Modeling and Optimization of a Two-Stage Mixed-Gas Joule-Thomson Cryoprobe System

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ABSTRACT

Cryosurgery is a technique for destroying undesirable tissue, such as cancers, by exposing it to a very low temperature. This paper presents a thermodynamic modeling tool that has been developed for a two stage, mixed gas Joule-Thomson (MGJT) cryoprobe used for cryosurgery. A conventional vapor compression (VC) cycle using a pure refrigerant precools the MGJT cycle providing refrigeration at the cryoprobe tip. The model is integrated with an optimization routine, in order to investigate the optimal mixture composition for different cryoprobe tip temperatures and identify the optimal precooling temperature. The cryoprobe system performance is reported in terms of cryoprobe conductance (related to size) and compressor displacement. The results for the two stage system are compared to the performance of a single stage MGJT cycle in order to demonstrate the benefits and limitations associated with the addition of the precooling cycle.

INTRODUCTION

Brief Overview of Cryosurgery

Cryosurgery is a technique for destroying undesirable tissue such as cancers by using a freezing process. Cryosurgery is used to ablate prostate and liver cancer tumors as well as a variety of dermatological and gynecological procedures. Cryosurgery relies on a cryosurgical probe that is inserted into the body to create the necessary cryogenic temperatures; the cryoprobe tip reaches approximately 150 K for most procedures.

Brodyansky et al.[1] showed that mixed gas Joule-Thomson (MGJT) system can provide substantially more cooling per unit mass than a single component J-T system, which leads to a relatively compact and convenient device that is more appropriate for a clinical environment. The pressure required by a Mixed Gas Joule-Thomson (MGJT) system (typically 1.5 MPa or 200 psi) is much lower than is required for a single component J-T system and, therefore, it is possible to recover the low pressure fluid leaving the probe and recompress it in a small, portable compressor in an operating room.

The limitations on the use of cryosurgery are primarily related to the cryoprobe refrigeration technology. For treatments that cover large regions deep within the body, current cryoprobe technology requires that multiple probes be inserted and precisely positioned to ensure complete cell death. Clearly, a single probe capable of providing more cooling power within the same geometric

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envelope is more desirable and less invasive. A recent advancement in cryosurgical probe technology addresses this need by improving the underlying thermodynamic cycle. Multistage J-T cycles are used to divide the large temperature range that must be spanned (from room temperature to approximately 150 K) into two smaller temperature spans that can each be supplied using a compact system. The result is a probe that can provide the necessary refrigeration in a more compact configuration. A two stage MGJT system model and the optimization study based on this model is presented here.

Two Stage Mixed Gas J-T Cycle

This paper describes a computational model of a cascaded (two-stage) J-T cycle for use in a cryosurgical probe. Figure 1 provides a schematic of the primary components in the entire system, including numbered thermodynamic states. A conventional vapor-compression cycle labeled "1st stage" provides precooling for the 2nd stage mixed gas J-T cycle. The working fluid for the 1st stage analysis is R22 whereas the working fluid for the 2nd stage is a mixture of nitrogen, ethane, methane, propane, isobutane, isopentane, and argon. The cryoprobe tip, which provides cooling to the tissue at the surgical sight, is represented by a heat exchanger. The refrigeration capacity of the cryoprobe is $\dot{Q}_{load}$ and the nominal tip temperature is T_7.

The purpose of the thermodynamic model presented here is to investigate cycle design issues. For example, the model will allow the determination of the optimal mixture composition for the 2nd stage J-T cycle as well as the optimum precooling temperature. This work is partially based on previous work at the University of Wisconsin at Madison that evaluated optimum gas mixtures for a single stage J-T cryosurgical system.[1] This initial work has been used to optimize the design of a single-stage system for cryosurgery.[2] This paper utilizes the same modeling methodology, but expands the approach to the two stage cycle shown in Figure 1. To our knowledge, the theoretical optimization of a cascade MGJT system for cryosurgery has not previously been reported. Therefore, the model is used to identify the merits as well as the potential drawbacks associated with using a cascaded system as compared to a single stage, mixed gas J-T cycle.

