Thermodynamic Comparison of Two-Stage Pulse Tube Refrigerators for Two Different Configurations

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ABSTRACT

Using exergy analysis, control thermodynamic models of two-stage Pulse Tube Refrigerators (PTRs) are developed for two different configurations. The models assume that phase shift controllers exist that control the phase shift between the mass flow and pressure in the pulse tubes. In one configuration, using thermally-coupled stages, separate gas circuits are used, thus requiring two compressors. This configuration provides flexibility for thermodynamic optimization. In another configuration, a conventional gas-coupled two-stage PTR is used where a constraint exists for mass flow allocations and the implementation of the phase shift control. The models include controllers for flow conductance, heat transfer effectiveness, and conduction heat transfer parameters in the regenerators in both stages for each configuration. The effects of the allocation of the values of flow conductance and ineffectiveness parameters in the regenerators, the mid-stage temperature, and the phase shift in each stage on the performance of the refrigerators are investigated. Important dimensionless parameters controlling the thermodynamic performance of the two-stage PTRs for each configuration is developed and discussed.

INTRODUCTION

Multi-stage Pulse Tube Refrigerators (MSPTRs) are attractive cooling systems for a variety of applications and their staging configurations provide the flexibility to conveniently change the various stage temperatures for different system applications.[1] Recently, design and development of high frequency MSPTRs have been reported.[2-5] In general, the model of control thermodynamics (Finite time thermodynamics) can be applied for analysis and optimization of multi-stage refrigerators. Different thermodynamic models to analyze and optimize multi-stage refrigerators exist in the literature and use different models to calculate the irreversibility for internal processes and external heat transfer with reservoirs,[6-8] to just name a few. Thermodynamic optimization of two-stage PTRs has been carried out assuming that the only irreversibility in the regenerator is due to conduction heat transfer.[9] Recently we used a control thermodynamic model to investigate the performance of a thermally-coupled Two-Stage Pulse Tube Refrigerators (TSPTRs).[10] In this study we develop the control thermodynamic model of a gas-coupled TSPTR and compare the performance of two configurations of TSPTRs. The model is based on the assumption that a general phase-

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shifter exists to control the phase shift between the mass flow rate and pressure at the cold sides of the pulse tubes. In addition, we include a more realistic model of the regenerator in both stages to include the effects of the void, pressure drop, heat transfer ineffectiveness, and conduction heat transfer in the regenerators. At least a first order thermodynamic model for the regenerator is necessary to be able to provide a more realistic thermodynamic model to account for its contribution to the irreversibility of TSPTRs. Important dimensionless parameters controlling the performance of the refrigerators are obtained and discussed. The advantage of the control thermodynamic model used in this study is to find important dimensionless parameters influencing the performance of TSPTRs and the development of analytical expressions to obtain thermodynamic bounds of the refrigerators.

Exergy is a function of both the state of a system and the state of the environment as it measures the departure of the state of the system from the state of the environment. In application to PTRs, for each component, considering one channel of heat transfer between the system and a thermal reservoir at the temperature T_R and one channel of inlet and exit mass transfer, the exergy balance for one cycle can be written as[11,12]

$$\dot{E}_D = \dot{M}\, h_i - (\dot{M}\, h)_e - T_o\left[(\dot{M}\, s)_i - (\dot{M}\, s)_e\right] - \dot{W} + (1 - \frac{T_o}{T_R})\dot{Q}_R \tag{1}$$

where $\dot{E}_D$ is the exergy destruction in the component, $\dot{M}$ is the mass flow rate, subscripts i and e denote the inlet and exit to each component, and subscripts o and R denote environment and reservoirs, respectively. Assuming the ideal gas law is valid and thermophysical properties are constant, the enthalpy and entropy in the above equation can be written, respectively as,

$$h = C_p(T - T_o) \tag{2}$$

$$s = C_p \ln(T/T_o) - R\ln(P/P_o) \tag{3}$$

MATHEMATICAL FORMULATION

Fig. 1 shows the two configurations of TSPTRs and the important parameters used for the thermodynamic modeling in this investigation. Exergy comes into the system from the compressors and is destroyed as it moves into the system. The product exergy is delivered to the cold reservoirs at the mid-stage and the second stage. We assume that the pressure and mass flow rate at any location at the inlet and exit of each component are given by

$$\dot{M}_j = \dot{m}_j\, Cos(\omega t + \phi_j) \tag{4}$$

$$P_j = P_a + p_j Cos(\omega t + \theta_j) \tag{5}$$

where ω is the angular frequency and ϕ_j and θ_j are the phase angle for mass flow and pressure at the inlet or exit of any component in the system, respectively. Using Eqs. (2) to (5), the pressure component of exergy at any location can be written as[10]

$$(\dot{E}_j)_p = (1/2)RT_o\, \dot{m}_j \frac{P_j}{P_a} Cos(\phi_j - \theta_j) \tag{6}$$

