Cryocooler Performance Estimator

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ABSTRACT

Several studies have been published recently that use a simple cryocooler model to perform trade studies between active and passive cryogenic propellant storage systems for space missions. This paper describes the cryocooler model. The model is based on published databases and performance correlations of commercial, space flight, and flight-like cryocoolers.

The objective was to develop a tool to estimate the cryocooler requirements and performance for concurrent engineering based feasibility studies. This environment requires rapid analyses and precludes using either full cryocooler optimization tools, or the real performance of the flight coolers that do not yet exist.

The model was based on empirical correlations of the then current state-of-the-art. As cooler technology progresses, the correlations used here will need to be updated to keep current.

NOMENCLATURE

Roman	
a	Coefficient
m	Mass
P	Power
Q	Heat flow
T	Temperature
Greek	
ε	Carnot efficiency
η	Efficiency (fraction of Carnot)
Superscript	
$*$	Modified
‘	Virtual
†	nominal (design point)

Subscript	
c	Coldest stage
$c\ min$	Cold stage – minimum value
h	Hot end, reject temperature
i	Input
m	mid (1st stage of 2-stage cooler)
$max\ eff$	Maximum efficiency operating point
N	Index
off	Nonoperating cooler
x	Variable (place holder)

INTRODUCTION

The model described here is a tool developed to estimate cryocooler efficiency, input power, and mass for concurrent engineering based space mission design studies using zero boiloff (ZBO) cryo-propellant storage.[1-4] These studies only permit rapid (time scale of seconds to minutes) analyses, which precludes using either full cryocooler optimization tools that typically require hours to days to run, or the real performance of the flight cooler as it may not exist. These latter options are reserved for the more detailed

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analyses that occur in later stages of developing a mission. The approach is to combine the underlying physics of coolers with empirical correlations. There is extensive experience with coolers for liquid oxygen, LOx, storage (80 K nominal). Unlike the LOx coolers, there is only limited data on low temperature flight-like coolers for liquid hydrogen, LH2, storage (20 K nominal). Data from detailed cryocooler models and from commercial coolers were used to supplement the available databases. The empirical correlations and model results are interpolated based on the Carnot efficiency relation.

While the models discussed here were developed for the storage of LOx and LH2, the models are quite general and can be easily extended to other applications requiring single or multistage coolers. These models were based on the then current state-of-the-art. As cooler technology progresses, the correlations used here will need to be updated to keep current.

APPROACH

First, the cryocooler design point for a particular mission scenario must be determined. This is usually the worse case; the case that results in the highest input power. Conditions that result in highest input power are:

1. the highest heat rejection temperature,
2. the largest heat load on the cooler,
3. the requirement to cool tank from a temperature above the required operating temperature range, i.e., the cooling power must exceed tank heat load,
4. the margin requirements, and
5. if redundant coolers are included, the requirement to operate with the redundant cooler off or failed.

These define the design point of the coolers: (Tc, Th, and Qc) for a single-stage cooler or (T_c, T_m, T_h, Q_m, and Q_c) for a two-stage cooler. Defining the design point may be iterative.

Once the cryocooler is sized, its mass and performance (Q_c as function of T_c, T_h, and P_i, or Q_m and Q_c as function of T_c, T_m, T_h, and P_i) are fixed for the rest of the mission.

MODELING A SINGLE-STAGE COOLER (80 -120 K)

Thermal – Determining the Efficiency and Input Power

The efficiency of a single stage cryocooler is η, where

$$\eta\,\varepsilon_c = Q_c/P_i\,, \tag{1}$$

and $\varepsilon_c = T_c/(T_h - T_c)$ is the Carnot efficiency. The heat rejected by the cooler is (by the first law)

$$Q_h = Q_c + P_i \tag{2}$$

Non-ideal effects are all combined into the efficiency term, η. The sources of the inefficiency are the result of many competing mass flow and heat transfer mechanisms inside the cooler.

One can estimate the temperature of peak efficiency as

$$T_{\text{max eff}} = (T_h\ T_{\text{c min}})^{1/2} \tag{3}$$

where $T_{c\,min}$ is T_c when $Q_c = 0.5$. For the 80 K cooler, one can expect to be operating near $T_{max\,eff}$, which, as will be seen later, simplifies the modeling task.

The efficiency at the design point can be estimated from historical data. In 1974, a survey, showed that the efficiency of commercial coolers depended on Q_c and was not a strong function of T_c.[6] There is no physical basis for this correlation – it was a snapshot in time. The survey data is shown in Fig. 1.

