Investigations on a Standing Wave Thermoacoustic Refrigerator

R. C. Dhuley, M. D. Atrey
Department of Mechanical Engineering,
Indian Institute of Technology Bombay,
Mumbai, India-400076

ABSTRACT

The dynamic pressure inside a Thermoacoustic Refrigerator (TAR) is an important parameter which governs the cold temperature and the cooling power. The present work aims to investigate the effect of two operating parameters: the resonant frequency and charging pressure on the dynamic pressure inside a TAR. A simple theoretical model is used to predict the behavior of dynamic pressure with charging pressure and resonant frequency. Experimental investigations have been carried out using a gas filled column driven by a moving coil loudspeaker. It has been observed that at small excitation levels, the results from theory and experiment closely match. Large deviations have been observed due to non linear effects at high excitation levels. One such non linear effect, periodic shocks, has been observed.

INTRODUCTION

A Thermoacoustic Refrigerator uses acoustic power to generate cold temperatures. Interaction of periodic compressions and rarefactions present in an acoustic wave with a porous medium sets up a heat flow across the porous medium. This causes one of its ends to cool down. Development and study of TARs has gathered much focus due to their inherent advantages like use of inert working medium, few moving parts, and many more.

The operating parameters that govern the performance of a given configuration of TAR are the charging pressure of the working medium, the driving frequency, the dynamic pressure, and the mean temperature. In standing wave TARs, the attainable cold temperature and the cooling power crucially depend on the dynamic pressure and operation near the resonant frequency of the system. In addition, off-resonance operation hampers the dynamic pressure drastically and hence, is detrimental to the TAR performance.

It is also advisable to keep the charging pressure in the TAR as high as possible, because the energy density of an acoustic field is directly proportional to the mean pressure of the medium. In loudspeaker driven TARs, the dynamic pressure obtained at the vibrating piston surface is dependent of the magnitude of the acoustic impedance which the load (the TAR) presents to the driver piston. For a given TAR geometry, this load impedance can be conveniently varied without disturbing the resonant frequency, by changing the charging pressure of the system.

The present work is carried out to investigate the effect of resonant frequency and charging pressure on the attainable dynamic pressure in a straight resonator standing wave TAR. The

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theoretical prediction of the dynamic pressure is done using the impedance transfer technique [1]. This technique enables one to calculate the effective acoustic impedance of the load at the driver piston, and its resonant frequency. The electrical network model of a moving coil loudspeaker is coupled with this acoustic impedance model so that the dynamic pressure can be directly computed as a function of input electrical voltage to the system. To determine the dynamic pressure experimentally, an acoustic driver is constructed out of a commercial moving coil loudspeaker. Dynamic pressure measurements are carried out in hollow columns of gas, resonant near a frequency of 400 Hz. The acoustic impedance is changed by varying the charging pressure and by changing the working gas as well.

THEORETICAL MODEL

Acoustic impedance of a gas filled column

Consider a straight hollow channel of length ‘L’ and uniform cross section area ‘A’, filled with a gas at pressure ‘p_m’ and temperature ‘T_m’. The speed of sound in the gas is ‘a’. A source of acoustic wave (acoustic driver) is attached to one end of the channel at x=0. When an acoustic wave with an angular frequency ‘ω’ propagates through this channel, the acoustic impedance at any location ‘x’ in the channel is given by:

$$Z_{ac}(x) = \frac{p_1(x)}{Au_1(x)} \tag{1}$$

where, $p_1(x)$ and $u_1(x)$ are the oscillatory pressure and velocity at location ‘x’. The transfer function giving the acoustic impedance at location ‘x’ in terms of acoustic impedance at any other location ‘x`’ is given by [1] :

$$Z_{ac}(x) = \frac{Z_{ac}(x')\cos k(x'-x) + jZ_c \sin k(x'-x)}{j\dfrac{Z_{ac}(x')}{Z_c}\sin k(x'-x) + \cos k(x'-x)} \tag{2}$$

