Are P-V and T-S Diagrams Meaningful for Regenerative Cryocoolers?

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ABSTRACT

P-V and T-S diagrams are commonly used tools to illustrate thermodynamic cycles. For recuperative cycles, it is easy to idealize a cycle so that the history of a gas element can be traced on P-V and T-S diagrams as it flows around the machine. In such cycles, dead volumes such as accumulators, reservoirs, and clearance space in piston compressors and expanders are not fundamental to the operation. However, such dead volumes do have practical purposes in controlling pressure variations in recuperative coolers. Regenerative cryocoolers also have dead volumes, which include the void volumes in heat exchangers, regenerators, and pulse tubes. Because of these volumes, gas elements do not traverse all components of the cooler. Rather, an element can remain in a single component. For the Stirling cycle, P-V and T-S diagrams can be constructed if the void volumes are ignored. However, what happens in a pulse tube cooler where the volumes of the pulse tube, inertance tube, and reservoir are fundamental to the coolers operation? P-V and T-S diagrams can still be constructed if they are reinterpreted to represent the envelope of the motion of all possible gas elements. This approach will be explored here.

INTRODUCTION

Pressure-volume (P-V) and temperature-entropy (T-S) diagrams are often used as teaching aids to describe refrigeration processes in introductory textbooks. They trace the path of a hypothetical element of gas as it moves through a system during a complete thermodynamic cycle. The usefulness of these diagrams depends on how accurately they capture the refrigeration process. Sometimes the process is idealized (simplified) during the construction of the diagrams. In a single-stage recuperative refrigerator, a gas element passes through all of the critical components during a cycle. Thus, the diagrams can account for the complete cycle. In multistage machines, diagrams for each stage can be superimposed. Occasionally, losses are incorporated into the diagrams. As an example of an ideal single-stage refrigerator, consider the ideal gas cycle shown in Fig. 1. This cycle has two isothermal steps, compression and expansion, and two isobaric steps, flow through the recuperator. The P-V and T-S diagrams for this cycle are illustrated in Fig. 2 where the numbers 1 and 2 refer to the entrance and exit of the compressor, respectively, and the numbers 3 and 4 refer to the entrance and exit of the expander, respectively. Reservoirs and other appendices such as fill lines, filters, and sorbents are not included in Figs. 1 and 2 because they are not fundamental to the operation. Rather, they serve practical purposes; e.g., reducing the pressure oscillations caused by the compressor and

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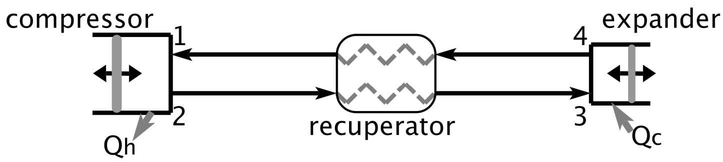

Figure 1. A schematic representation of the recuperative refrigerator discussed in the text with an isothermal compressor and expander and isobaric flows through the recuperator. The valves on the compressor and expander are not shown, nor are the timing of the valves and the motion of the compressor and expander. The numbers show the locations used in Fig. 2.

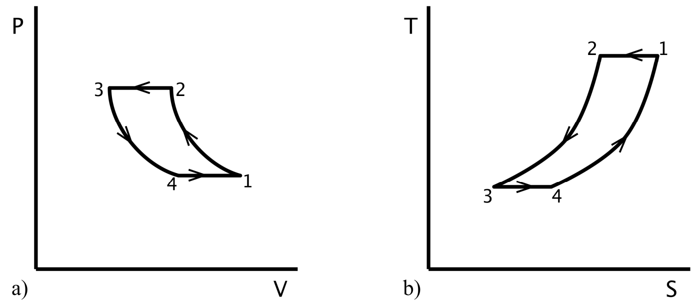

Figure 2. The a) P-V and b) T-S diagrams for the recuperative cycle shown in Fig. 1. The pressure oscillations have been ignored. The numbers refer to the locations in Fig. 1.

expander, and reducing the pressure changes caused by temperature changes during start up and shut down of the refrigerator.

