Design of Standing Wave Type Thermoacoustic Prime Mover for 300 Hz Operating Frequency

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ABSTRACT

Thermoacoustically driven pulse tube cryocoolers are gaining significant interest in the recent time due to the key advantage of complete absence of moving components for the entire system. The present work gives a simple design procedure for a 300 Hz Standing Wave-Type Thermoacoustic Prime Mover having a parallel plate type stack. The necessary code is written in MATLAB for solving the Rott's wave equation. The code is validated by comparing the results with those available in the literature.

The work further reports the effect of operating and geometrical parameters on the performance of the standing wave-type thermoacoustic prime mover. The system performance, mainly in terms of acoustic power and pressure ratio, is influenced significantly by the operating and geometrical parameters. The operating parameters are heat input, hot end temperature, frequency, filling pressure. The geometrical parameters are the stack length, the stack position, the resonator length and the dimensions of the acoustic amplifier, etc.

INTRODUCTION

A pulse tube cooler driven by a thermoacoustic prime mover, without any moving components at both ambient and cryogenic temperatures, has a the potential for long-life with high stability and reliability. Thermoacoustic engines are promising in practical applications for the merits of their simple configuration, reliable operation and an environmentally friendly working gas. Progress has been made in recent years, in order to enhance the performance of thermoacoustic engine. In order to improve the performance, great efforts should be placed on optimizing the match between the thermoacoustic engine and the pulse tube cryocooler. An acoustic amplifier can increase the output pressure amplitude of a thermoacoustic engine and improve its match with the load.[1] High frequency operation of such a system can lead to a reduction in size, which is very attractive in small-scale cryogenic applications.

The first high frequency thermoacoustically driven pulse tube cryocooler was established in 1996 by Godshalk, et al.[2] However, the cooler just reached its lowest no-load temperature of 147 K with a frequency of 350 Hz. Most recently, a 300 Hz thermoacoustically driven pulse tube cryocooler was developed and studied by W. Dai et al.[3] It utilizes the invention of a acoustic pressure amplifier tube to couple the standing wave engine with a pulse tube cooler. The lowest no load temperature of 56 K[4] and 68 K[5] were reached. An acoustic amplifier dissipates some amount of

Cryocoolers 16, edited by S.D. Miller and R.G. Ross, Jr.

acoustic power while amplifying the pressure amplitude. Therefore it is necessary to find ways to improve its performance by reducing its power consumption.

The present work reports on the investigations of the design parameters, like the stack length, the positioning of the stack, the buffer length, the length and diameter of the acoustic amplifier and the operating parameters like the heat input, hot end temperature, mean pressure, and their effect on the pressure ratio and acoustic power. Results for the optimum stack length and stack position for the maximum pressure amplitude and the maximum acoustic power generation are obtained. Effects of the hot end temperature, the heat input and the mean pressure on these results are discussed. Engine performance characteristics are studied in terms of the non-dimensional parameters.

MODEL DESCRIPTION AND SOLUTION

Theoretical modeling of thermoacoustic devices plays an important role in the design and development of a thermoacoustic prime mover. In order to formulate a design algorithm capable of modeling the performance of a thermoacoustic prime mover, the independent parameters which may be allowed to vary during the operation of the device must be identified, and a system of equations that relates these variables should be developed.

The sound wave equation is a second order equation which can be considered as a combination of two first order equations of volume velocity and pressure amplitude. Equation (1) gives velocity at different points in the system.[6]

$$dU_1 = \frac{i\omega A dx}{\gamma P_m}\left[1+(\gamma-1)f_k\right]p_1 + \frac{(f_k - f_v)}{(1-f_v)(1-\sigma)}\frac{dT_m}{T_m}U_1 \tag{1}$$

The momentum equation, as given in equation (2), gives knowledge about the pressure and the viscous force.[6]

$$dp_1 = -\frac{i\omega\rho_m dx / A}{1-f_v}U_1 \tag{2}$$

Equations (1) and (2) may be considered as principal tools of thermoacoustic analysis. The f_v and f_k factors in above equations arise from thermal and viscous contact of gas with the walls of the duct.