The refrigeration capacity of a J-T cycle is fundamentally limited by the Joule-Thomson effect associated with the working fluid. The Joule-Thomson effect is related to the isothermal enthalpy difference between the high and low pressure streams in the recuperator. Keppler et. al[3] and many others have demonstrated that the isothermal enthalpy difference exhibited by any pure fluid is large only over a small temperature span near the vapor dome. The vapor dome associated with a mixture of gases tends to extend over a larger temperature range corresponding to a temperature that is near the lowest boiling point of the components to one that is near the highest boiling point component. Therefore, the use of gas mixtures significantly extends the temperature range over

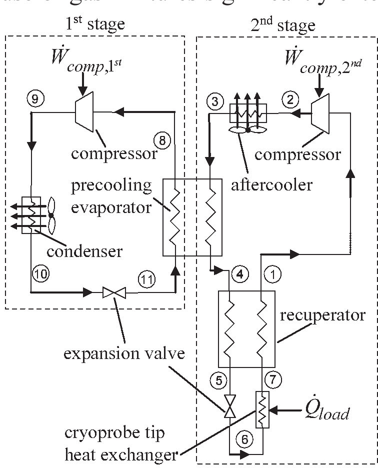


Figure 1. Schematic of two stage refrigeration cycle showing the thermodynamic states associated with each stage.

which the isothermal enthalpy difference is large and therefore enhances the performance of the J-T cycle.

There is a tradeoff between the maximum cooling power that can be provided and the temperature range that must be spanned by the recuperator. For example, consider two 7-component mixtures that could be used in the 2nd stage of the cascaded system in Figure 1a where the load temperature is 140 K, and the high and low pressures are 1000 kPa and 100 kPa. The composition of mixtures A and B are summarized in Table 1 (by mole fraction); these mixtures have been optimized to produce the maximum J-T effect over two different temperature spans, but both mixtures have the same constituents. Mixture A is carefully optimized for a temperature span of 285 K to 140 K, which would be typical of a single stage J-T cycle (i.e., Figure 1 with the 1st stage removed). Mixture B is optimized for a temperature span of 238 K to 140 K, which is typical of a J-T cycle with some precooling that lowers the recuperator hot inlet temperature to 238 K. The maximum cooling effect that can be produced per unit of mass flow rate (i.e., the minimum value of the isothermal enthalpy change) over the temperature span for mixture A is 73 W/(g/s), whereas the maximum cooling effect for mixture B over its temperature span is 60% larger, 115 W/(g/s). In this example, by reducing the temperature range that must be spanned by the mixed gas J-T system, it is possible to achieve a 60% increase in the amount of refrigeration provided by the J-T cycle.

A cryoprobe must be compact; that is, a surgically useful cryoprobe will provide a large amount of cooling while still being physically small and subsequently surgically ergonomic, minimally invasive, and easy to control. Cryosurgical procedures requiring a single probe can be carried out more quickly and planned with greater precision than those requiring several probes. In a single stage system, the recuperative heat exchanger is rigidly coupled to the shaft of the cryoprobe and therefore affects the overall cryoprobe size. In the two stage system, both the recuperative and precooling heat exchangers are coupled to the cryoprobe. Therefore, the benefit of precooling must be evaluated based on whether the increase in cooling power that can be obtained is worth more than the increase in overall cryoprobe size associated with the addition of the precooling heat exchanger. A Figure of Merit that describes the compactness of the cryoprobe relative to its performance is the ratio of the refrigeration load to the total heat exchanger conductance ($\dot{Q}_{load}/UA_{total}$), because the conductance is closely related to the size of the heat exchangers. The total conductance of the two stage system must include the recuperator and precooler, the conductance of the single stage system only includes the recuperator.

The Optimization Results section demonstrates that the cascaded system does offer a more compact cryoprobe for a given refrigeration power over a reasonable range of precooling temperatures, provided the refrigerants used in both stages are correctly optimized. Other secondary parameters that must be considered when comparing the single- and two-stage systems include the overall compressor size and power consumption, contamination control, and the complexity and reliability of the system. The compressor requirements (power and displacement) can be precisely evaluated using the model discussed in this paper; the Optimization Results section shows that the two stage system can be implemented without a significant change in total compressor displacement although it is likely that the two compressors will take up substantially more space than a single compressor with the same displacement.