where,

$$Z_c = \frac{\rho_m \omega}{Ak(1-f_v)} \tag{3}$$

Here, ‘k’ is the wave number and ‘f_v’ is the complex Rott’s viscosity function denoting the loss of acoustic power at the walls of the channel. A rigidly sealed end at x=L would result in a location of infinite acoustic impedance. In this case, the acoustic impedance at the driver piston would be:

$$Z_{ac} = -j\frac{\rho_m \omega}{Ak(1-f_v)}\cot kL \tag{4}$$

This complex acoustic load resonates when its imaginary part is zero. The relation between the channel length, the frequency and the sound speed for the fundamental resonance mode is:

$$L = \frac{\pi a}{\omega} \tag{5}$$

As can be seen from Eq.(4), the acoustic impedance of a given geometry depends on the density of the working medium. Thus, it can be varied by changing the mean pressure, mean temperature or the medium itself. However in practice, it is not feasible to vary the mean temperature of the working medium.

Electrical Network Model of a Moving Coil Loudspeaker

A simple moving coil loudspeaker consists of an electrical conductor coil suspended in a radial magnetic field. When excited by an alternating current, the coil reciprocates perpendicular to the plane containing the magnetic field and current, producing a Lorentz's force. A diaphragm or a piston attached to the reciprocating coil causes periodic compressions and rarefactions in the surrounding medium, thereby producing an acoustic wave. A representation of a moving coil loudspeaker in an electrical network form is shown in Figure 1.

Traversing from right to left in Figure 1, the first element is the acoustic impedance of the load at the driver piston. The centre block represents the mechanical part consisting of the moving mass M_m, stiffness K_m and mechanical resistance R_m of the loudspeaker. The leftmost block is the electrical part, comprising of the electrical resistance R_e and inductance L_e of the coil. The transduction coefficient 'Bl' converts the electrical energy to mechanical energy, while the piston surface area 'S' converts mechanical energy into acoustic energy. The electro-acoustic efficiency of a loudspeaker can be maximized by making the mechanical part and the acoustic part resonate at a same drive frequency [2].

The total electrical impedance of the loudspeaker circuit between the two terminals of an AC power source is given by:

$$Z_e = R_e + j\omega L_e + \frac{(Bl)^2}{R_m + j\left(\omega M_m - \frac{k_m}{\omega}\right) + S^2 Z_{ac}} \tag{6}$$

Using the standard transduction relations, the expression for the pressure amplitude generated at the piston surface can be written as:

$$p_1 = V \frac{S(Bl)(Z_{ac})}{Z_e\left[R_m + j\left(\omega M_m - \frac{k_m}{\omega}\right) + S^2 Z_{ac}\right]} \tag{7}$$

where 'V' is the voltage input to the system.

EXPERIMENTAL SETUP

An experimental setup consisting of an acoustic driver and a straight hollow stainless steel tube (resonator) is built to investigate the effect of charging pressure and resonant frequency on the dynamic pressure. The acoustic driver is constructed from the electrodynamic motor of a commercial loudspeaker. The 10" paper cone and the spider suspension which came along with the speaker are cut and a thin aluminum cone is glued to the voice coil. The cone provided the reduction of cross section from 52 mm voice coil to 32 mm resonator. A 0.7 mm thick rubber sheet is used to suspend the voice coil in the magnet gap. The rubber sheet also provides sealing between the driver chamber and the resonator.

The driver assembly is mounted on a flange and enclosed by a cylindrical jacket made of stainless steel. The resonator is bolted on to the other face of the mounting flange. In order to study the effect of working gas on dynamic pressure, two different resonators are built.

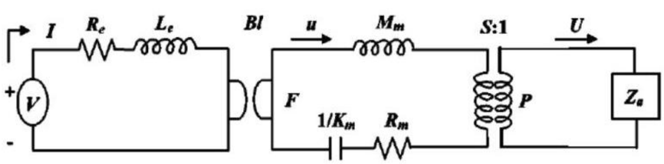

Figure 1. Electrical circuit representation of a loudspeaker with an acoustic load.