For the solution of the theoretical model, one-dimensional equations are integrated, in a geometry given by the user as a sequence of segments.[7] Differential equation (3) for Tm(x) can be written using Rott's energy equation as;

$$\frac{dT_m}{dx} = \frac{H_2 - \frac{1}{2}\mathrm{Re}\left[p_1 U_1\left(1-\frac{T_m\beta(f_k - f_v)}{(1+\epsilon_s)(1+\sigma)(1-f_v)}\right)\right]}{\frac{\rho_m c_p |U_1|^2}{2\omega A_{fluid}(1-\sigma)|1-f_v|^2}\mathrm{Im}\left[f_v + \frac{(f_k - f_v)(1+\epsilon_s f_v / f_k)}{(1+\epsilon_s)(1+\sigma)}\right] - A_{fluid}K - A_{solid}K_{solid}} \tag{3}$$

A schematic diagram of a thermoacoustic prime mover is shown in Figure 1. The sound equation is solved in each component, such as the buffer, the heat exchanger, the stack and the acoustic amplifier. The appropriate geometrical parameters of the components are given as input parameters. The theoretical model is solved using Euler's method considering helium gas as the working fluid. Algorithms are developed for the solution of the model and the programming is done using MATLAB. It is validated by comparing obtained results with those published in literature.[6]

RESULTS AND DISCUSSION

Results are obtained for a half wavelength standing wave-type thermoacoustic prime mover with a 300 Hz operating frequency and a parallel plate type stack. Initial performance characteristics of the prime mover are studied in terms of the normalized heat input and the normalized output pressure amplitude. This is followed by a study of the approximate sizing and designing of the prime mover. The results are discussed below.

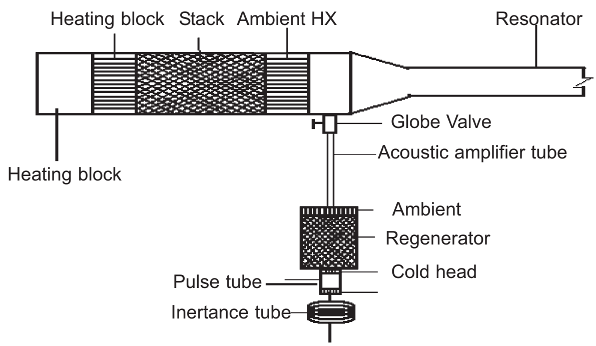
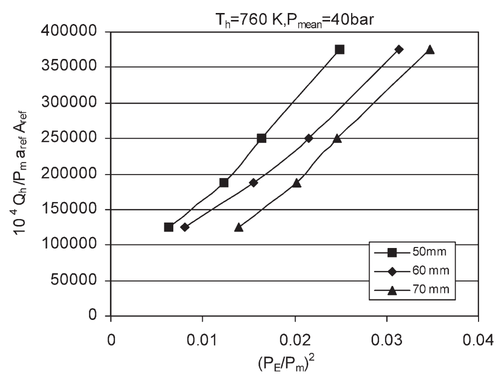


Figure 1. Thermoacoustic Prime Mover

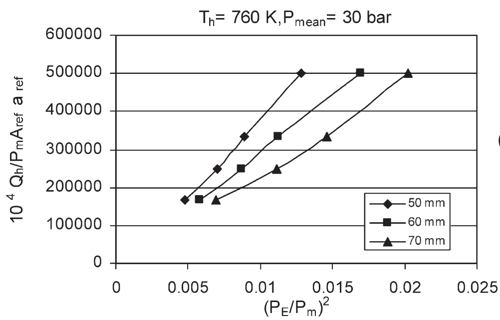
Performance Characteristics of Prime Mover

It is necessary to present the results in a non-dimensional form as there are large numbers of variables that affect the characteristics of thermoacoustic prime mover. The efficiency of a prime mover is studied in terms of the normalized heat input and the output pressure amplitude as presented in the literature.[8,9] Results are plotted for the available pressure amplitude by varying the heat input for different stack length depicting the effect of mean pressures in Figure 2a and b and the effect of heat input temperatures as shown in Figure 2a & 3.