The thermal exergy transfer at the cold side of the nth regenerator, due to conduction heat transfer and ineffectiveness of the regenerator, can be written as

$$(\dot{E}_n)_{th} = (\dot{Q}_{regn} + \dot{Q}_{condn})(1 - \frac{T_o}{T_{cn}}) \tag{7}$$

where $\dot{Q}_{regn}$ can be estimated using the ineffectiveness of the nth regenerator defined by[11]

$$\lambda_n = \frac{\dot{Q}_{regn}}{\Delta H_n} = \frac{\dot{Q}_{regn}}{(1/\pi)C_p(T_{hn}\,\dot{m}_{hn} - T_{cn}\,\dot{m}_{cn})} \tag{8}$$

where the subscripts hn and cn denote the hot side and cold side of the nth regenerator and λ_n is its ineffectiveness. The denominator of Eq. (8) represents the maximum rate of enthalpy transfer into the regenerator. Conduction heat transfer to the cold heat exchanger can be estimated assuming a linear temperature profile in the regenerator,

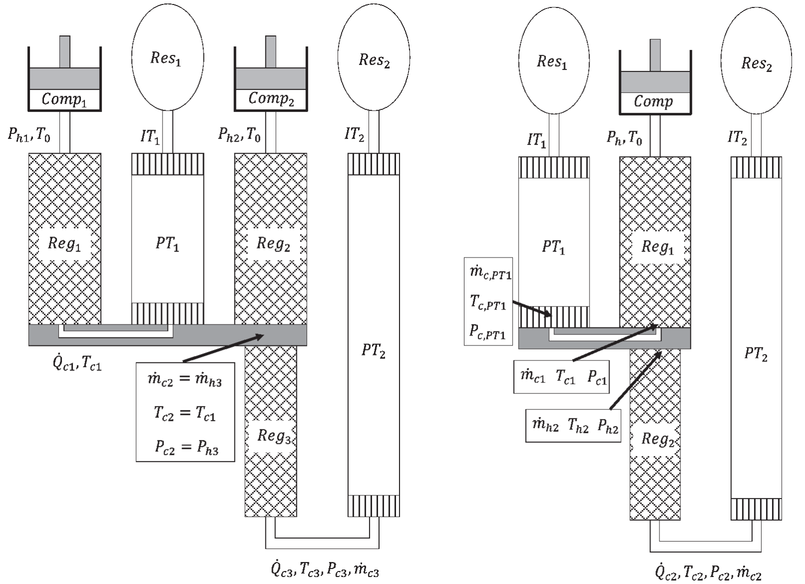

Figure 1. Schematic of the two configurations of TSPTRs and the parameters used in the models.

$$\dot{Q}_{condn} = (KA/L)_n (T_{hn} - T_{cn}) \tag{9}$$

where $(KA/L)_n$ is the effective thermal conductance of the n^{th} regenerator and its shell.

Oscillating flow of heat and mass transfer in regenerators is very complicated. For thermodynamic analysis we consider a simple model for the regenerator to find the bounds for cooling capacity and efficiency of TSPTRs. It is assumed that the mass flow rate at the cold side of the regenerator can be obtained using a linear relation based on a given, appropriately average flow conductance.

$$\dot{m}_{n+1} Cos(\omega t + \phi_{n+1}) = C_n \left[p_n Cos(\omega t + \theta_n) - p_{n+1} Cos(\omega t + \theta_{n+1}) \right] \tag{10}$$

where C_n is the average flow conductance in the n^{th} regenerator. In addition, a simple model for the conservation of mass in the regenerator can be written as,

$$\dot{m}_n Cos(\omega t + \phi_n) = \dot{m}_{n+1} Cos(\omega t + \phi_{n+1}) - V_n\, \omega \left[p_n Sin(\omega t + \theta_n) + p_{n+1} Sin(\omega t + \theta_{n+1}) \right] \tag{11}$$

where V_n is the properly averaged effective regenerator void volume including the influence of temperature distribution in the n^{th} regenerator. Expanding Eqs. (10) and (11) and applying trigonometric identities gives a set of equations that uniquely determine the relations between the phase shifts and amplitudes of pressure and mass flow rates at the cold and hot sides of the regenerator. For the thermally-coupled TSPTRs expansion of Eqs. (10) and (11) for the three regenerators gives 12 equations relating the amplitude of mass flow, amplitude of the pressure, and the phase angles at the inlet and exit of each regenerator. For given phase shift at the cold sides of pulse tubes assumed to be controlled by the phase shifters and given amplitude of pressure at the driver sides, the twelve equations can be used to find the amplitudes of pressure, flow rates and the phase shifts at all other locations. Assuming exergy destruction in the heat exchangers at the mid-stage and cold stage is zero, the balance of exergy give the cooling capacity for the mid-stage and the cold stage of the thermally-coupled TSPTRs.[10]