Table 1. Mixture operating temperatures and compositions

Mixture	A	B
Low Temp	140 K	140 K
High Temp	285 K	238 K
Nitrogen	0.11 %	0.0%
Methane	43.3 %	50.1 %
Ethane	40.3 %	39.3 %
Propane	0.06 %	1.17 %
Isobutane	6.67 %	9.38 %
Isopentane	9.49 %	0.01 %
Argon	0.07 %	0.0 %

An optimized mixture for a cryosurgical system will provide a large J-T cooling effect but require a relatively small system of heat exchangers in the cryoprobe. It is not possible to analytically or intuitively select: (1) a 2nd stage mixture, (2) precooling temperature, and (3) recuperator and precooling evaporator pinch point temperatures for the two stage system that yield an optimum ratio of cooling power and heat exchanger size, as well as a system with a practical compressor sizes. The relationship between mixture composition and J-T effect is affected by the complex mixture equation of state. The heat exchanger size that is required depends strongly on the specific heat capacity of the mixture, which varies substantially as a function of temperature, pressure, and mixture composition. Methods have been developed[4,5] for selecting a mixture to yield the largest cooling effect. However these optimization methods don't consider the heat exchanger size. Brodyansky et. al[1] demonstrated that a cryoprobe which uses a mixture that is optimized for maximum cooling (or maximum efficiency) may be more than twice the size of a cryoprobe using a mixture that is optimized for cooling per heat exchanger size. Therefore, a numerical optimization technique described here is used to design a cryosurgical probe that is designed based on minimization of heat exchanger size.

OPTIMIZATION MODEL

This section presents the details of the thermodynamic model that provides the basis for the optimization of the 2nd stage mixture and precooling temperature for the two stage cryosurgical system. The correlations used to evaluate the mixture properties are discussed. A freezing point model is described; the freezing point model is incorporated into the optimization routine in order to provide a constraint on the optimization so that a mixture that may freeze and clog the system is not considered. The numerical parameters that are required by the model and the optimization algorithm are investigated and appropriate parameters are selected. Finally, the optimization algorithm that is used to select an optimal mixture for a given load temperature is presented.

Thermodynamic Model

The two stage refrigeration cycle shown in Figure 1 is evaluated using a numerical modeling tool discussed in this section. The Engineering Equation Solver (EES) software[6] is used to solve the governing system of equations that describe the performance of the system for a particular set of operating conditions and geometry. The modeling tool is used with an optimization algorithm in order to maximize the system performance in terms of the previously discussed figure of merit, the cryoprobe refrigeration load per total heat exchanger conductance ($\dot{Q}_{load}/UA_{total}$).

A variety of pure fluids or mixtures can be used in the 1st stage; the working fluid analyzed here is R22. Property data for R22 are provided in EES. The 2nd stage hydrocarbon mixture property data are obtained from the NIST4 (also called SUPERTRAPP) database.[7] A mixture of synthetic refrigerants was also considered, but is not discussed in this paper. The numerical EES model is interfaced with the FORTRAN routines provided in the SUPERTRAPP program from the National Institute of Standards and Technology (NIST) with a separate interface routine.[6] A comparison of the NIST4 and REFPROP[8] mixture property data over the range of temperatures/pressures considered in this paper is described in the paper by Brodyansky, et al.[1] The model is designed with the flexibility to use a mixture or a pure fluid in the 1st stage, however for this paper, only pure refrigerants are used in the 1st stage.

The inputs to the model are chosen based on the operating conditions that are achievable using conventional equipment. The 2nd stage aftercooler and 1st stage condenser are not explicitly modeled, rather they are assumed to be sufficiently large that the fluid exiting the compressors is cooled to ambient temperature (T_{amb}). Additionally, the 1st stage refrigerant leaving the precooling evaporator (at state 8) is assumed to be saturated vapor. The pressure drop in the heat exchangers is neglected; therefore the working fluids change pressure only across the compressors and expansion valves. Operating pressures representing the high and low pressures of each cycle are defined: ($P_{high,1st}$, $P_{low,1st}$, $P_{high,2nd}$, $P_{low,2nd}$). Other inputs to the model include: 2nd stage fluid compositions ($\overline{y}_{2nd}$, a vector of molar concentrations of each component that will be controlled and adjusted, eventually, by the optimization algorithm), the ambient temperature, the load temperature (T_{load}), and the precooling and recuperative heat exchangers pinch-point temperature differences ($\Delta T_{pp,pc}$ and $\Delta T_{pp,rec}$).

