}, C_j = \frac{A\frac{l}{n}}{\gamma RT_h}, R_j = \frac{2X\mu\frac{l}{n}}{\pi d^4 \rho}, Y_j = \frac{4\left|\dot{m}_j(t)\right|}{\pi d\mu}, X = \left(64^{a_1} + D_1^{a_1} Y_j^{a_1(1-D_2)}\right)^{\frac{1}{a_1}} \tag{9}$$

$$L_j = \frac{4l}{\pi d^2}, V_{ir} = \frac{V_r + A\frac{l}{2n}}{\gamma RT_h} \tag{10}$$

where the variables P_l corresponds to the pressure that comes from the hot heat exchanger, A is the area of the cross section of the inertance tube, d is the diameter of the inertance tube, C_j are the capacitance coefficients, l is the length of the inertance tube, n is the number of times the inertance tube is split, T_h is the hot temperature, R_j are the resistive coefficients, μ is the gas viscosity, ρ is the gas density, L_j are the inductance coefficients, V_{ir} is the volume of the reservoir plus the last piece of the unaccounted inertance tube volume a_1, Y_j, X, D_1, and D_2 are the laminar/turbulent coefficients described in.[3] The mass flow rate from the pulse tube is linked to the inertance tube by the conservation of mass and energy for the hot heat exchanger.

RESULT AND DISCUSSION

Simulations were run to test the effect of pulse tube adiabatic efficiency on cold and hot side mass flow rates, pressures and temperatures. The important geometric and flow parameters for the ITPTR system using helium as the working fluid are shown in Table 1. Pulse tube efficiencies of 1, .85 and .7 were investigated using a frequency of 60 Hz, a fixed compressor stroke length, and a

Table 1. Important geometric and flow parameters for the ITPTR system

Component	Important data
Compressor	$V_s = 8.2cc$, $V_0 = 8.85cc, \eta_c = .85$ -stroke volume, dead volume, compressor efficiency
Aftercooler	$L = .02$ m, $D = .02$ m
Regenerator	400 mesh, $L = .06$ m, $D = .02$ m
CHX	$L = .01$ m, $D = .02$ m
Pulse Tube	$L = .1$ m, $D = .01$ m
HHX	$L = .01$ m, $D = .01$ m
Inertance Tube	$D = 2.5$ mm, L=1.5 m, distributed model parameters are $a_1 = 12, D_1 = .184$ and $D_2 = .2$

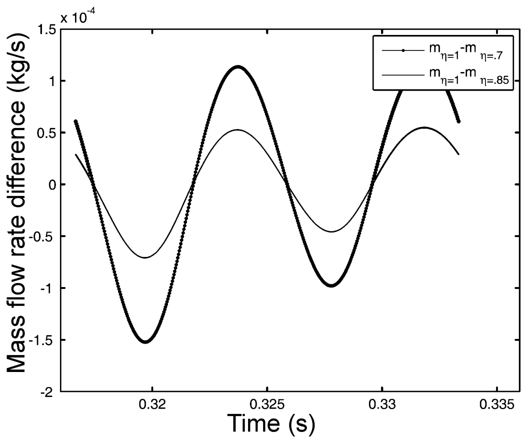

Figure 2. Plots of the differences in cold end mass flow rates for two different pulse tube adiabatic efficiencies for T=80K

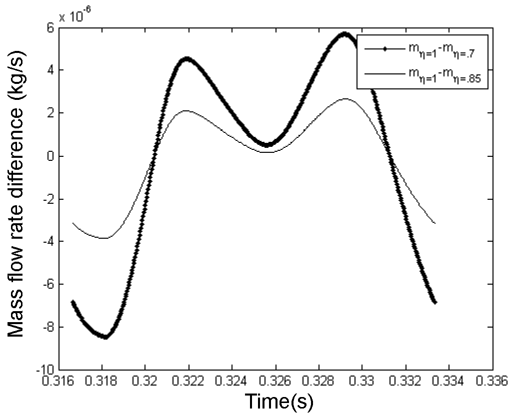

Figure 3. Plots of the differences hot end mass flow rates for two different pulse tube adiabatic efficiencies for T=80 K