In 2001, 80 K cryocoolers were surveyed (see Fig. 2).[7] For the most part, the 1974 and 2001 data lie below a line we will call the ter Brake limit.

In 2004, the Air Force Research Laboratory (AFRL) surveyed space flight cryocoolers.[8] These have been designed for flight, but not all have been flight qualified.

The correlations from these three data sets are summarized in Fig. 3.

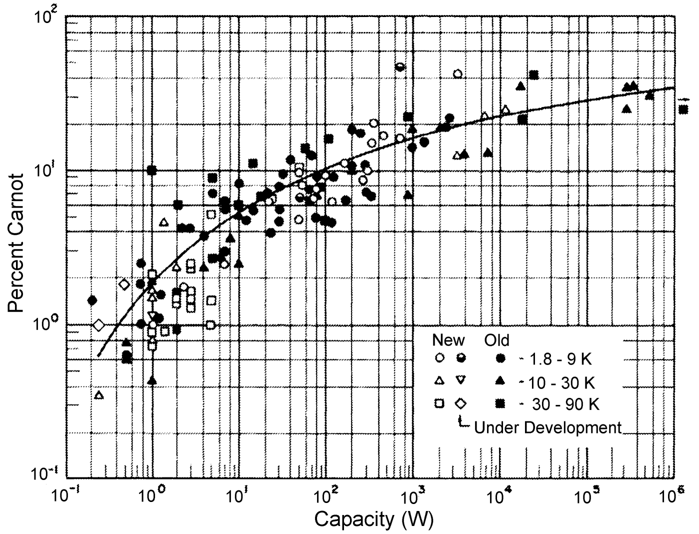

Figure 1. Strobridge efficiency vs. cooling power data.

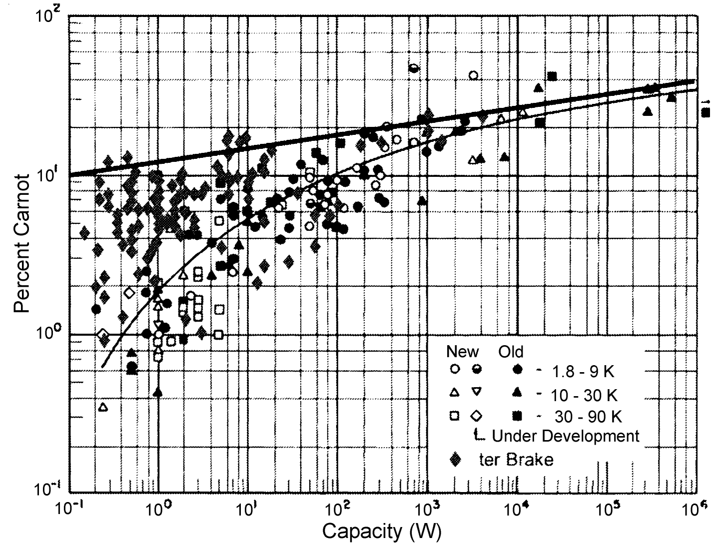

Figure 2. The ter Brake study data superimposed on the Strobridge data. The solid straight line is the ter Brake limit.

Summary of Correlations (as Fraction of Carnot). The following equations summarize the three correlations:

AFRL:

$$\eta = 10^\wedge[\,-1.26281 + 0.45936 \log Q_c - 0.08743\,(\log Q_c)^2] \tag{4}$$

3x Strobridge:

$$\eta = 10^\wedge(\Sigma a_n (\log Q_c)^n) \tag{5}$$

where the a_n are given in Table 1.

ter Brake limit:

$$\eta = 10^\wedge(\,-0.92237 + 0.07763 \log(1+Q_c)) \tag{6}$$

In previous studies,[9-17] 2.4x the Strobridge correlation was recommended as the efficiency estimate. This underestimates the AFRL data, which is about 3x higher than the mean efficiency found by Strobridge.