1st Stage Analysis

An iterative process is required to solve the governing equations to determine the performance of the cycle. The iteration procedure begins by considering the enthalpy of the fluid entering the 1st stage expansion valve (h_{10}), which is computed according to:

$$h_{10} = enthalpy\left(T_{amb}, P_{high,1^{st}}, \bar{y}_{1^{st}}\right) \tag{1}$$

where *enthalpy* represents the correlation in NIST4 or EES that evaluates the specific enthalpy at the given state. The enthalpy (h_{11}) and temperature (T_{11}) for the 1st stage fluid entering the precooling heat exchanger can be computed assuming isenthalpic expansion across the valve:

$$h_{11} = h_{10} \tag{2}$$

$$T_{11} = temperature\left(h_{11}, P_{low,1^{st}}, \bar{y}_{1^{st}}\right) \tag{3}$$

where *temperature* represent the correlations in NIST4 or EES that evaluates the temperature at the given state. An assumed cold-end temperature difference for the precooling evaporator ($\Delta T_{cold,pc}$) is systematically varied until the specified precooling evaporator pinch-point temperature ($\Delta T_{pp,rec}$) difference is achieved. The pinch point temperature difference is defined below. The temperature (T_4) and enthalpy (h_4) of the 2nd stage fluid leaving the precooling evaporator are calculated:

$$T_4 = T_{11} + \Delta T_{cold,pc} \tag{4}$$

$$h_4 = enthalpy\left(T_4, P_{high,2^{nd}}, \bar{y}_{2^{nd}}\right) \tag{5}$$

The 1st stage working fluid (which is assumed here to be a pure refrigerant) is assumed to exit the precooling evaporator as a saturated vapor, so the enthalpy (h_8) is computed as:

$$h_8 = enthalpy\left(x_8 = 1, P_{low,1^{st}}, \bar{y}_{1^{st}}\right) \tag{6}$$

The enthalpy (h_3) of the 2nd stage fluid entering the precooling evaporator is calculated using:

$$h_3 = enthalpy\left(T_{amb}, P_{high,2^{nd}}, \bar{y}_{2^{nd}}\right) \tag{7}$$

The ratio of the mass flow rate in the 1st to the mass flow rate in the 2nd stage (MR) is defined as:

$$MR = \dot{m}_{1^{st}} / \dot{m}_{2^{nd}} \tag{8}$$

and is computed using an energy balance on the precooling evaporator:

$$MR = \left(h_3 - h_4\right) / \left(h_8 - h_{11}\right) \tag{9}$$

The rate of precooling heat transfer as well as all subsequent energy transfer rates are computed on a per unit of 2nd stage mass flow rate basis.

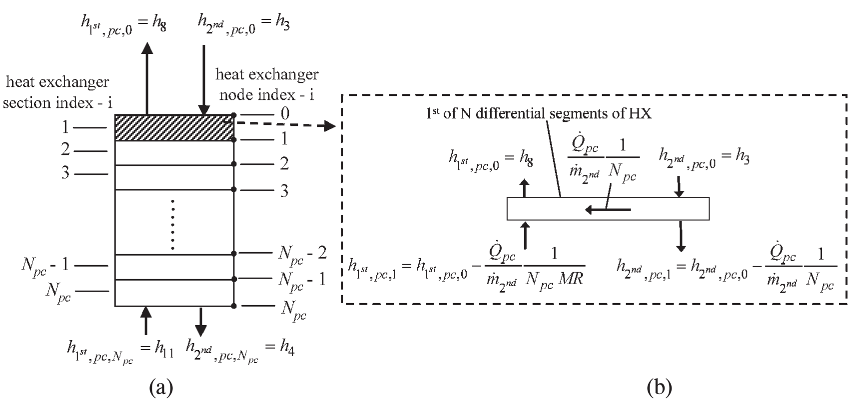


Figure 2. (a) Precooling heat exchanger divided into N_{pc} sections and (N_{pc} + 1) nodes. (b) First differential heat exchanger element.

$$\dot{Q}_{pc}/\dot{m}_{2^{nd}} = MR\left(h_8 - h_{11}\right) \tag{10}$$

The precooling heat exchanger is divided into a number of small heat exchangers(N_{pc}), as shown in Figure 2a, where each section transfers an equal fraction ($1/N_{pc}$) of the total precooling load. Dividing the heat exchanger into equal heat transfer segments rather than equal physical sizes facilitates direct computation of the enthalpy distribution in the heat exchangers and significantly improves computation speed and convergence. The first heat exchanger section is located at the hot end of the precooling evaporator and is shown in Figure 2b. The enthalpy of the 1st stage working fluid leaving the precooling evaporator is equal to the enthalpy of the 1st stage fluid at the first node of the heat exchanger.