For gas-coupled TSPTRs there are two regenerators and eight equations can be obtained from Eqs. (10) and (11). Assuming no pressure and temperature change at the gas coupling, the pressure amplitude and temperature are assumed to be the same at mid-stage. However, Eq. (4) and the conservation of mass must be used to find the relations between the mass flow amplitudes and the phase shifts at the mid-stage. These relations are given below.

$$\frac{\dot{m}_{c1}}{\dot{m}_{c,PT1}} = \sqrt{1 + R_m^2 + 2R_m(X_{c,PT1}X_{h2} + Y_{c,PT1}Y_{h2})} \tag{12}$$

$$Y_{c1} = \frac{Y_{c,PT1} + R_m Y_{h2}}{[(X_{c,PT1} + R_m X_{h2})^2 + (Y_{c,PT1} + R_m Y_{h2})^2]^{1/2}} \tag{13}$$

where $Y_i = Sin(\phi_i - \theta_i)$, $X_i = Cos(\phi_i - \theta_i)$ and subscript i corresponds to the subscripts used in Eqs. (12) and (13). In addition, $R_m = \dot{m}_{h2}/\dot{m}_{c,PT1}$ is the ratio of the amplitude of mass flow at the hot side of second regenerator to amplitude of mass flow at the cold side of the first pulse tube. It is assumed that the mass flow ratio and the phase shifts at the cold sides of pulse tubes are the control parameters. Assuming exergy destruction in the heat exchangers at the mid-stage and cold stage is zero, the balance of exergy gives the cooling capacity for the mid-stage and the cold stage of the gas-coupled TSPTRs respectively as

$$\dot{Q}_{c1} = \frac{(1/2)RT_o \frac{p_{c1}}{P_a}[\dot{m}_{c1}\,Cos(\phi_{c1} - \theta_{c1}) - \dot{m}_{c,PT1}\,Cos(\phi_{c,PT1} - \theta_{c,PT1})]}{\frac{T_o}{T_{c1}} - 1 + \frac{1}{\eta_{PT1}}} - (1/\pi)\lambda_1 C_p(T_o\dot{m}_{h1} - T_{c1}\dot{m}_{c1}) - (KA/L)_1(T_o - T_{c1}) \tag{14}$$

$$\dot{Q}_{c2} = \frac{(1/2)RT_o\,\dot{m}_{c2}\frac{p_{c2}}{P_a}Cos(\phi_{c2} - \theta_{c2})}{\frac{T_o}{T_{c2}} - 1 + \frac{1}{\eta_{PT2}}} - (1/\pi)\lambda_2 C_p(T_{c1}\dot{m}_{h2} - T_{c2}\dot{m}_{c2}) - (KA/L)_2(T_{c1} - T_{c2}) \tag{15}$$

where T_{c1} and T_{c2} are the load temperatures of mid-stage and second stage reservoirs, respectively. Eq. (9) can be used to find the power input to each compressor of the two configurations of TSPTRs.

$$\dot{W}_{compj} = (1/2)RT_o\,\dot{m}_{hj}\frac{p_{hj}}{P_a}Cos(\phi_{hj} - \theta_{hj}) \tag{16}$$

The exergetic efficiency of the TSPTRs in general can be calculated as the ratio of the total exergy delivered to the cold reservoirs divided by the total compressor power,

$$\eta_{ex} = \frac{\dot{Q}_{c1}(T_o/T_{c1} - 1) + \dot{Q}_{c2}(T_o/T_{c2} - 1)}{\sum_j W_{compj}} \tag{17}$$

For gas-coupled TSPTRs the cooling capacities are given by Eqs. (14) and (15) and there is only one compressor. For the thermally-coupled TSPTRs the total input power of two compressors should be used.[10]

