A Model for Exergy Efficiency and Analysis of Regenerators

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ABSTRACT

One of the major losses in Stirling-Type Cryogenic Refrigerators (STCRs) is due to heat transfer and fluid flow irreversibility in the regenerative heat exchanger. Accurate numerical modeling of these losses requires extensive numerical simulation of coupled mass, energy and momentum conservation equations. In this study, a first order model is presented relating the mass flow rate and pressure at the hot and cold sides of the regenerator. Heat transfer and pressure drop in the regenerator are obtained assuming reasonable correlations for fluid friction and heat transfer in the regenerator. Therefore, exergy analysis of the regenerator can be performed and the exergy at the cold and hot sides of the regenerator can be related. A performance criteria based on exergetic efficiency of the regenerator is defined and evaluated. The performance of the regenerator and its optimization, based on the exergetic efficiency, is evaluated. The exergy destruction due to heat transfer and pressure drop is determined and discussed. The effect of important parameters on the exergetic efficiency of the regenerator is presented and discussed.

INTRODUCTION

The regenerator is an essential component in the design of STCRs. The efficiency of the refrigerator depends in large part on the efficiency of the regenerator. Therefore, several analytical and numerical methods have been used to model the regenerator in application to STCRs[1-7], to just name a few. The National Institute of Standards and Technology (NIST) code, REGEN 3.2[2], is a powerful code for analysis of regenerators in application to STCRs. Exergy analysis is a powerful method for the design and analysis of thermal systems[8]. Exergy analysis has been used to quantify exergy flow and exergy destruction in PTRs[9-11]. Using a first order model of the regenerator, the focus of this paper is to identify exergy contributions due to: (1) fluid friction causing pressure drop, (2) heat transfer between the regenerator matrix and working fluid, and (3) conduction heat transfer in the fluid, matrix, and regenerator shell. Given the

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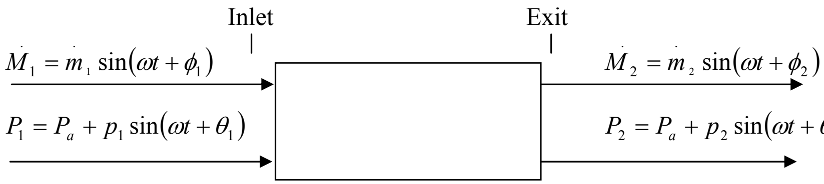

Figure 1. Regenerator schematic with important modeling parameters

geometry of regenerator and the temperatures at the cold and hot sides, the first order model is used to find the exergy at the inlet and exit of the regenerator. The geometry and important parameters of the regenerator are given in Figure 1.

From a thermodynamic point of view, the exergetic efficiency provides a true measure of the regenerator performance. Exergetic efficiency of the regenerator, in application to STCRs, is defined by the exergy at the cold side of the regenerator divided by the exergy delivered to the regenerator at the hot side.

$$\eta_{ex} = \frac{\dot{E}_2}{\dot{E}_1} = \frac{\dot{E}_2}{\dot{E}_2 + \dot{E}_D} = \frac{\dot{E}_2}{\dot{E}_2 + \dot{E}_{D,th} + \dot{E}_{D,p}} = \frac{1}{1 + \frac{\dot{E}_{D,th}}{\dot{E}_2} + \frac{\dot{E}_{D,p}}{\dot{E}_2}} \tag{1}$$

where $\dot{E}_D$ is the exergy destruction (irreversibility) of the regenerator and it can be written as the sum of the exergy destruction due to heat transfer ($\dot{E}_{D,th}$) and fluid friction ($\dot{E}_{D,p}$). One goal of this study is to use a first order model to quantify the exergetic efficiency of the regenerator and the components of exergy destruction due to heat transfer and fluid friction.