$$h_{1^{st},pc,0} = h_8 \tag{11}$$

The enthalpy of the mixture entering the precooling evaporator is equal to the enthalpy for the mixture at the first node of the heat exchanger.

$$h_{2nd,pc,0} = h_3 \tag{12}$$

The enthalpies of the hot and cold exit streams at the interface of each segment are computed using an energy balance.

$$h_{1^{st},pc,i} = h_{1^{st},pc,i-1} - \dot{Q}_{pc}/\left(\dot{m}_{2^{nd}} N_{pc}\, MR\right) \qquad i = 1 \ldots N_{pc} \tag{13}$$

$$h_{2^{nd},pc,i} = h_{2^{nd},pc,i-1} - \dot{Q}_{pc}/\left(\dot{m}_{2^{nd}} N_{pc}\right) \qquad i = 1 \ldots N_{pc} \tag{14}$$

The temperatures at the inlet and exit of each side of each section (i.e., heat exchanger node index in Figure 2a are computed based on the enthalpy and pressure:

$$T_{1^{st},pc,i} = temperature\left(h_{1^{st},pc,i}, P_{low,1^{st}}, \overline{y}_{1^{st}}\right) \tag{15}$$

$$T_{2^{nd},pc,i} = temperature\left(h_{2^{nd},pc,i}, P_{high,2^{nd}}, \overline{y}_{2^{nd}}\right) \qquad i = 0 \ldots N_{pc} \tag{16}$$

The pinch-point temperature difference is defined as the minimum temperature difference between the 1st and 2nd stage streams anywhere within the precooling heat exchanger.

$$\Delta T_{pp,pc} = \min\left(T_{2^{nd},pc,i} - T_{1^{st},pc,i}\right) \tag{17}$$

The size of the heat exchangers is a function of the pinch-point temperature difference; a smaller pinch point temperature corresponds to a larger value of overall conductance (UA, the overall heat transfer coefficient-area product). The cryoprobe tip load also depends on the pinch point temperatures; as the pinch point temperature differences in either the recuperator or precooling evaporator decreases, the cryoprobe tip load increases. Therefore a compact cryoprobe system (where $\dot{Q}_{load}/UA_{total}$ is maximum) results by optimizing the pinch point temperature difference in order to balance the heat exchanger size against the cryoprobe load. The model uses 2 K as the pinch point temperatures for both the precooling evaporator and the recuperator based on previous observation that the optimal pinch point temperature for cryoprobes is about 2-6 K.

The conductance of the precooler (UA_{pc}) can be calculated using an effectiveness-*NTU* relationship for a counterflow heat exchanger[9] if the specific heat capacities of the fluids are constant throughout the heat exchanger. However, the specific heat of the mixture is very sensitive to the temperature and therefore it varies significantly within the heat exchanger. If a sufficient number of heat exchanger sections are used (i.e., if N_{pc} is large) then the specific heat capacity within each section is very nearly constant and so the effectiveness-*NTU* solution can be used to compute the conductance of each section. A numerical study ensured that N_{pc} was sufficiently large that the results are insensitive to this parameter. The total heat exchanger conductance is calculated by summing the conductance of each section. The fluid specific heat within a section is represented by an average specific heat defined for each stage as:

$$\overline{c}_{1^{st},pc,i} = \left(h_{1^{st},pc,i-1} - h_{1^{st},pc,i}\right)/\left(T_{1^{st},pc,i-1} - T_{1^{st},pc,i}\right) \qquad i = 1 \ldots N_{pc} \tag{18}$$

$$\overline{c}_{2^{nd},pc,i} = \left(h_{2^{nd},pc,i-1} - h_{2^{nd},pc,i}\right)/\left(T_{2^{nd},pc,i-1} - T_{2^{nd},pc,i}\right) \qquad i = 1 \ldots N_{pc} \tag{19}$$

The effectiveness of each segment () is defined as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate that could occur in that section. The maximum heat transfer rate in each section occurs when the outlet temperature of the minimum capacity rate stream reaches the inlet temperature of the maximum capacity rate stream.

$$\varepsilon_{pc,i} = \left[\frac{\dot{Q}_{pc}}{\dot{m}_{2^{nd}}}\frac{1}{N_{pc}}\right] \Big/ \left[\min\left(\bar{c}_{2^{nd},pc,i}, \bar{c}_{1^{st},pc,i}\,MR\right)\left(T_{1^{st},pc,i-1} - T_{2^{nd},pc,i}\right)\right] \quad i = 1 \ldots N_{pc} \tag{20}$$