Both the resonators are sealed at one end. The length is calculated as per Eq.(5), so that they resonate near 400 Hz. The lengths for Helium and Nitrogen resonators respectively, are 1275 mm and 445 mm. The dynamic pressure is measured by means of a piezoresistive pressure transducer attached to the resonator, 30 mm from the driver piston surface.

RESULTS AND DISCUSSION

Driver Parameters

In the course of adapting the commercial speaker to our purpose, the mechanical parameters of the driver got modified and are determined experimentally. The voice coil, the cone and the suspension are accurately weighed to find the new M_m. The new suspension stiffness k_m is determined by measuring the resonance frequency in vacuum. Bl is calculated by measuring the magnetic gap flux and the active coil length. The electrical parameters of the coil *viz.* R_e and L_e remain unchanged during modification of the speaker. The mechanical resistance R_m is calculated from the measured value of Z_{et} at resonance in vacuum, Bl and R_e. The electrical reactance being small compared to R_e, is neglected while calculating R_m. The driver parameters are given in Table 1.

Figure 2 shows the current drawn by the driver-resonator assembly over a frequency range when the input voltage is kept constant at 5 V. The measurement in vacuum (10^{-3} mbar) shows only one minimum of current where the system is resonant (driver mechanical resonance). When the system is charged with Helium or Nitrogen at 9.5 bar, two current minima are observed. The driver resonance frequency still remains close to 85 Hz as in vacuum, while the acoustic load (charged gas column) resonates close to 400 Hz as predicted by Eq.(5). This ascertains the absence of a possible stiff gas cavity in the driver back chamber, which would shift up the driver resonance frequency [1]. This makes the driver parameters independent of the gas used or the charging pressure. Hence, the driver is suitable for the present investigations.

Effect of Operating Frequency on Dynamic Pressure

In standing wave TAR systems, the phasing between the dynamic pressure and oscillatory velocity is very crucial for production of low temperatures. The proper phasing can be achieved by operating the system near the resonant frequency of the acoustic load. Furthermore, when the acoustic load resonates, the dynamic pressure obtained is also large. This can be seen from Figure 3. It shows the dynamic pressure as the operating frequency is varied around the resonance at a constant supply voltage of 5 V. In case of both, the theoretical prediction and the experiments, the dynamic pressure increases as the operating frequency approaches the load resonant frequency. It exhibits a maxima peak near the resonant frequency and again starts to diminish after the resonance cross-over.

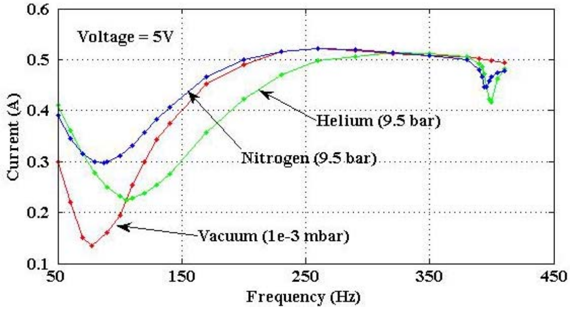

Figure 2. Variation of current with operating frequency

Table 1. Driver Parameters

R_e	7.2 Ω
L_e	1.03 mH
Bl	10.0 T-m
M_m	0.026 kg
k_m	7000 N/m
R_m	4.58 N-s/m
S	0.0008 m^2
f_{res} in vacuum	78 Hz

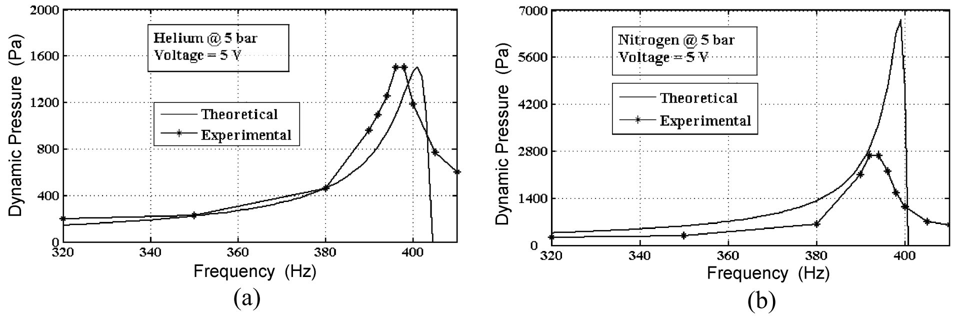

Figure 3. The dynamic pressure in the resonator as a function of operating frequency **a)** with Helium; **b)** with Nitrogen.