It can be seen from these results that the available pressure amplitude increases with the heat input and also with the stack length. The effect of the heat input is more at higher mean pressure due to the change in the fluid properties with pressure. The performance of a prime mover improves in terms of the output pressure amplitude with the decrease of heat supply temperature for a given heat input.



(a)



(b)

Figure 2. (a) Performance characteristics of thermoacoustic prime mover for different stack length for 40 bar mean pressure and (b) for 30 bar mean pressure

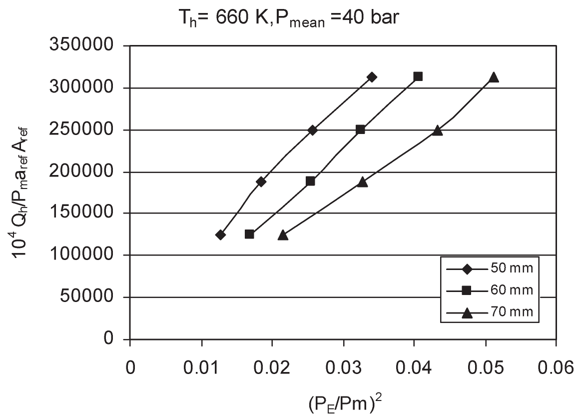


Figure 3. Performance characteristics of thermoacoustic prime mover for different stack length at 660 K hot end temperature

Approximate Sizing and Designing

The thermoacoustic prime mover can be designed for the desired output in terms of the pressure amplitude or the acoustic power. Both are significantly influenced by three parameters, i.e. stack length, stack position as length of buffer, and length and diameter of acoustic amplifier. The effect of the stack length and the length of buffer on the output pressure amplitude and the acoustic power is shown in Figure 4 through 9 for different heat inputs and heating temperatures.

It can be observed from Figures 4, 6 and 8 that within the range of the present investigation, the normalized pressure amplitude reduces and the acoustic power increases with an increase in the stack length. Deciding the stack length for a higher pressure amplitude will result in a reduced acoustic power and a lower efficiency where as, deciding the stack length for a higher acoustic power (thermodynamic efficiency) will result in a lower pressure amplitude making it less suitable for an operating pulse tube cryocooler. Hence, the stack length should be selected corresponding to the intersection point of the pressure amplitude and the acoustic power variation.

The next step is to decide the stack position in terms of the length of the hot buffer. Variation of the normalized pressure amplitude and the acoustic power against the length of the hot buffer is similar to those for the stack length and can be seen from Figures 5, 7 and 9. Based on similar arguments presented earlier, length of buffer should be selected corresponding to the intersection point of curves for the normalized pressure amplitude and the acoustic power. It can be seen in Figures 4, 6 and 8 that the optimum stack length is higher for higher heating temperature and lower heat input. Similarly it can be seen from Figure 5, 7 and 9 that the optimum buffer length is higher for higher heating temperature and higher heat input.

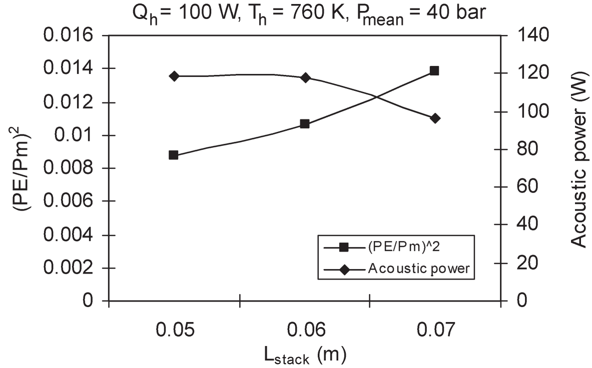


Figure 4. Normalized pressure amplitude and acoustic power variation along the stack length for 760 K hot end temperature