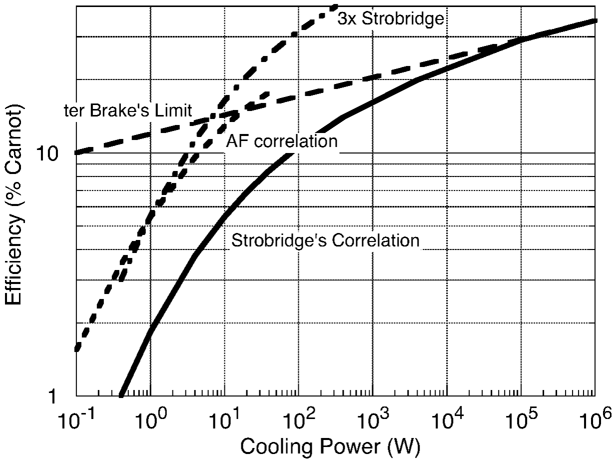

Figure 3. Efficiency correlations from the data of Strobridge, ter Brake, and AFRL.

Table 1. The coefficients a_n.

n	a_n	n	a_n
0	-1.25874	3	0.021323
1	0.59998	4	-0.00125
2	-0.1474		

For this model, for a single stage cooler, the lesser of the AFRL or the ter Brake limit was used. This results in an efficiency that represents the mean of the AFRL data and does not exceed the ter Brake limit. The selected correlation can be used to determine P_i and Q_h from T_c, T_h, and Q_c. The nominal thermal requirements of the cooler have now been defined. These characteristics will be used later to determine the performance of the cooler when it is operating in conditions other than those of its design point.

Correlation's Validity Range. The correlations mentioned above are not based on physical principles; they are purely empirical. The AFRL correlation is from their published work. The other two are our fits to published data. Their use should be limited to the range of their respective data sets. These ranges are given in Table 2.

Mass

Previously studies[9-17] used a mass estimate based on Strobridge's study[6] and a few data points from flight coolers. ter Brake found a correlation for 80 K cryocoolers; that the mass depends primarily on the input power. There is some basis for this simplicity. Most of the mass of Stirling and Pulse Tube coolers is in the compressor, which is sized for the input power. In Braytons, the mass resides in the heat exchangers, which are also sized by the input power. The Air Force study has collected a significant quantity of data on flight type cryocoolers. The Air Force data for cryocoolers operating at > 65 K is in good agreement with ter Brake's correlation. These data and correlation are shown in Fig. 4.

For single stage coolers operating above 65 K, ter Brake's correlation has been chosen

$$m = 0.0711\,P_i^{0.905} \qquad (7)$$

where the nominal (design point) P_i is used to determine the mass.

Table 2. Validity range of efficiency correlations

Correlation	Range
Strobridge	0.2 – 10^6 W
AFRL	0.02 – 20 W
ter Brake	0.1 - 10^6 W

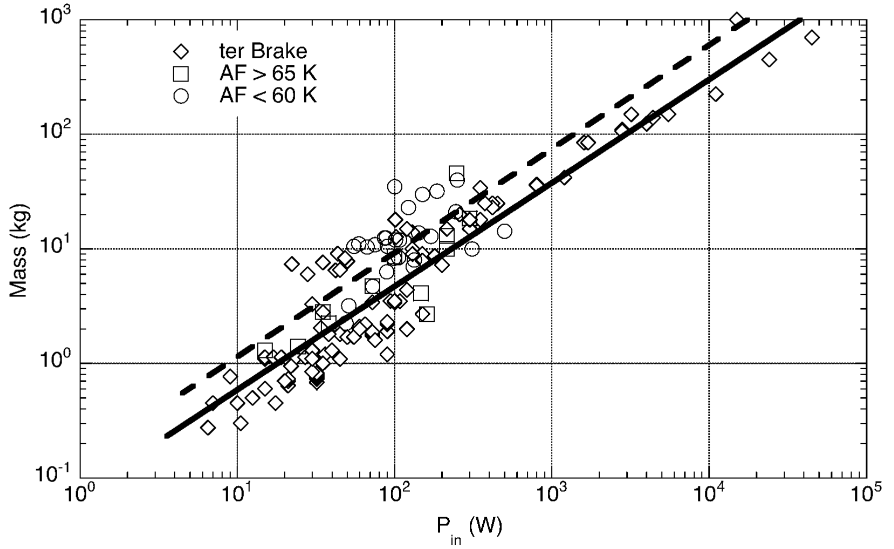

Figure 4. ter Brake's data with the Air Force's data superimposed. Both ter Brake's correlation (solid line) and 2x that correlation (dashed line) are shown. Not shown are the data from other studies that were include in ter Brake's analysis.

Off Nominal Performance

After the cooler has been sized, all modeling is based on the off-nominal performance. The off-nominal performance of coolers is not easy to accurately estimate over their complete range. However, near their peak efficiency, the efficiency only varies slowly. This peak efficiency usually occurs at a higher temperature than the design point of the cooler. For a given input power, peak efficiency is given by Eq. (3).