3.3 MPa charge pressure. Pressure changes are neglected in the pulse tube and imply that the hot and cold side pressures are the same. Figures 2, 3, and 4 show that the effects on the cold and hot side mass flow rates as well as the pressure are negligible due to changes in pulse tube adiabatic efficiency. The main effect of changing the pulse tube adiabatic efficiency is the modification of the transient temperature distributions for both the cold and hot side of the pulse tube. This effect can be seen in Figures 5 and 6, respectively. The change in the transient temperature distribution is given in Figures 5 and 6, and results in a change in the enthalpy flow in the pulse tube. The enthalpy flow through the pulse tube decreases as the adiabatic efficiency of the pulse tube is reduced. Therefore, for a given rate of heat transfer from the regenerator, the cooling capacity of the refrigerator decreases for lower values of pulse tube adiabatic efficiency.

Another set of simulations was run to test the effect of changing pulse tube efficiency on the cold end cooling load. For these simulations, pulse tube adiabatic efficiencies of 1, .85, .7 and .55 were used with a frequency of 60 Hz, a fixed compressor stroke length, and a 3.3 MPa charge pressure. Figure 7 shows the load curve for different values of the pulse tube adiabatic efficiency. The effect of the efficiency on the no-load temperature is clearly seen. For a given temperature, the cooling capacity increases with an increase in the adiabatic efficiency. Enthalpy, entropy and exergy flow rates at the hot and cold ends of the pulse tube for different pulse tube adiabatic efficiencies for T= 80 K are shown in Table 2.

Different parameters can be used to characterize and define the losses in the pulse tube. Figure of Merit (FOM) for the pulse tube is defined as the ratio of the enthalpy flow through the pulse tube

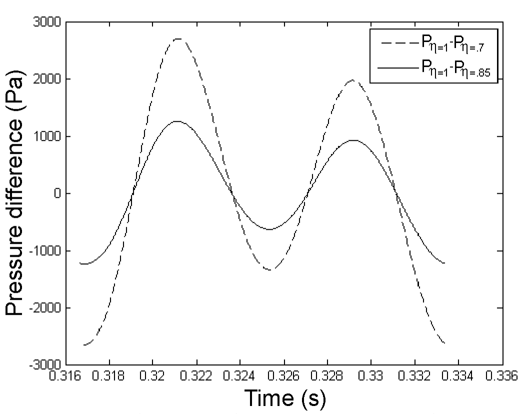

Figure 4. Plots of the differences in pressures for two different pulse tube adiabatic efficiencies for T=80 K

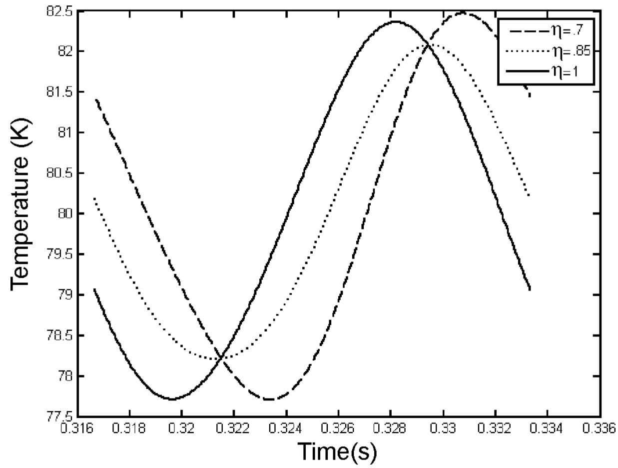

Figure 5. Plots of cold end temperatures for various pulse tube adiabatic efficiencies for T= 80 K

divided by the acoustic power.[1]

$$FOM = \frac{\langle \dot{H} \rangle}{\langle P_d \dot{V} \rangle} \tag{11}$$

where $\dot{H}$ is the enthalpy flow, P_d is the dynamic pressure and $\dot{V}$ is the volume flow rate. The losses in the pulse tube can be defined based on the exergy destruction in the pulse tube. Therefore, the second law efficiency (exergetic efficiency) of the pulse tube is defined previously.[11]