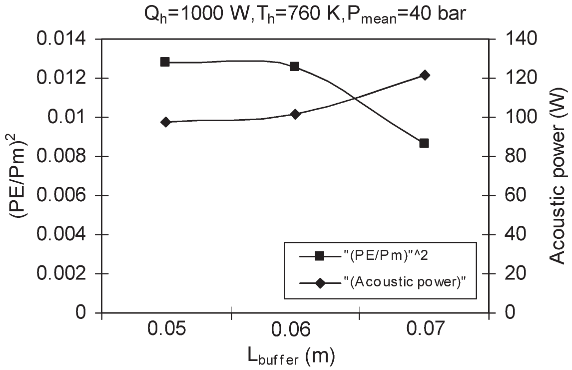


Figure 5. Normalized pressure amplitude and acoustic power variation along the buffer length for 760 K hot end temperature

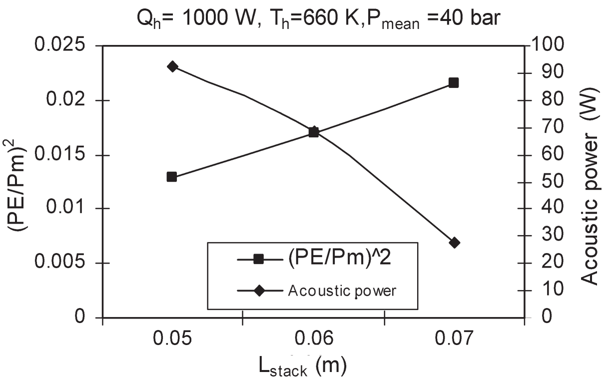


Figure 6. Normalized pressure amplitude and acoustic power variation along the stack length for 660 K hot end temperature

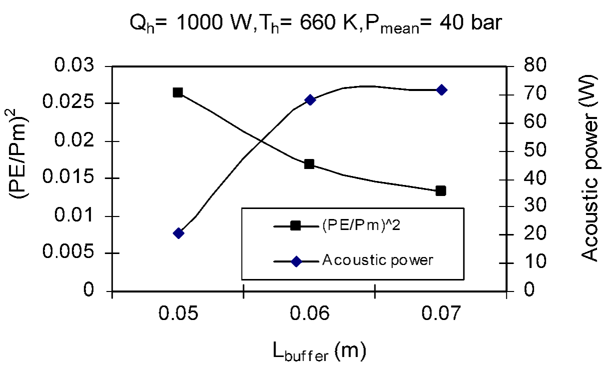


Figure 7. Normalized pressure amplitude and acoustic power variation along the buffer length for 660 K hot end temperature

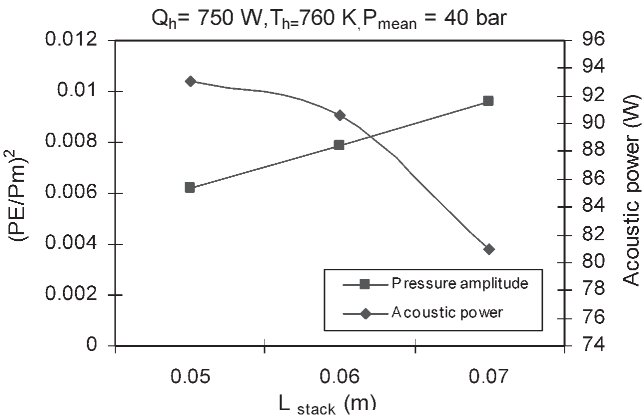


Figure 8. Normalized pressure amplitude and acoustic power variation along the buffer length for 750 W heat input

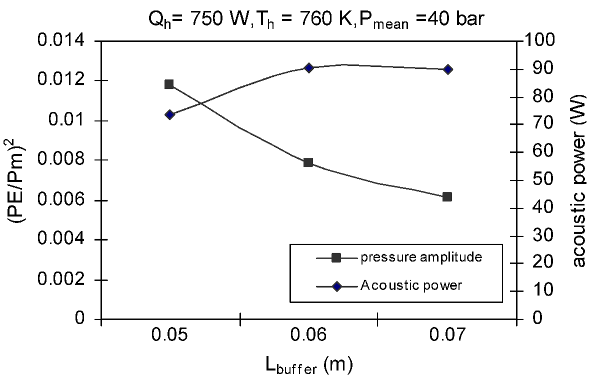


Figure 9. Normalized pressure amplitude and acoustic power variation along the buffer length for 750 W heat input

Figure 10a to 10d shows the effect of mean (filling) pressure on stack length and stack position. There is no significant effect of mean pressure on optimum stack length as seen from Figure 10a and 10c where the optimum length of buffer is higher for higher mean pressure.