For LOx coolers, we assume the cooler is operating near the peak cooler efficiency. An operating point where η is a slowly varying function of conditions (see Fig. 5). Once a cooler is sized, its performance can be reasonably approximated by Eq. (1) by assuming constant efficiency, η. This will not be true for a 20 K cooler, as they generally operate well below their peak efficiency. Fig. 5 shows data for a pulse tube.[18] For this cooler, peak efficiency of 15 % of Carnot occurs for the lowest line (120 K). The linearity and overlap of the lines near peak efficiency indicate that the performance of the cooler can be approximated as having constant efficiency in that region. At 95 K, the efficiency has dropped to about 14 % of Carnot, a 7 % decrease. Assuming constant efficiency results in an error that is small compared to the overall accuracy of the model.

Accuracy of Model

Absolute Accuracy. The absolute accuracy is poor. The efficiency is estimated from Eqs. (4) or (6). Eq. (4) is the median value for the flight coolers in the Air Force's database. Half of the coolers are more efficient and half are less efficient. The efficiencies vary by a factor of 5 from worse to best. By choosing the median, we are choosing a cooler that has an efficiency x2 lower than the best that have been built.

Realism Model. There is a high probability that a flight cooler could be built to this estimate because 50% of flight coolers have been built with greater efficiency – a considerable design margin.

Accuracy of Assuming Constant Eficiency. Based on a single cooler[18], the efficiency varies about 7 % (between 14 and 15 % of Carnot) over the range of our interest in this study. Trying to extrapolate this behavior to other coolers operating in this range is probably on the order of 10 % if the coolers have no load temperatures below 50 K. This error is less than the error in η at the design point or the error in estimating the design point.

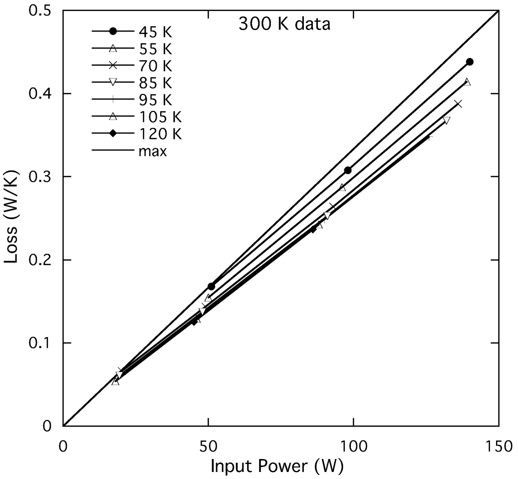

Figure 5. Plot of losses (entropy generated vs. input power) in a typical cooler. The line labeled "max" represents 100% loss (no cooling).

MODELING A MULTISTAGE COOLER (6-40 K)

A 20 K cooler is assumed to have two or more stages. The upper stages may be used to cool the LOx tank, a cooled shield, or both. There are two topology choices: separate coolers for LOx and LH2 or an integrated cooler servicing both tanks. Separate coolers are slightly less efficient, facilitate independently controlling the cooling power delivered to the tanks, and are easier to integrate. With a multistage cooler, changing the input power to change the cooling to one tank may change the cooling to the other tank. A two stage cooler with cooling at 95 and 20 K results in rather poor performance in the second (20 K) stage. A three stage cooler with a 35-45 K stage may be more efficient. A 3-stage cooler also has the advantage of isolating the performance of the first and last stages. Therefore, changes in the first stage temperature have less effect on the performance of the final stage.

After selecting the design approach, the second step is to determine the design point (T_c, T_m, T_h, Q_m, and Q_c) of the cryocooler. This is usually the worse case; the case that results in the highest input power. Conditions that result in highest input power were discussed above. The design process may be iterative.

Once the cryocooler is sized, its mass and performance (Q_m and Q_c as function of T_c, T_m, T_h, and P_i) are fixed for the rest of the mission.