EXERGY FLOW THROUGH THE REGENERATOR

The exergy transfer at any section in the regenerator is defined by:

$$\dot{E} = \frac{1}{\tau}\int_0^{\tau}\left[h(t) - h_0 - T_0(s(t) - s_0)\right]\dot{M}(t)dt \tag{2}$$

where h is the specific enthalpy, s is the specific entropy and τ is the period of the oscillating flow through the regenerator. The subscript 0 denotes the property at the environmental condition. Using the ideal gas law with constant specific heat for helium as the working fluid, equation (2) can be written as

$$\dot{E} = \frac{1}{\tau}\int_0^{\tau}\left[C_p\left(T(t) - T_0\right) - R\ln(T(t)/T_0)\right]\dot{M}(t)dt - \frac{RT_0}{\tau}\int_0^{\tau}\left[P(t)/P_a)\right]\dot{M}(t)dt \tag{3}$$

where the first term and the second term on the RHS of equation (3) denote the thermal and pressure components of exergy transfer by mass at any location over a cycle, respectively. Using the sinusoidal function of mass flow rate given in Figure 1 and the definition of the regenerator ineffectiveness, the thermal component of exergy due to the heat transfer between the matrix and the gas at the cold side and the hot side of regenerator can be written, respectively as[12],

$$\dot{E}_{2th} = \frac{\lambda C_p}{\pi}\left(\dot{m}_1 T_h - \dot{m}_2 T_c\right)\left(1 - \frac{T_0}{T_2}\right) \tag{4}$$

$$\dot{E}_{1th} = \frac{\lambda C_p}{\pi}\left(\dot{m}_1 T_h - \dot{m}_2 T_c\right)\left(1 - \frac{T_0}{T_1}\right) \tag{5}$$

where λ is the ineffectiveness of the regenerator and $\bar{T}_1$ and $\bar{T}_2$ are the average temperatures of the hot and cold sides, respectively. Using the sinusoidal functions for mass flow rate and pressure shown in Figure 1, the pressure component of exergy given by the second term of equation (2) for the cold and hot sides of regenerator can be written, respectively as

$$\dot{E}_{2p} = \frac{p_2}{2P_a} RT_0 \dot{m}_2 \cos(\phi_2 - \theta_2) \tag{6}$$

$$\dot{E}_{1p} = \frac{p_1}{2P_a} RT_0 \dot{m}_1 \cos(\phi_1 - \theta_1) \tag{7}$$

The thermal exergy due to conduction heat transfer in the matrix and shell of the regenerator for the cold and hot sides of the regenerator can be written, respectively as

$$\dot{E}_{2,cond} = \frac{k_{eff} A}{L} (\bar{T}_1 - \bar{T}_2)(1 - T_0 / \bar{T}_2) \tag{8}$$

$$\dot{E}_{1,cond} = \frac{k_{eff} A}{L} (\bar{T}_1 - \bar{T}_2)(1 - T_0 / \bar{T}_1) \tag{9}$$

where A is the area of the regenerator matrix, L is the length of the regenerator, and k_{eff} is the effective thermal conductivity of the regenerator and the shell based on the cross sectional area of the matrix. A linear temperature distribution is assumed for the conduction heat transfer. The total thermal component of exergy at the inlet and exit of the regenerator is the sum of equations (5) and (9) and equations (4) and (8), respectively. In order to calculate the thermal and pressure components of exergy at the inlet and exit of the regenerator, the quantities given in equations (4) to (9) must be known. A first order model of flow through the regenerator is developed to find the quantities at the inlet and exit of the regenerator. The average temperature of the cold and hot sides of the regenerator can be obtained using the well established correlation for heat transfer and the NTU method for the heat exchanger analysis[1,3]. The first order model of conservation of mass for the regenerator can be written as,

$$\dot{M}_1 = \dot{M}_2 + \int_0^L \frac{RA_f}{T(x)} \frac{dP(x,t)}{dt} dx \tag{10}$$

where A_f is the flow area, $T(x)$ is the average temperature at the axial location, and $P(x, t)$ is the pressure variation at the axial location in the regenerator. The pressure is assumed to be a sinusoidal function with amplitude and phase shift as a function of x. Therefore, the integral in equation (10) can be approximated using properly averaged values of the functions over x in the regenerator. Trigonometric identities of sinusoidal functions in equation (10) give two equations relating the amplitudes and phase shift of mass flow at the inlet and exit of the regenerator.