$$\eta_{2-PT} = \frac{\langle \dot{E}_5 \rangle}{\langle \dot{E}_4 \rangle} = 1 - \frac{\langle \dot{I}_{PT} \rangle}{\langle \dot{E}_4 \rangle} \tag{12}$$

where $\dot{E}_4$ *and* $\dot{E}_5$ are the rate of exergy flow at the cold end and the hot end of the pulse tube, respectively. $\dot{I}_{PT}$ is the rate of irreversibility in the pulse tube. For a reversible adiabatic pulse tube, $\dot{I}_{PT}=0$, corresponds to $\eta_{2-PT}= 1$. Figure 8 shows *FOM* and η_{2-PT} as a function of the adiabatic efficiency of the pulse tube (η) for two cold end temperatures of 60 K and 80 K.

Due to the definition of *FOM*, the value does not significantly depend on the cold end temperature and decreases as the adiabatic efficiency decreases. For a fixed adiabatic efficiency, the second law efficiency of the pulse tube is a function of the cold end temperature. For comparison, the

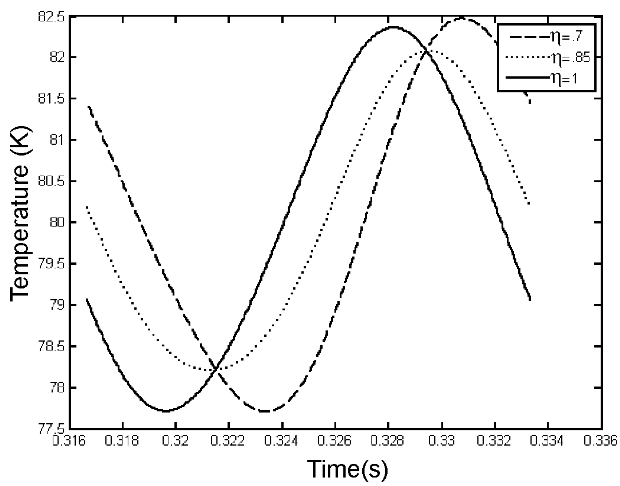

Figure 6. Plots of hot end temperatures for various pulse tube adiabatic efficiencies at T=80 K

Table 2. Enthalpy, entropy and exergy flow rates at the hot and cold ends of the pulse tube for different pulse tube adiabatic efficiencies for T = 80 K

$\eta = .7$				
	Enthalpy flow rate [W]	Entropy flow rate [W]	Exergy flow rate [W]	Acoustic Power flow [W]
Cold end	17.675	-0.057	34.696	22.215
Hot end	17.675	-0.015	22.234	22.211
$\eta = .85$				
Cold end	20.984	-0.028	29.308	22.260
Hot end	20.984	-0.007	23.208	22.259
$\eta = 1$				
Cold end	23.241	0.000	23.241	22.295
Hot end	23.241	0.000	23.241	22.295

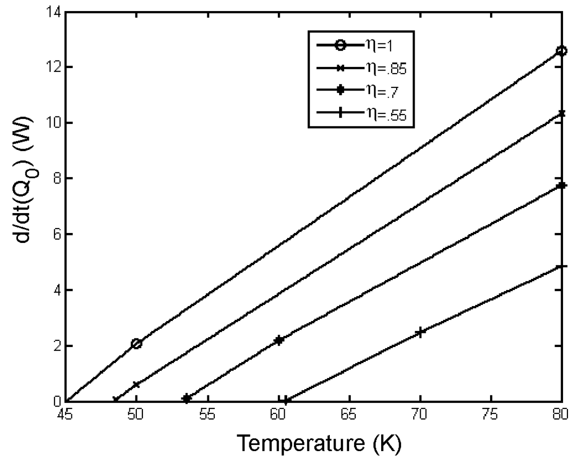

Figure 7. Plots of cooling load versus the cold end temperature for various pulse tube adiabatic efficiencies