Acoustic Amplifier Design

The acoustic amplifier is used generally with high frequency prime mover for increasing pressure amplitude making suitable for operating the high frequency pulse tube cryocooler. This amplification is achieved with some dissipation of acoustic power.[10] Variation of pressure amplitude and power with acoustic amplifier length for its fixed diameter is shown in Figure 11 and 12. It can be seen from these figures that the pressure amplitude increases with length of acoustic amplifier but there is simultaneous drop of acoustic power. Hence, for given diameter of acoustic amplifier the optimum length should be selected corresponding to the intersection point of two curves. The optimum length of amplifier increases with its diameter, as seen from Figures 11 and 12. It can also be seen from these figures that smaller diameter of amplifier gives better pressure amplitude.

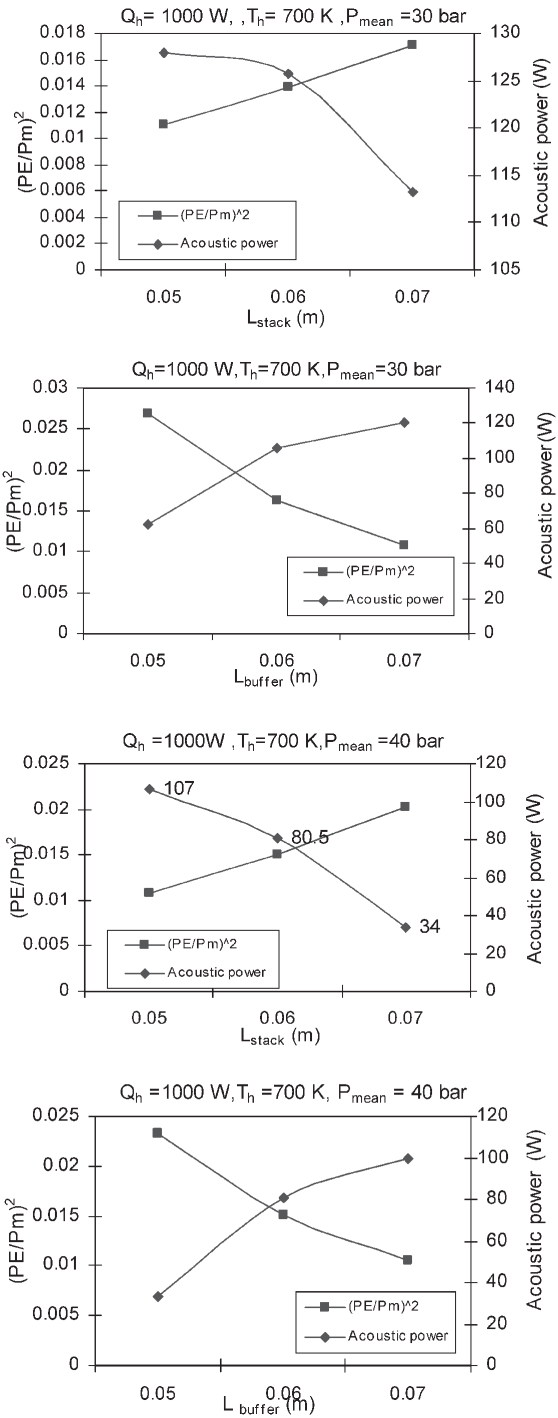


Figure 10 (a) and (b) Normalized pressure amplitude and acoustic power variation along the stack length and buffer length for mean pressure of 30 bar. (c) and (d) Normalized pressure amplitude and acoustic power variation along the stack length and buffer length for mean pressure of 40 bar.