$$\dot{m}_1^2 = \dot{m}_2^2 + \left(\frac{A_f \omega L}{R\bar{T}} \right)^2 \left(\overline{PS}^2 + \overline{PC}^2 \right) - \frac{2\dot{m}_2 A_f \omega L \overline{PS}}{R\bar{T}} \tag{11}$$

$$\tan\phi_1 = \frac{A_f \omega L \overline{PC}}{2\dot{m}_2 R\bar{T} - A_f \omega L \overline{PS}} \tag{12}$$

where the average temperature $\bar{T} = (T_h - T_c) / \ln(T_h / T_c)$, $\overline{PC}$ and $\overline{PS}$ are defined by,

$$\overline{PC} = \int_0^L p(x) \cos[\theta(x)] dx \tag{13}$$

$$\overline{PS} = \int_0^L p(x) \sin[\theta(x)] dx \tag{14}$$

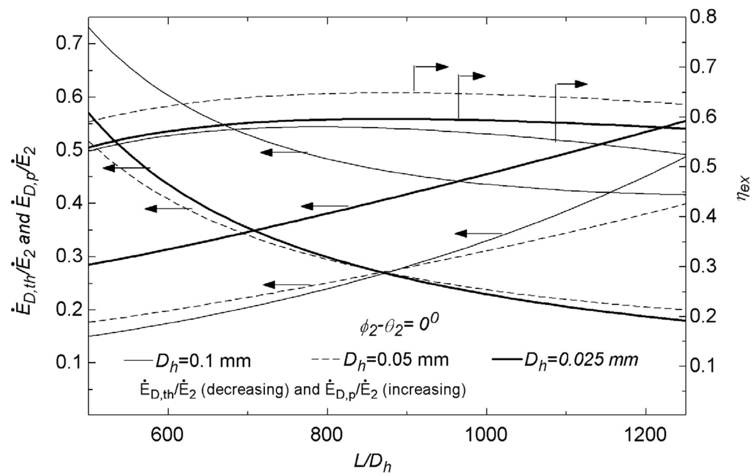

Figure 2: Exergetic efficiency (right axis) and the ratio of thermal and fluid friction exergy destruction to the exergy at the cold side (left axis) as a function of length to hydraulic diameter ratio for φ_2-θ_2=0.

In this study, a linear function is assumed to estimate the integrals given in equations (13) and (14). The relation between the pressure amplitudes and phase shifts at the inlet and exit of the regenerator is obtained using the definition of the Darcy friction factor and the pressure drop in the regenerator.

$$P_1 = P_2 + \frac{Lf\rho u_{avg}\left|u_{avg}\right|}{2D_h} \tag{15}$$

where f is the Darcy friction factor, u_{avg} is the average velocity, ρ is the density, and D_h is the hydraulic diameter. In this study, we assume a general correlation function for the friction

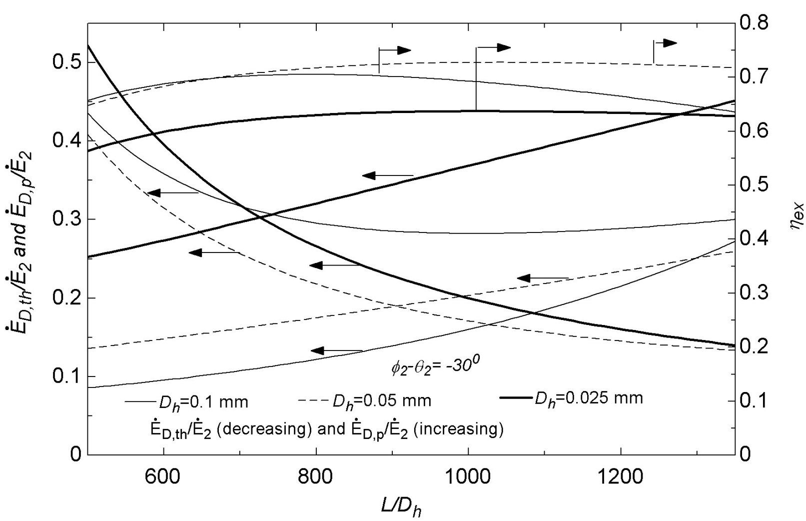

Figure 3: Exergetic efficiency (right axis) and the ratio of thermal and fluid friction exergy destruction to the exergy at the cold side (left axis) as a function of length to hydraulic diameter ratio for φ_2-θ_2=-30.