Note that the capacity of the 1st stage fluid stream must be scaled by MR in order to compare the capacity rates of the two streams. The conductance of each section is calculated:

$$\frac{UA_{pc,i}}{\dot{m}_{2^{nd}}} = \min\left(\bar{c}_{2^{nd},pc,i}, \bar{c}_{1^{st},pc,i}\,MR\right) \ln\left(\frac{\varepsilon_{pc,i} - 1}{\varepsilon_{pc,i}\,C_{r,pc,i} - 1}\right) \Big/ \left(C_{r,pc,i} - 1\right) \quad i = 1 \ldots N_{pc} \tag{21}$$

where $C_{r,pc,i}$ is the capacity ratio characterizing the section:

$$C_{r,pc,i} = \min\left(\bar{c}_{2^{nd},pc,i}, \bar{c}_{1^{st},pc,i}\,MR\right) \Big/ \max\left(\bar{c}_{2^{nd},pc,i}, \bar{c}_{1^{st},pc,i}\,MR\right) \quad i = 1 \ldots N_{pc} \tag{22}$$

The overall conductance of the precooler per unit of 2nd stage mass flow rate is computed by summing the conductances of each of the segments.

$$\frac{UA_{pc}}{\dot{m}_{2^{nd}}} = \sum_{i=1}^{N} \frac{UA_{pc,i}}{\dot{m}_{2^{nd}}} \qquad i = 1 \ldots N_{pc} \tag{23}$$

2nd Stage Analysis

The thermodynamic states of the 2nd stage, the cryoprobe load, and the temperature distribution in the recuperator are solved using a process similar to that described in the 1st stage analysis. The solution process is nearly the same as that of the single stage MGJT cycle presented in Reference 1. The only difference is that here, the high pressure gas mixture is precooled before entering the recuperator, whereas in the single stage system the high pressure gas mixture enters the recuperator near ambient temperature.

The recuperator hot end temperature difference $(T_4\text{-}T_1)$ is iteratively adjusted to achieve a specified recuperator pinch point temperature difference ($\Delta T_{pp,rec}$). The load temperature (T_7) is specified and isenthalpic expansion is assumed across the expansion valve. A numerical model of the recuperator was created by dividing it into sections of equal heat transfer to calculate an enthalpy distribution. The enthalpies at states 1, 4, 5, and 7 are used as boundary conditions for the numerical recuperator model. The enthalpy distribution and recuperator pressures ($P_{h,2nd}$ and $P_{l,2nd}$) are used to calculate a temperature distribution that facilitates the calculation of mixture heat capacities and recuperator conductance (UA_{rec}).

Overall Thermodynamic Analysis – Figures of Merit

The overall system performance can be quantified using several Figures of Merit of importance to a cryosurgical probe system. From a surgical procedure standpoint, an optimal cryoprobe is small and generates a large amount of cooling power; such a probe will produce the largest possible cryolesion (i.e., frozen tissue). Larger cryolesions reduce the number of surgical sites and/or cryoprobes that are required to treat a given volume of tissue. A small cryoprobe is ergonomic, can accommodate other instrumentation given surgical site space constraints, is less invasive, can be precisely controlled, and requires less planning. These factors contribute to reduce the overall procedure time, complication rates, and expense. Therefore an appropriate Figure of Merit[1] that is used to optimize the system is the total cryoprobe cooling load provided per total heat exchanger conductance, which is indicative of the cryoprobe size.

$$\dot{Q}_{load} / UA_{total} = \left(\dot{Q}_{load} / \dot{m}_{2^{nd}}\right) \Big/ \left(UA_{rec} / \dot{m}_{2^{nd}} + UA_{pc} / \dot{m}_{2^{nd}}\right) \tag{24}$$

It is also of interest to reduce the size of the other hardware required; particularly the compressors. The compressors can be connected to the cryoprobe heat exchangers via flexible tubing and physically decoupled from the cryoprobe. Therefore, the size of the compressors is less important than the size of the cryoprobe. However, the size of the compressors largely dictates the size and weight of the cabinet that houses the compressors, as well as the 2nd stage aftercooler, and 1st stage condenser. Smaller compressors will therefore lead to a small, portable cryosurgical unit that is easy to handle and can be deployed in a variety of settings. The compressor suction side flow rate determines the required displaced volume and therefore, to first order, the size of the compressor. The figure of merit that captures the combined compressor size is the refrigeration load per unit of total compressor displacement:

$$\dot{Q}_{load} / \dot{v}_{total} = \left(\dot{Q}_{load} / \dot{m}_{2^{nd}}\right) \Big/ \left(\dot{v}_1 / \dot{m}_{2^{nd}} + \dot{v}_8 / \dot{m}_{2^{nd}}\right) \tag{25}$$