The usefulness of the P-V and T-S diagrams is less obvious for regenerative machines. There, void volume effects prevent a single element from traversing the whole refrigerator during a cycle. This difficulty is often overcome by assuming no void volumes in the components; e.g., heat exchangers, regenerator, compressor, and expander. When void volumes are included, some gas elements may spend a complete cycle within a single component while others cross the boundaries between components. Each gas element follows a slightly different P-V and T-S path.[1] All of the possible paths can be superimposed on the same P-V and T-S diagrams. This set of diagrams is bounded by curves similar to diagrams developed for the no-void-volume assumption. Thus, for regenerative coolers the familiar P-V and T-S diagrams can be used if they are interpreted as the envelope of all possible paths of gas elements. Multistage machines can be treated with the same approach.

A second difficulty in regenerative systems is, in generating the P-V and T-S diagrams, it is assumed the motion of the pistons in the compressor and expander have instantaneous changes in velocity. This permits the diagrams to have sharp corners and the gas elements to follow paths having a constant thermodynamic quantity; e.g., an isobar. In practice, the piston velocity is continuous with approximately sinusoidal motion. The resulting gas motions trace distorted ellipses in the P-V and T-S planes. They do not follow a path of any constant thermodynamic quantity. It is difficult to distinguish the different thermodynamic cycles form the resulting P-V and T-S diagrams, defeating their usefulness as teaching aids. Following the usual practice, this difficulty will be ignored here for P-V and T-S diagrams, and we will keep the sharp corners.

As an example, consider the ideal single-stage regenerative cycle shown in Fig. 3. This cycle has two isothermal steps, compression and expansion, and two isochoric steps, flow through the recuperator. This is the idealized Stirling cycle. Assume that the three components have void volumes; e.g., the compressor and expander do not completely expel the working fluid and the regenerator has passages with finite volume. In the small amplitude limit, a gas element in the compressor traces *path* (1-2-3-4) in Fig. 4, while an element in the expander traces *path* (1’-2’-3’-4’). A typical path inside the regenerator is also shown in Fig. 4. The isochoric

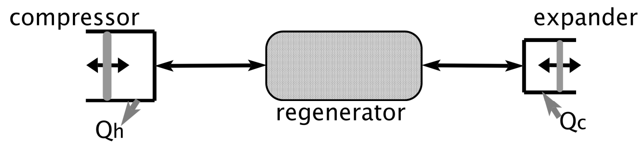

Figure 3. A schematic representation of the regenerative (Stirling) refrigerator discussed in the text with isothermal compression and expansion and isochoric flow in the regenerator. The two pistons operate synchronously and there are no valves.

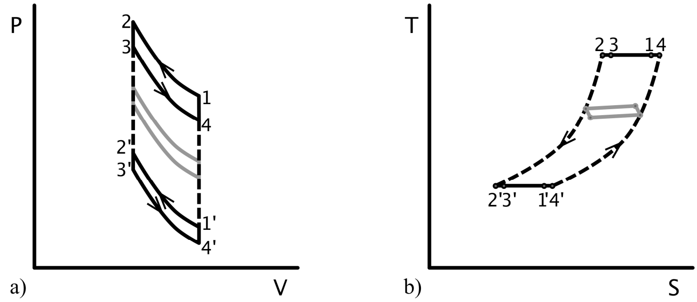

Figure 4. The a) P-V and b) T-S diagrams for the regenerative cycle shown in Fig. 3 with void volumes in the small amplitude limit. The paths and numbered points are discussed in the text.

condition is imposed as a boundary condition on the regenerator by the synchronized motions of the pistons during the time mass is flowing through the regenerator. (To achieve the sharp corners in Fig. 4, the piston motions are segments linear in time.) The pressure changes during the isochoric steps because the gas temperature changes as it moves through the regenerator. During these steps, the specific volume of the gas elements remains constant. The path of a gas element inside the regenerator has a different shape than the paths in the compressor and expander because its temperature changes as it moves.

The limits of these P-V and T-S diagrams are shown by the dotted line in Fig. 4. The shape of the limiting envelope is similar to the diagram for the no-void-volume case shown in textbooks.