RESULTS AND DISCUSSIONS

The results for thermally-coupled TSPTRs have been reported previously.[10] In this study we report the results of calculations for the gas-coupled TSPTRs for similar cases used previously. There are several parameters that can be controlled or used as variables to perform parametric studies and optimization of the TSPTRs. For the regenerators of both configurations of TSPTRs the important dimensionless parameters that results from Eqs. (10) and (11) are $\alpha_n = V_n\omega/C_n$, $\beta_n = \dot{m}_{cn}/C_n p_{hn}$, $mr_n = \dot{m}_{cn}/\dot{m}_{hn}$, and $Pr_n = p_{cn}/p_{hn}$. For gas-coupled TSPTRs an additional free parameter R_m is used in Eqs. (12) and (13). One of the important performance characteristics of the cryogenic refrigerators is the no-load temperature. The no-load temperatures of the first and second stage of the gas-coupled TSPTRs can be obtained from Eqs. (14) and (15) when the cooling capacities are zero. The equations clearly show the importance of the ineffectiveness of the regenerators on the no-load temperatures. In fact, the no-load temperatures approach zero for the thermally perfect regenerators. In the design of regenerators for PTRs the magnitude of conduction heat transfer does not often have a significant effect on the no-load temperatures and usually is much smaller that the effect of the regenerator ineffectiveness. The effect of conduction heat transfer in the regenerator on the performance of a two-stage PTR has been the subject of a previous investigation.[9] Assuming conduction heat transfer is zero and the mid-stage temperature is given, Eq. (15) gives the no-load temperature,

$$\frac{2P_a\lambda_2}{\pi\, p_{c2}}\frac{\gamma}{\gamma-1}\left(\frac{T_o}{T_{c02}}-1+\frac{1}{\eta_{PT2}}\right)\left(\frac{T_{c1}}{T_o mr_2}-\frac{T_{c02}}{T_o}\right)=Cos(\phi_{c2}-\theta_{c2}) \tag{18}$$

where γ is the specific heat ratio of the working fluid and T_{c02} denotes the no-load temperatures of the second stage. For the practical range of parameters the effect of efficiency of the pulse tube on the no-load temperature is small and the approximate solution to the above quadratic equation can be written as

$$\frac{T_{c1}}{T_{c02}}\approx mr_2\left[1+\frac{\pi}{2\lambda_2}\frac{p_{c2}}{P_a}\frac{\gamma-1}{\gamma}Cos(\phi_{c2}-\theta_{c2})\right] \tag{19}$$

Eqs. (18) and (19) are also applicable to the thermally-coupled TSPTRs using parameters corresponding to the regenerator 3 instead of regenerator 2. The above equations clearly show the effect of important parameters on the no-load temperature of the second stage. It should be pointed out that the thermally-coupled TSPTRs have two separate gas circuits resulting in additional degrees of freedom compared to the gas-coupled TSPTRs. Therefore, the average pressure, input amplitude, frequency and even the working fluid for the two separate gas circuits can be different.

Fig. 2 shows the cooling capacity of the second stage and the exergetic efficiency of the gas-coupled STPTRs as a function of the pressure amplitude ratio across the regenerator of the second stage. In this calculation no load is applied to the mid-stage ($\dot{Q}_{c1}$ = 0). In the calculation it is assumed that the mass flow and pressure are in-phase at the cold sides of both pulse tubes and the values of ineffectiveness for the regenerators are fixed. The values of important control and free parameters associated with these calculations are given in the figure. The important free parameter in the thermally-coupled TSPTR analysis is the pressure amplitude ratio across the regenerator of the auxiliary pulse tube while the free parameter for the gas-coupled TSPTR used in this study is the mid-stage mass amplitude ratio R_m. The results of the two configurations of TSPTRs are qualitatively similar for the cases studied.[10] Fig. 3 shows the cooling capacity and the exergetic efficiency as a function of the cold end temperature of the second stage for fixed values of pressure amplitude ratio across the second regenerator and the mid-stage temperature. The results are given for three values of second stage regenerator ineffectiveness while the regenerator ineffectiveness of the first stage is fixed. The cooling capacity is approximately a linear function of the load temperature as typically occurs in practice. In addition, the no-load temperature obtained from Eqs. (18)

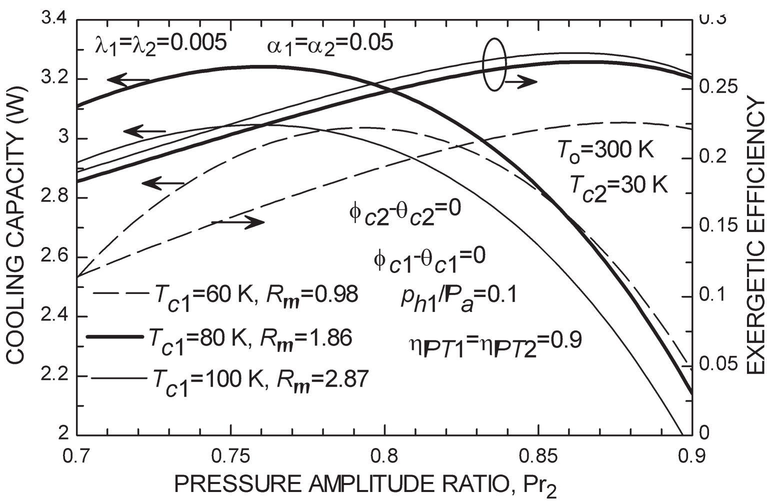

Figure 2. Cooling capacity and exergetic efficiency as a function of Pr_2.