A comparison between theory and experiment shows a very good agreement in case of Helium (Figure 3(a)). However, in the case of Nitrogen, as seen in Figure 3(b), very high dynamic pressure is predicted around resonance by the theory as compared to that observed during the experiments. The discrepancy may be attributed to the non-linear effects presented in a later section.

Effect of Charging Pressure and Working Gas on Dynamic Pressure

The density of working gas, which appears in the expression of acoustic impedance (Eq.4), is one of the parameters that governs the dynamic pressure in a TAR. The density of a gas is proportional to its pressure, and it is more intuitive to study this dynamic pressure dependency in terms of the charging pressure of the gas. Figure 4 shows a comparison between the theoretically predicted and experimentally obtained dynamic pressure at resonance for several input voltage levels. The test is carried out for Helium and Nitrogen at three different values of charging pressure *viz.* 5 bar, 8 bar and 9.5 bar. In case of both the gases, the dynamic pressure for a given value of input voltage increases with the charging pressure. This can be interpreted in the following manner. In an electric circuit where the components are in series, the voltage drop across a certain component will increase if its impedance is somehow made to increase. In a similar fashion, the dynamic pressure, which is analogous to the 'voltage drop' across the acoustic impedance, should increase with an increase in the acoustic impedance. This is predicted theoretically as well as observed in experiments. Furthermore, it can be seen that for a certain value of charging pressure and voltage, Nitrogen induces larger dynamic pressures. This is because Nitrogen being heavier that Helium, has more density at the same pressure and temperature. As a result, Nitrogen offers a greater impedance to the driver.

Non-Linear Effects

Referring to Figure 4, there is a fair match between theoretical and experimental values of dynamic pressure when the input voltage is small. When the voltage is increased, the dynamic pressure is expected to increase linearly as predicted by theory. However, this is very unlike what is observed from experiments. Further, at large voltages, the deviation from theory is quite large. This deviation can be accounted for by the non-linear effects that are prevalent in resonators with uniform cross-sections [1] when driven at large voltage excitation. Straight resonators with uniform cross sections have resonant modes that are integral multiples of the fundamental mode. In the non-linear regime, the acoustic energy that the driver pumps into the acoustic load, is transferred to higher harmonics of the fundamental. In this way, the energy of the fundamental mode is lesser than it could have been in case of linear response. One such non linear effect observed in our experiments was the phenomenon of periodic shocks [3]. Such shocks exist when a straight gas column is excited with large amplitudes of the piston near its resonant frequency. The images in Figure 5 show the waves observed on an oscilloscope which