Thermal – Determining the Efficiency and Input Power

The efficiencies of a two stage cryocooler are η_m and η_c, where

$$P_i = Q_m/\eta_m\varepsilon_m + Q_c/\eta_c\varepsilon_c \qquad (8)$$

and $\varepsilon_x = T_x/(T_h\text{-}T_x)$ is the Carnot efficiency. The heat rejected by the cooler, Q_h, is

$$Q_h = Q_c + Q_m + P_i \qquad (9)$$

One might model a 2-stage cooler as two 1-stage coolers operating in series. In this approach, a virtual piston is placed between the two stages. The "piston" acts as a compressor for the second stage and an expander at the first stage. One can then apply the correlations used for single stage coolers. Unfortunately, this approach leads to the efficiencies of the two stages being multiplied, resulting in a very low over all efficiency, much lower than experience suggests. The assumption would be fine if this virtual piston had the same losses as a real compressor and real expander. However, it is not a real piston; the virtual piston should be treated as an ideal loss less device.

There is very little experience with flight-like coolers at 20 K. In addition to the correlations in Fig. 3, Ball 2-stage and 3-stage Stirlings[19], an Astrium Stirling [19], a Pulse Tube model [20], a reverse Brayton model [21], the design point for NASA's ACTDP 6 K coolers[22], and the published performance of a 6 K pulse tube cooler [23] were used to estimate the efficiency of a 20 K cooler. This fits the pulse tube model if one assumes a combined efficiency of 0.7 for the compressor and electronics. The Creare data points are for two different coolers: 1 W @ 18 K + 1 W @ 60 K cooler and 25 W @ 25 K + 10 W @ 110 K. The available data fits the AFRL correlation. The AFRL correlation appears to be a good fit down to 6 K and is valid to lower cooling powers than the Strobridge correlation.

In the discussion on modeling a single stage 80K cooler, an efficiency correlation was suggested based on the smaller of a) the AFRL correlation, or b) the ter Brake limit. This recommendation is still reasonable (see Fig. 6).

Summary of Correlations. The correlations for a multistage cooler are the same as for a single-stage cooler: Eqs. (4) and (6). The efficiency of the first stage is expected to be better than that calculated for the 80 K cooler. This stage not only removes the external heat load, Q_m, but it also intercepts some of the losses of the 20 K stage. Thus, it is a bigger cooler than one needed to just absorb the external load. This effect is not large because the efficiency does not vary rapidly for coolers bigger than 10 W. We will use a modified Eq. (8):

$$P_i = Q_m/\eta^*_m\varepsilon_m + Q_c/\eta_c\varepsilon_c \tag{10}$$

where η^*_m is found by using

$$Q^*_m = Q_m + Q_c\frac{\eta_m T_m}{\eta_c T_c} \tag{11}$$

in the efficiency correlation. The effective heat lift of the upper stage, Q^*_m, is only used in determining η^*_m, the power is still calculated from the real heats.

The selected correlation can now be used to determine P_i and Q_h. The nominal thermal requirements of the cooler have now been defined.

Origin of Eq. (11). $Q_c/\eta_c T_c$ is the entropy generated by Q_c and all of the losses associated with the second stage. A heat load Q'_m absorbed at T_m would generate the same entropy if

$$Q'_m/\eta_m T_m = Q_c/\eta_c T_c. \tag{12}$$

Then $Q'm$ is the amount of heat that must be removed if all of the losses associated with Q_c were removed at T_m by a virtual piston. Also, $Q'_m/\eta_m T_m$ is the entropy generated by Q'_m and its associated losses. Thus, the first stage acts as if $Q_m{}^* = Q_m + Q'_m$ is the heat removed at T_m. Solving Eq. (12) for Q'_m results in $Q'_m = Q_c\eta_m T_m/\eta_c T_c$.

Mass

For the 80 K cooler we used a mass correlation based on ter Brake and a few data points from flight coolers; that the mass depends primarily on the input power. The Air Force data for cryocoolers operating at < 65 K is greater than ter Brake's correlation. When all the Air Force's data is included, the mass of flight coolers is closer to 2x ter Brake's correlation. This is shown in Fig. 4.

The 20 K model uses 2x ter Brake's correlation:

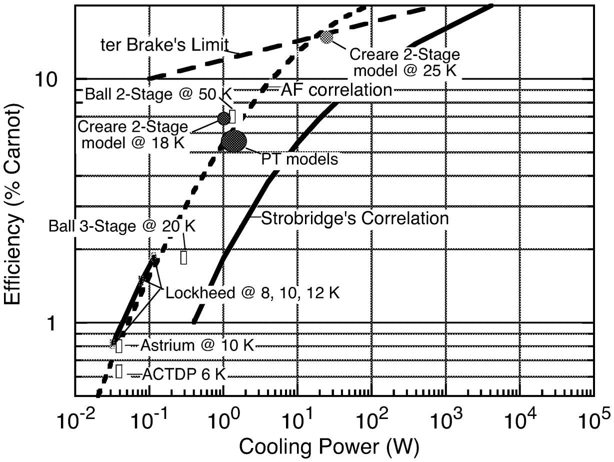

Figure 6. Efficiency correlation for a 20 K stage. The data presented here assumes that all of the input power is used to provide the final stage cooling; i.e., the upper stages are assumed to carry no loads. This is a consistent assumption, but underestimates the true efficiency of the second stage.

$$m = 0.1422\, P_i^{0.905} \qquad (13)$$

where the nominal (design point) P_i is used to determine the mass. Once the mass is found, it is fixed for the mission.