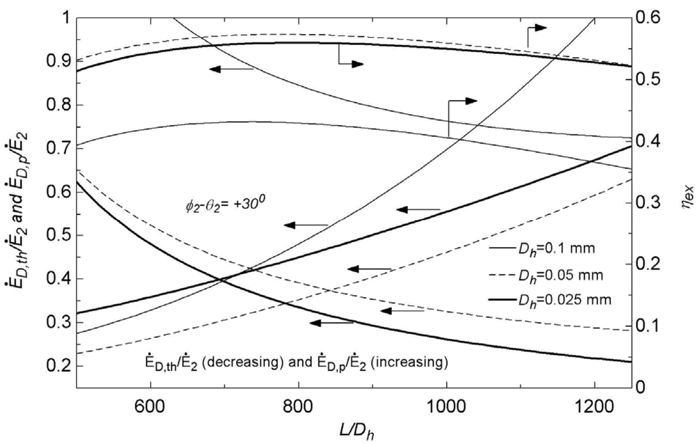

Figure 4: Exergetic efficiency (right axis) and the ratio of thermal and fluid friction exergy destruction to the exergy at the cold side (left axis) as a function of length to hydraulic diameter ratio for φ_2-θ_2=+30.

factor, $f = \sum_i b_i / (\mathrm{Re}_{D_h})^{c_i}$. The parameters b_i and c_i are obtained from experimental data[13-17]. Using the definitions of friction factor and Reynolds number, equation (15) can be written in terms of average mass flow rate as,

$$P_1 = P_2 + \left[\sum_i \frac{b_i L(\bar{\mu})^{c_i} \left|\dot{M}_{avg}\right|^{(1-c_i)}}{2\rho D_h^{(1+c_i)} A_f^{(2+c_i)}} \right] \dot{M}_{avg} \tag{16}$$

where $\bar{\mu}$ is the average viscosity of the fluid in the regenerator. Using functional relations for pressure and mass flow rates given in Figure 1 and the trigonometric identities, the following equations relate the amplitudes of mass flow, amplitudes of pressure, and the phase shifts at the inlet and exit of the regenerator.

$$p_1^2 = p_2^2 + G^2\left(\dot{m}_1^2 + \dot{m}_2^2 + 2\dot{m}_1\dot{m}_2\cos\phi_1\right) + 2G\left[p_2\cos\theta_2\left(\dot{m}_1\cos\phi_1 + \dot{m}_2\right) + p_2\dot{m}_1\sin\theta_2\sin\phi_1\right] \tag{17}$$

$$\tan\theta_1 = \frac{P_2\sin\theta_2 + G\dot{m}_1\sin\phi_1}{P_2\cos\theta_2 + G\left(\dot{m}_1\cos\phi_1 + \dot{m}_2\right)} \tag{18}$$

where G is the bracket in equation (16) and the magnitude of the average mass flow in the equation is estimated from the mass flow at the inlet and exit of the regenerator, $(\dot{m}_1 + \dot{m}_2)/\pi$. In equations (17) and (18) the base phase is assumed to be the phase of the mass flow at the cold side, $\phi_2 = 0$ degrees.

Heat transfer analysis in the regenerator starts with assuming a correlation for the heat transfer coefficient between the gas and matrix in the regenerator. In this study, the following correlation for the Nusselt number is used[15]

$$Nu = \frac{hD_h}{k_f} = C_1 + C_2(\mathrm{RePr})^{C_3}\left[1 - C_4(1-\varepsilon)\right] \tag{19}$$

where ε is the porosity, h is the heat transfer coefficient, k_f is the thermal conductivity of the working fluid, Pr is the Prandtl number, and C_1 to C_4 are the correlation coefficients[15]. Equation (19) can be used to find the Number of Transfer Units (NTU) in the regenerator heat exchanger.

$$NTU = \frac{2k_f NuV\varepsilon}{D_h^2 C_p \left|\dot{M}_{avg}\right|} \tag{20}$$

Another important parameter is the matrix capacity ratio defined by,

$$CR = \frac{(MC)_m}{C_{\min}} = \frac{V\rho_m C_m (1-\varepsilon)}{\tau C_p \left|\dot{M}_{avg}\right|} \tag{21}$$

where V is the volume, ρ_m and C_m are the density and specific heat of the matrix in the regenerator. The regenerator effectiveness is given as a function of NTU and CR in graphical and tabulated forms [1, 3, 16].