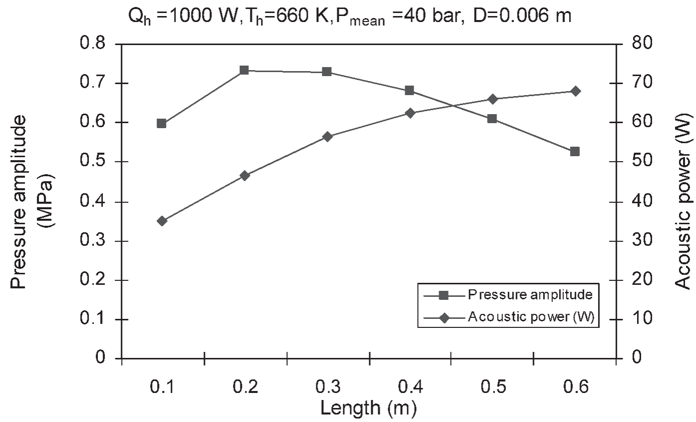


Figure 11. Pressure amplitude and acoustic power along the length of amplifier

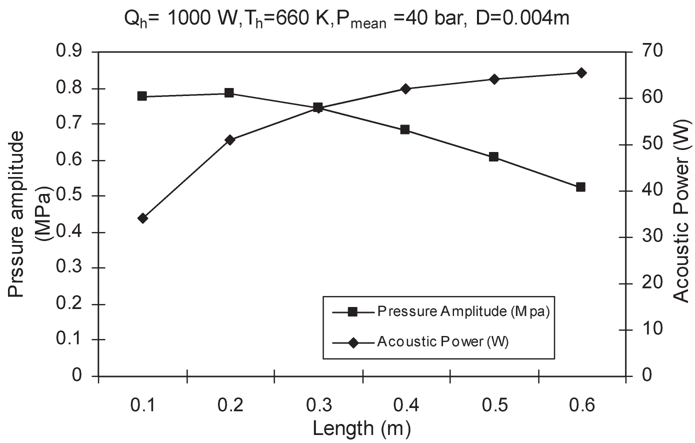


Figure 12. Pressure amplitude and acoustic power along the length of amplifier

CONCLUSION

Results for the performance characteristics of thermoacoustic prime mover are obtained in terms of non-dimensional parameters. It can be concluded that efficiency of the thermoacoustic prime mover is improved with an increase in heat input. It is also observed that for a given heat input, performance is improved with increase in heat supply temperature up to a certain limit.

Results are obtained for the optimum stack length and optimum stack position for maximum acoustic power and maximum pressure amplitude for different heat inputs, different hot end temperatures and different mean pressures. It is concluded that with an increase in the heat input, the optimum stack length decreases. Also for a given heat input, as the hot end temperature increases beyond a certain limit, the available pressure amplitude and acoustic power are less for the optimum stack length.

Results for stack length at different mean pressures show that optimum stack length is more for lower filling pressure and available pressure amplitude is less for corresponding stack length as the mean pressure decreases.

The results are obtained for pressure amplitude for different lengths and different tube diameters of acoustic amplifier. It can be concluded that there is an increase in pressure amplitude with increase in length of acoustic amplifier up to certain extent. It can also be noted that the optimum length of acoustic amplifier increases with the tube diameter increases. It can be concluded that the length of acoustic amplifier should be decided for maximum pressure amplitude at exit of amplifier and maximum acoustic power.

ACKNOWLEDGMENT

Authors are thankful to the BRNS, Department of atomic energy, Government of India, (No.2007/34/33/-BRNS/2908) for funding the research project under which present work is carried out.

Nomenclature

A	area	U	volumetric velocity	μ	dynamic viscosity
a	sound speed	u	x component of velocity	ν	kinematic viscosity
D	diameter	V	volume	π	perimeter
E	acoustic power	v	y component of velocity	ρ	density
f	frequency	W	work	ρ	Prandtl Number
H	total energy	w	z component of velocity	ω	angular frequency
i	$(-1)^{1/2}$	Z	acoustic impedance		
L	length	β	thermal expansion coefficient		
m	mass	γ	ratio, isobaric to isochoric specific heats		
p	pressure	δ	penetration depth		
Q	heat	η	efficiency		
T	temperature	k	thermal diffusivity		
t	time	λ	wavelength		

Subscripts and superscripts

k thermal
v viscous
1 first order
2 second order
f frequency
m mean value

Special symbols

Im [ ] **Imaginary** part of
Re [ ] Real part of

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