CONSTRUCTING P-V AND T-S DIAGRAMS FOR AN OPTR

Orifice pulse tube refrigerators, OPTRs, have large gas volumes in the pulse tube and reservoir. It is not possible to idealize an OPTR by setting these volumes to zero, as they are essential to the refrigerator's operation. One cannot create a no-void-volume approximation. Thus, we must define the diagrams to be the envelope that encompasses all possible gas elements. For simplicity, we will use the small amplitude limit. The components of an OPTR along with the external heat and work flows are shown in Fig. 5.

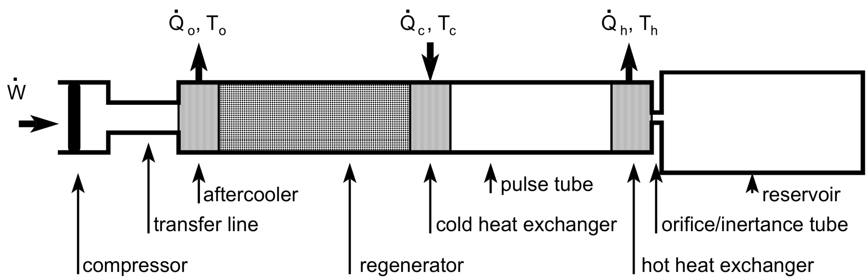

Figure 5. Sketch of the components of an OPTR. In the text, the compressor, transfer line, and after cooler are treated as a single component.

To account for the effect of the volume of the pulse tube, a dimensionless parameter, Ψ, will be used. Inside the pulse tube, the flow that contributes to the cooling is in phase with the pressure and changes occur at constant volume. The volume flow in the pulse tube at the cold heat exchanger is the same as the volume flow through the orifice at the hot heat exchanger. (This effect is the result of mass accumulation and is discussed elsewhere.[2]) Since the hot heat exchanger is at higher temperature, T_h, than the cold heat exchanger, T_c, the mass flow amplitude, $\dot{m}$, for an ideal gas decreases as T^{-1} along the pulse tube. This can be characterized within the pulse tube by a dimensionless mass flow parameter:

$$\Psi = \frac{\dot{m}(T)}{\dot{m}(T_c)} = \frac{T_c}{T}, \tag{1}$$

where $\dot{m}$ is the amplitude of the oscillating mass flow and T is the mean temperature. Both $\dot{m}$ and T are functions of position.

Then, an entropy change, Δs, with a mass flow, $\dot{m}(T_c)$, at the cold heat exchanger, results in a heat flow of $\dot{m}(T_c)T_c\Delta s$. At a higher temperature within the pulse tube, $\Psi T\Delta s = T_c\Delta s$. Thus, the same entropy change results in a heat flow at the hot heat exchanger of

$$\dot{m}(T_h)T_h\Delta s = \dot{m}(T_c)T_c\Delta s. \tag{2}$$

In the rest of the PTR, from the cold heat exchanger to the compressor, Ψ is assumed to be 1. This is equivalent to assuming that the void volumes of the regenerator and heat exchangers are small compared to the pulse tube volume. If the void volumes are included, then there is a slight gradient in Ψ in the regenerator and Fig. 6 is slightly changed.

The OPTR cycle is a four-step process. We will make the same assumptions as are usually made in textbooks. The compressor, as for the Stirling cycle discussed above, is isothermal rather than adiabatic followed by an aftercooler. To maintain the sharp corners of the diagrams, the pressure waveform in the compressor can be approximated by straight lines: two isothermal legs linear in time, and two isobaric legs. Fig. 6. shows the paths of gas elements in the compressor, cold heat exchanger, hot heat exchanger, and regenerator as solid lines. The dashed lines connect these paths. Within the regenerator, the dashed lines correspond to a constant temperature gradient.[2] Within the pulse tube, the dashed line corresponds to Eq. (1). The path of a gas element within the pulse tube will be discussed later. The process starts at A (compressor), E (cold heat exchanger), and I (hot heat exchanger). The cycle is