RESULTS AND DISCUSSION

In order to evaluate the first order model and the exergy based criteria for performance evaluation of regenerators, several studies were performed some of which are reported in this section. Most of the calculations are for a frequency of 50 Hz, average pressure of 3 MPa, hot reservoir temperature of $T_0 = T_h =$ 300K, and cold reservoir temperature T_c=70 K. In order to calculate conduction heat transfer in the regenerator a conduction degradation factor of 0.1 was assumed[1]. The regenerator matrix and shell are assumed to be made of stainless steel. Figure 2 shows the ratio of thermal and fluid friction components of exergy destruction to the total exergy at the cold side of the regenerator as a function of the ratio of the length of the regenerator to the hydraulic diameter (L/D_h) for three values of hydraulic diameter D_h of 0.1 mm, 0.05 mm and 0.025 mm. The value of the phase shift between pressure and mass flow at the cold side is kept constant at zero degrees. The value of the pressure ratio at the cold side of the regenerator is kept constant at the value of $PR = (P_a+p_2)/(P_a-p_2) = 1.213$. Therefore, using equation (6) the value of the pressure component of exergy at the cold side of the regenerator is kept constant at 60 watts. From the figure, it can be seen that as the length of the regenerator increases, the thermal component of exergy destruction is reduced while the pressure component increases. As the length of the regenerator increases, its effectiveness increases, the conduction heat transfer goes down, and the pressure drop due to fluid friction goes up. The compromise between the exergy destruction due to heat transfer and fluid friction results in a minimum for the total exergy destruction in the regenerator. For each value of the hydraulic diameter, this minimum total exergy destruction corresponds to the optimum exergetic efficiency as defined in equation (1). The values of exergetic efficiency are shown on the right y-axis of the Figure. Since the pressure component of exergy at the cold side, as given by equation (6) does not change, the total exergy delivered at the cold side of the regenerator increases as its length increases. The input exergy

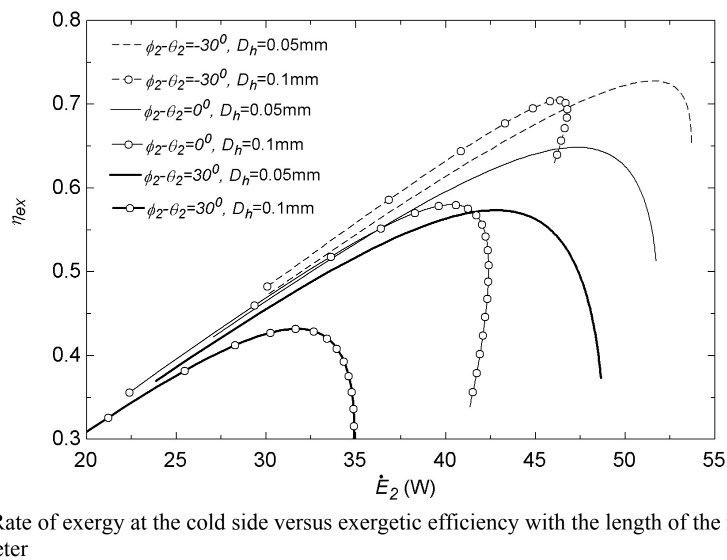

Figure 5: Rate of exergy at the cold side versus exergetic efficiency with the length of the regenerator as a parameter

also increases with the length of the regenerator, resulting in an optimum exergetic efficiency of the regenerator. It should be pointed out that since the average temperature at the hot side of the regenerator is close to the temperature of the hot reservoir, the thermal exergy at the hot side of the regenerator is negligible. Figures 3 and 4 show the same results as Figure 2 for different phase shifts between the mass flow and pressure at the cold side of the regenerator of -30 degrees (mass flow lagging the pressure) and +30 degrees (mass flow leading the pressure), respectively. Comparing Figures 3 and 4, it can be seen that when mass flow is lagging pressure the exergetic efficiency of the regenerator is higher than when mass flow is leading the pressure. This result shows the advantage of the inertance tube in controlling the phase shift at the cold side of the regenerator, as compared to the orifice, as it is well known in the design of inertance tube pulse-tube refrigerators. In calculations of Figures 2 to 4, the mass flow rates are kept constant and as the phase shift changes in each figure, the corresponding pressure ratio at the cold side is changed such that the pressure component of exergy at the cold side of regenerator is the same in all Figures.