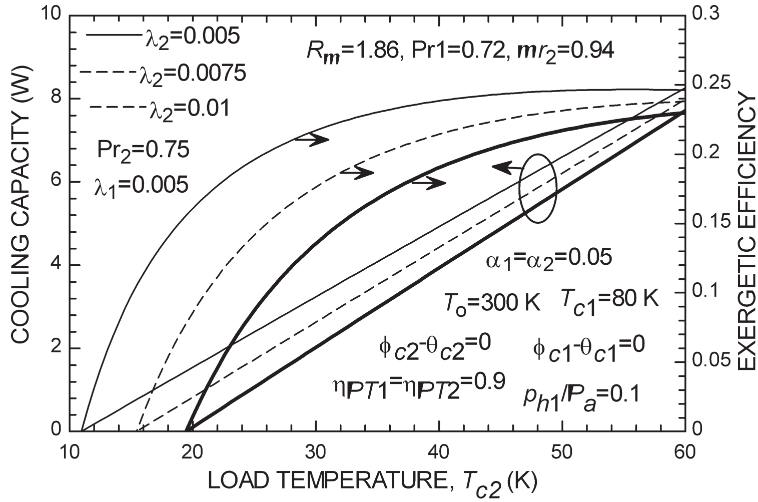

Figure 3. Cooling capacity and exergetic efficiency as a function of T_{c2}.

and (19) are good approximation to the numerical calculations of the models used in this study for both configurations.[10] Fig. 4 shows the cooling capacity and efficiency diagram corresponding to Fig. 2 when the pressure amplitude ratio across regenerator 2 is used as a parameter. In this calculation the ineffectiveness of regenerator 2 is used as a control parameter to show its effect on the performance of the gas-coupled TSPTR. The values of other parameters are given in the figure. The loop-shaped curves in the figure indicate a compromise between the cooling capacity and efficiency of the gas-coupled TSPTR. Similar results are obtained for the thermally-coupled TSPTRs.[10]

CONCLUSIONS

Using exergy analysis the control thermodynamic models for two configurations of TSPTRs are compared. A simple model for the regenerators was developed based on given flow conductance. Important dimensionless parameters for parametric study and optimization of the refrigera-

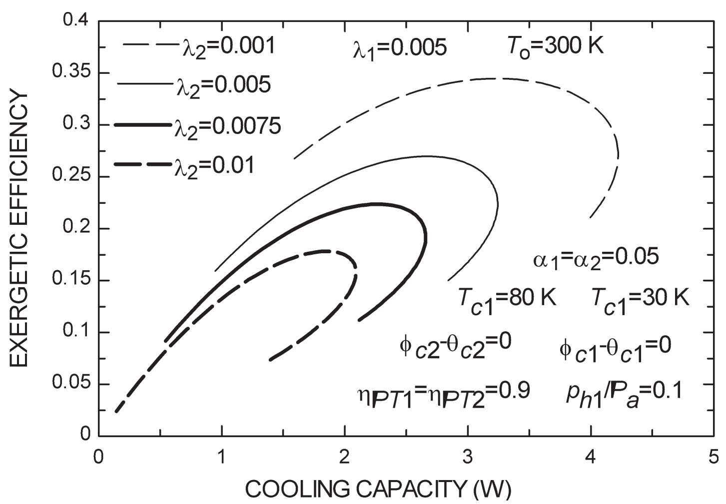

Figure 4. Cooling capacity and efficiency diagram for different values of λ_2.

tor were developed. Thermally-coupled TSPTRs have more degrees of freedom for optimization of the refrigerators because of their separate gas circuits. For gas-coupled TSPTRs the mass amplitude ratio parameter at the mid-stage is an important control parameter. The effect of important parameters on the thermodynamic bound for the no-load temperature at the second stage was developed. The cooling capacity and efficiency diagram for the refrigerators is presented and the effect of the regenerator ineffectiveness and mid-stage temperature on cooling capacity and efficiency is evaluated. It is shown that depending on the constraints imposed on the system, a compromise between cooling capacity and efficiency is possible.

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