Optimization Algorithm

The optimization process for the results presented in this paper identify the optimal mixture composition in the 2nd stage (y_{2nd}, the vector of compositions of each mixture component) for a particular set of operating conditions. The total cryoprobe compactness figure of merit ($\dot{Q}_{load}/UA_{total}$) calculated by the thermodynamic model varies significantly as the mixture mole fraction vector (y_{2nd}) changes, so it is necessary to evaluate a wide range of mole fraction combinations. It is not computationally efficient to parametrically evaluate the mole fractions for all possible y_{2nd} combinations. Therefore, an optimization algorithm that is able to select the optimal mixture using significantly less computations than a parametric study is utilized here.

The optimization routine selected is the PIKAIA 1.2[10] genetic algorithm that finds the maximum of the objective function using an algorithm that mimics biological evolution. A detailed description and demonstration of the routine as it is used to select a best mixture is found in Brodyansky[1]. As shown by Brodyansky[1], other optimization techniques such as the direct search and variable metric strategies do not reliably converge because of the sharp discontinuities in mixture properties near phase boundaries as well as other constraints that are placed on the mixture. Note that the reliability of the genetic optimization routine comes at the expense of computation speed; the genetic algorithm should only be used when other, faster, routines have failed.

The optimization routine excludes mixtures based on two practical considerations: 1) A mixture is excluded from further consideration if the temperature at the exit of the 2nd stage expansion valve is below the freezing point temperature, and 2) Mixtures in a saturated or liquid state leaving the recuperator (state 1) are excluded to avoid the introduction of liquid into the 2nd stage compressor. The freezing point is calculated as the weighted average of the triple point of the mixture constituents, as described in Brodyansky, et. al.[1]

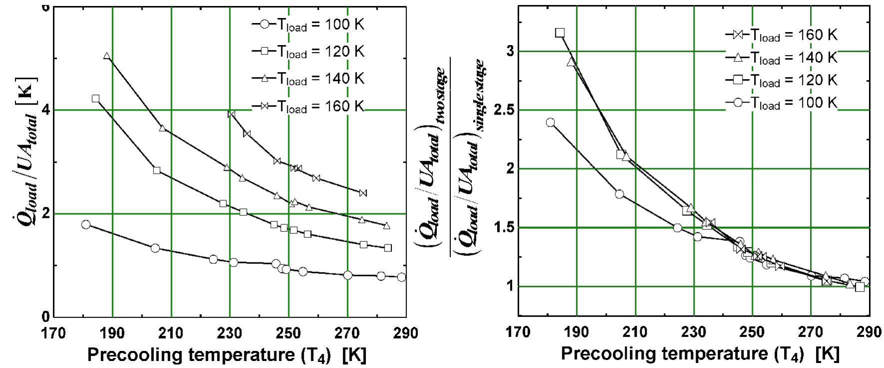
OPTIMIZATION RESULTS

The results of the optimization model using a pure refrigerant in the 1st stage and a hydrocarbon based mixture in the 2nd stage are presented in this section. The refrigerant used for analysis in the 1st stage is R22. The figure of merit is optimized in order to yield the cryoprobe that provides the most cooling for a given geometric size for load temperatures spanning 100 K to 180 K. The effect of the precooling temperature (T_4) on $\dot{Q}_{load}/UA_{total}$ is studied in order to show the optimal balance between the precooler and the recuperator. The other figure of merit, the load specific compressor volumetric flow rate is not explicitly optimized but is reported as it is important in the design of a practical system. The performance of the two stage system is normalized against the performance of a single stage system in order to show the relative benefit and penalty associated with the addition of precooling.

The $\dot{Q}_{load}/UA_{total}$ for a two stage system in which an optimal mixture has been selected for each precooling and load temperature is shown in Figure 3a. As the precooling temperature is reduced, the temperature range that must be spanned by the recuperator decreases and the performance of the 2nd stage cycle increases. The overall cryoprobe heat exchanger size for an optimized system remains relatively constant over the range of precooling temperatures studied here. Therefore, as the precooling temperature is reduced, $\dot{Q}_{load}/UA_{total}$ increases due to the improvement in the efficiency of the 2nd stage cycle.