Figure 5 shows exergetic efficiency as a function of exergy delivered at the cold side of the regenerator for two values of hydraulic diameter and three values of phase shift between the mass flow and pressure at the cold side. The length of the regenerator is a variable in all calculations. As the length of the regenerator increases, the thermal exergy entering the cold side of the regenerator is reduced. Therefore, the exergy delivered at the cold side is increased. The exergy input to the regenerator goes up when the length of the regenerator increases. This results in a loop shaped curve for some of the curves shown in the Figure. For the cases studied, all curves show optimum exergetic efficiency. The value of the optimum increases when the phase of mass flow is lagging the pressure at the cold side of the regenerator. For example, for the hydraulic diameter of 0.05 mm, the optimum efficiency is reduced by about 20 percent when the phase shift between the mass flow and pressure changes from -30 to +30 degrees. The reduction is about 40 percent for hydraulic diameter of 0.1 mm.

Figure 6 shows how changing the cross sectional area and the length of the regenerator, affects its performance while its volume is kept fixed at 5 cc. The results are given for a fixed mass flow rate, pressure ratio, and phase shift between the mass flow and pressure at the cold side of the regenerator. In this manner, the pressure component of exergy at the cold side of the regenerator is kept constant. The fluid mass flux will change as the area of the regenerator is

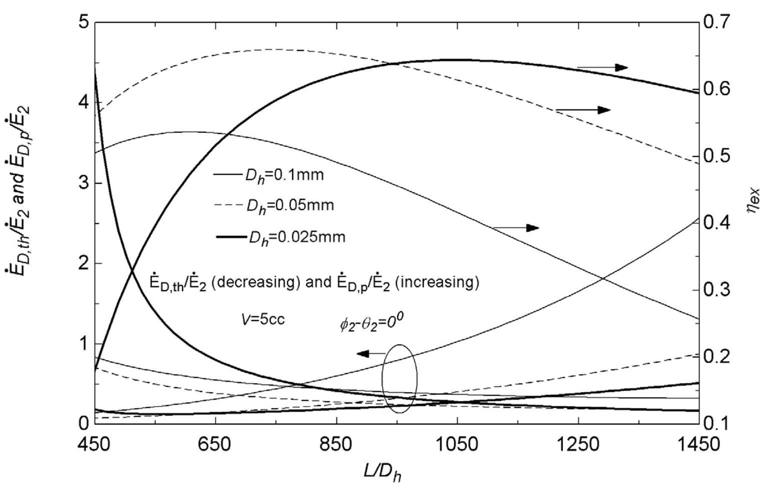

Figure 6: Exergetic efficiency and the ratio of thermal and fluid friction exergy destruction to the exergy at the cold side as a function of length to hydraulic diameter ratio with volume constraint.

changed. As in Figures 2 to 4, the left axis corresponds to the ratio of thermal and pressure components of exergy destruction to exergy delivered at the cold side of the regenerator while the right axis denotes the exergetic efficiency of the regenerator. Three values of hydraulic diameter of 0.1 mm, 0.05 mm, and 0.025 mm are selected for these calculations. The values of optimum exergetic efficiency given in Figure 6, indicate that the losses in regenerators are a substantial fraction of the losses of STCRs.

CONCLUSIONS

Criteria for performance evaluation for the regenerator of STCRs, based on the exergy flow in the regenerator, are proposed. A first order model is developed for the regenerator to assess the performance criteria for design analysis and optimization of the regenerator. The components of exergy destruction in the regenerator due to fluid friction and heat transfer are determined. The effect of important regenerator design parameters on the components of exergy destruction and exergetic efficiency of the regenerator are evaluated. The effect of phase shift between the mass flow and pressure at the cold side of the regenerator on its performance is presented and discussed. It is shown, that even under optimum condition, the exergetic efficiency of the regenerator is not very high and losses in the regenerator are major contributors in reducing the overall efficiency of STCRs. Depending on the constraints on the regenerators, a compromise between the exergy delivered at the cold side of the regenerator and its efficiency is present. This, in turn, can influence the power and efficiency of STCRs. The first order models and the performance criteria of regenerators are convenient in reducing the parametric design space of regenerators and their optimization. Subsequently, more extensive computational and experimental effort is required for the final design of the regenerator and its interaction with other components in the STCR system.

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