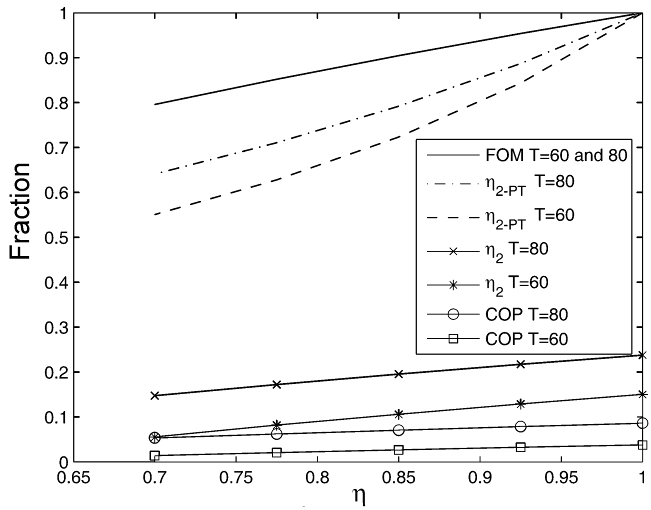

Figure 8. Figure of Merit (FOM), pulse tube efficiencies, 2nd law efficiencies and Coefficient of Performances (COPs) of the ITPTR for various pulse tube adiabatic efficiencies at T = 60K and T = 80K

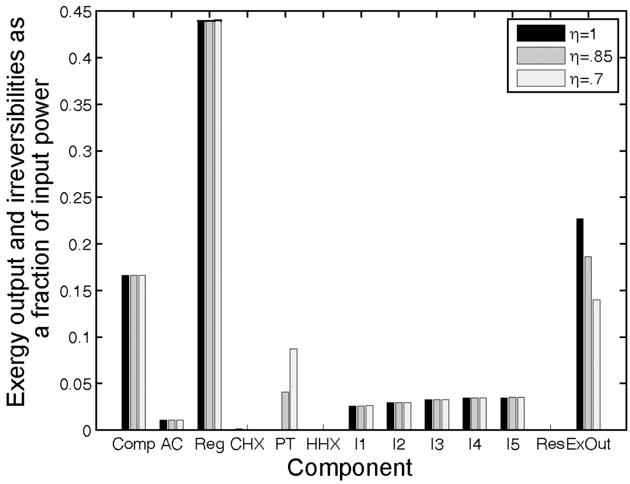

Figure 9. Exergy and irreversibilities for various pulse tube adiabatic efficiencies at T=80 K

second law efficiency (η_2) and the coefficient of performance (COP) for the ITPTR system is also given in Figure 8 . The effect of the adiabatic efficiency of the pulse tube on the system efficiency is clearly seen. For example, comparing to $\eta = 1$ to $\eta = .7$, the second law efficiency decreases from $\eta_2 = .2376$ to $\eta_2 = .1474$ (a reduction of 38%) for the cold end temperature of 80 K.

Figure 9 shows the irreversibility distribution for the important components of the ITPTR for three values of pulse tube adiabatic efficiency. As expected the regenerator is the major component in terms of system losses. The irreversibility in the inertance tube is given for the five sections of the tube modeled in the study and are represented by I1 to I5. As expected, a change in pulse tube adiabatic efficiency has a major impact on the pulse tube irreversibility and energy delivered to the cold reservoir while it has negligible effect on the other components.

CONCLUSIONS

We introduced an adiabatic efficiency parameter for the compression and expansion in the pulse tube to simulate the rate of irreversibility in the pulse tube. In addition we incorporated a distributed parameter model of the inertance tube to model an ITPTR. We showed that the main effect of changing the pulse tube adiabatic efficiency is to change the transient temperature distribution on the hot and cold ends of the pulse tube with a negligible effect on the values of mass flow rate and pressure. We showed that the irreversibility in the pulse tube results in the reduction of the enthalpy flow rate in the pulse tube. The effect of pulse tube adiabatic efficiency on the Figure of Merit (FOM) for the pulse tube, the exergetic efficiency of the pulse tube, the efficiency of the ITPTR and the COP is evaluated.

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