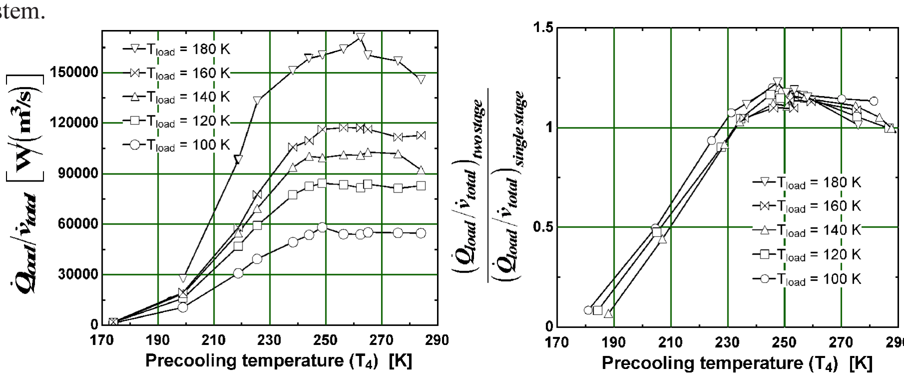
Figure 3b shows the results in Figure 3a normalized by the cryoprobe load per heat exchanger size for an optimized single stage system, $(\dot{Q}_{load}/UA_{total})_{\text{single stage}}$ as reported in Brodyansky[1]. Figure 3b shows clearly that the two stage system offers a more compact cryoprobe compared to the single stage system and this advantage increases as the precooling temperature is reduced.

Figure 3 suggests that the precooling temperature should be made as low as possible in order to achieve an optimized cryoprobe. However, other considerations related to the compressor power and size limit the range of practical precooling temperatures. Figure 4a shows the ratio of the cryoprobe refrigeration to the total volumetric flow rate at the suction to the compressors, $\dot{Q}_{load}/\dot{v}_{total}$ (which provides an indication of the size of the compressors required). The required compressor sizes increase as the precooling temperature is reduced. The suction pressure ($P_{low,1st}$) for the R22 must be



(a) (b)

Figure 3. a) $\dot{Q}_{load}/UA_{total}$ for the two stage system over a range of precooling and load temperatures. b) $\dot{Q}_{load}/UA_{total}$ for the two stage system normalized by the $\dot{Q}_{load}/UA_{total}$ of a single stage system.



(a) (b)

Figure 4. (a) $\dot{Q}_{load}/\dot{v}_{total}$ for the two stage system over a range of precooling and load temperatures. (b) $\dot{Q}_{load}/\dot{v}_{total}$ for the two stage system normalized by the $\dot{Q}_{load}/\dot{v}_{total}$ of a single stage system.

reduced in order to achieve the desired precooling temperature; the specific volume of the R22 at the compressor suction side increases and the compressor power and size therefore increases. The selection of precooling temperature must balance the reduction in cryoprobe size that can be achieved against the increased compressor size that is required. Figure 4b shows the $\dot{Q}_{load}/\dot{v}_{total}$ for the two stage system normalized by these quantities for a single stage system. Note that the increased volumetric flow rate requires additional power and the overall system COP decreases; the trend very nearly follows $\dot{Q}_{load}/\dot{v}_{total}$ trend.

CONCLUSION

A thermodynamic model was developed to evaluate the theoretical performance of the two-stage mixed gas J-T cycle used for cryosurgery. The model was interfaced with the genetic optimization algorithm in order to select the 2nd stage mixture composition that maximizes the cryoprobe load per heat exchanger conductance. The primary advantage associated with precooling is a reduction in the temperature span of the recuperator which allows a mixture with a higher J-T effect (more cooling capability) to be selected for the second stage J-T cycle. The increased cooling capability results in a more compact cryoprobe, as the overall cryoprobe heat exchanger size remains about the same with the addition of precooling. The two stage results compared with the single stage system performance show that the cryoprobe can be made significantly smaller with the addition of precooling. The model also demonstrates how the compressor size limits the range of practical precooling temperatures. The model is therefore a useful tool for designing a two stage system that balances the cryoprobe and compressor hardware parameters.

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