Off Nominal Performance

The off-nominal performance of coolers is not easy to accurately estimate over their complete range. This is particularly true for the 20 K cooler. There is no database for flight like coolers in this range. In addition, there is only limited modeling of off-nominal performance. This modeling is not verified against the performance of actual coolers; thus, it is suspect. There is, however, data on the performance of the Ball 2-stage (a flight like cooler) and on several commercial pulse tube coolers at lower temperatures. (A note of caution: all flight like coolers suitable for this application are low-pressure ratio machines, <1.4. The commercial coolers are high-pressure ratio machines, >2. In addition, the commercial coolers' compressors run open loop, with no control on the input power. The data on these coolers only give peak power, not the actual input power at each data point. For these reasons, the commercial machines may not mimic flight coolers.)

Commercial Coolers. (Cryomech PT403, PT405, PT 805, and Sumitomo DE202) These coolers use compressors running open loop. The cooling powers of the stages are nearly independent of each other. For a generic model of these coolers, it would be reasonable to assume this independence.

Ball Stirling.[19] For *Tc* > 50 K, this cooler shows good independence between the stages.

Below 50 K, the second stage is not sensitive to the first stage, but the first stage is very sensitive to the second. This behavior may reflect the peak efficiency of the second stage being in the 40- 50 K range.[5]

Reverse Turbo Brayton Model.[21] The model produces a load map with the two stages linearly dependent of each other.

Pulse Tube Model.[20] Complete load maps were not available for this model. The stages show a dependence similar to Stirlings.

Summary. Because the Brayton is a likely candidate for ZBO and because the cooling powers are linearly related and thus simple to model, a linear model was used as the basis of off axis modeling. The load lines for constant input power were approximated as

$$Q_m/Q_m^{\dagger} = 1 + 0.03178\,(T_m - T_m^{\dagger}) + 0.04503\,(T_c - T_c^{\dagger}) \qquad (14)$$

$$Q_c/Q_c^{\dagger} = 1 - 0.00544\,(T_m - T_m^{\dagger}) + 0.08349\,(T_c - T_c^{\dagger}) \qquad (15)$$

where the daggered (†) quantities represent the nominal performance. The off nominal performance should not be extended beyond ± 50% of $Q_m^{\dagger}$ or $Q_c^{\dagger}$.

Accuracy of Model

Absolute accuracy. The absolute accuracy is poor. The efficiency is estimated from Eqs. (10) or (11). Eq. (10) is the median value for the flight coolers in the Air Force's database. Half of the coolers are more efficient and half are less efficient. The efficiencies vary by a factor of 5 from worse to best. By choosing the median, we are choosing a cooler that has an efficiency x2 lower than the best that have been built.

Realism of Cryocooler Specifications. We can be confident that at flight cooler could be built to our requirements because flight-like coolers have been built with similar efficiencies.

Accuracy of Off Nominal Performance. Poor because it is based on modeling of only a single cooler and the accuracy and scalability of this model have not been verified and can to be extended to Pulse Tube or Stirlings.

DISCUSSION

A model has been developed for estimating cryocooler performance for quick turn around design studies of space missions using cryocoolers for storing cryo-propellants. The model is based on the empirical correlations of the current state-of-the art cryocooler performances. As the technology advances and cryocooler performance improves, the model will need to be updated. The model estimates the efficiency, input power, and mass of a cryocooler given a nominal design point: required cooling power at a specific temperature and for a given heat rejection temperature. For a LOx cooler, the off-nominal cryocooler performance is also estimated. There is insufficient experience with flight coolers in the 20-30 K range to develop an estimation of the off-nominal performance of a LH2 cooler. Adding this feature must wait until there is more data on the performance of such coolers. Overall, the model, which represents the mean of what has been built in the past without exceeding the best that have been built, estimates cryocooler specifications for a mission designer.

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