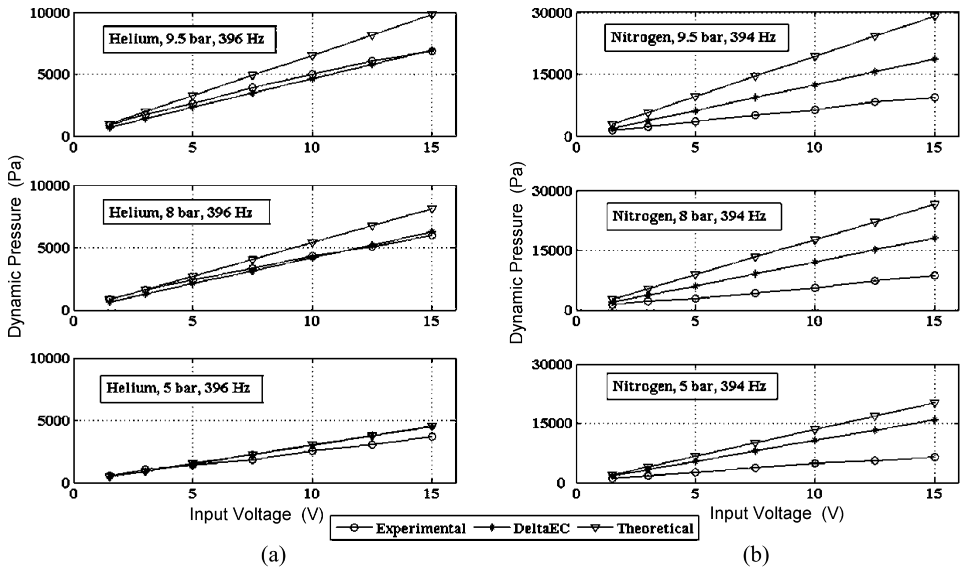

Figure 4. The dynamic pressure near the driver piston as a function of input voltage at various charging pressures when operated at resonance. **a)** with Helium **b)** with Nitrogen as working gas.

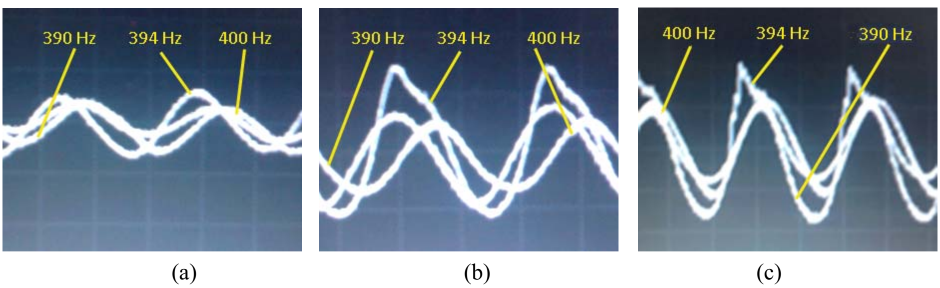

Figure 5. The waveforms observed on oscilloscope representing dynamic pressure in Nitrogen resonator at voltage levels- **a)** 5 V; **b)** 10 V; **c)** 15 V.

represent the dynamic pressure in a half-wavelength Nitrogen resonator charged at 9.5 bar pressure. At all the three voltage levels (5 V, 10 V and 15 V), the dynamic pressure wave is very much sinusoidal when the system is operated slightly away from resonance (390 and 400 Hz). Very close to resonance, periodic shocks set in and disturb the perfect sinusoidal nature of the waveform. It can also be seen that the shocks are more pronounced at higher voltage levels. Hence, the deviation of experiments from theory is large at higher level of input voltages.

To further account for the non-linear effects, a DeltaEC [4] model of the setup has been made and the simulations for the tests described in Figure 4 were carried out. The dynamic pressure predicted by DeltaEC in case of Helium follows very closely with the experimental findings. Deviations are still observed in case of Nitrogen, mainly due to the existence of periodic shocks in the resonator. However, the DeltaEC predictions follow the experimental results more closely as compared to those given by the impedance transfer technique.

CONCLUSIONS

The effect of the resonant frequency and the charging pressure on the dynamic pressure obtained in a gas filled column has been investigated in the present work. From resonant frequency measurements, it can be concluded that the impedance transfer technique predicts the resonant frequency of a given TAR configuration very accurately. The dynamic pressure predictions by this technique show a fair agreement with experimental findings at low levels of excitation. However, due to non linear effects, large deviations are observed when the input excitation is high. A quantitative estimation of these non-linear effects could not be given at this stage.

It can also be concluded that a larger dynamic pressure can be obtained for a given input voltage and operating frequency by imposing a larger acoustic impedance at the driver piston. This can be done by using a denser working gas or by using a higher charging pressure. However, the sound speed of a denser working gas is less and hence, this would decrease the energy density of the TAR configuration. Besides, the dynamic pressure which a given gas can generate, the other thermodynamic properties *viz.* the volumetric heat capacity and thermal diffusivity, which govern the temperature lift and cooling power in a TAR should be carefully studied before making an intelligent choice.

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