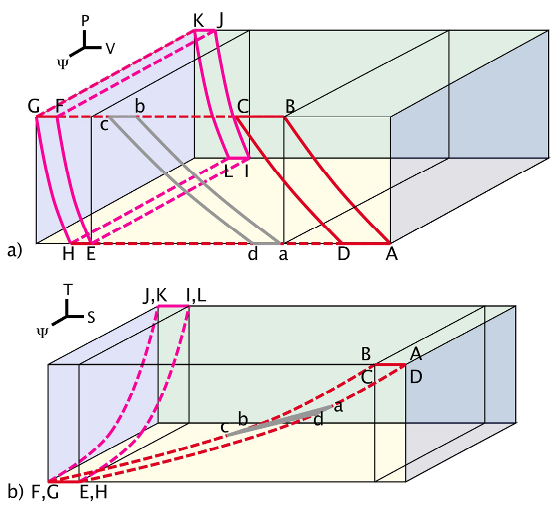

Figure 6. 3-D representation of the pulse tube cycle for an OPTR is shown in a) P-V-Ψ and b) T-S-Ψ space. The dimensionless quantity Ψ is discussed in the text. The paths of gas elements in the compressor (A-B-C-D), the cold het exchanger (E-F-G-H), and the hot heat exchanger (I-J-K-L) are shown as solid lines. The path within the regenerator (a-b-c-d) is shown grey.

1) Isothermal compression: A→B, E→F, and I→J. The temperatures are T_h, T_c, and T_h, respectively. The pressure changes throughout the system (except for the reservoir) from P_{low} to P_{high}. The changing pressure in the pulse tube requires mass to flow into it from the cold heat exchanger.
2) Continued displacement of the piston from B→C and isobaric flow through the lossless regenerator, which results in a volume flow from F→G at the cold heat exchanger and an equal volume flow through the orifice into the reservoir from J→K. The pressure remains constant at P_{high}.
3) Isothermal expansion: C→D, G→H, and K→L. The pressure changes throughout the system (except for the reservoir) from P_{high} to P_{low}.
4) Continued displacement of the piston from D→A and isobaric flow through the lossless regenerator, which results in a volume flow from H→E at the cold heat exchanger and an equal volume flows through the orifice from the reservoir from L→I. The pressure remains constant at P_{low}.

During steps 1 and 2, there is adiabatic compression and expansion, respectively, in the pulse tube. Being adiabatic, there is no direct heat transfer in the pulse tube. There is convective heat transfer at the heat exchanger to pulse tube boundary.[3] The heat transfer is into the hot heat exchanger during compression and from the cold heat exchanger during expansion. For the most part, these transfers occur within the respective heat exchangers. The heat exchangers are assumed isothermal at all times. Thus, for the purposes here, all of the external heat transfers (heat and entropy flows) are considered isothermal and occur at the heat exchangers or at the compressor. This is similar to assuming, for the Stirling cycle in Fig. 4, that the expansion is isothermal rather than a more realistic adiabatic piston coupled to an isothermal heat exchanger.

Over a cycle, the compressor does work $W = path$ (A-B-C-D) which is rejected as heat $W = Q_o = T_h S_{AB}$. At the cold heat exchanger, expansion work = *path* (E-F-G-H) is done absorbing heat $Q_c = T_c S_{EF}$, where $S_{AB} = S_{EF}$. At the orifice, work = *path* (I-J-K-L) is dissipated and rejected as heat $Q_h = \Psi T_h S_{IJ} = T_c S_{IJ}$, and $S_{IJ} = S_{EF}$.

REDUCING THE 3-D PTR DIARGAMS TO 2-D

The 3-D P-V-Ψ and T-S-Ψ diagrams of Fig. 6 can be projected onto 2-D P-V and ΨT-S diagrams – see Fig. 7. The projection superimposes *path* (I-J-K-L) onto *path* (E-F-G-H). These diagrams are similar to the P-V and T-S diagrams of the Ericsson cycle.

DISCUSSION

The above discussion shows that with no more idealization than used for the Stirling cycle, the OPTR refrigeration cycle can be reduced to an Ericsson cycle. The final step in this development was the suppression of the contribution of the pulse tube in Fig. 7. In addition, the path of gas elements within the pulse tube was not included in Figs. 6 and 7. To a certain extent,

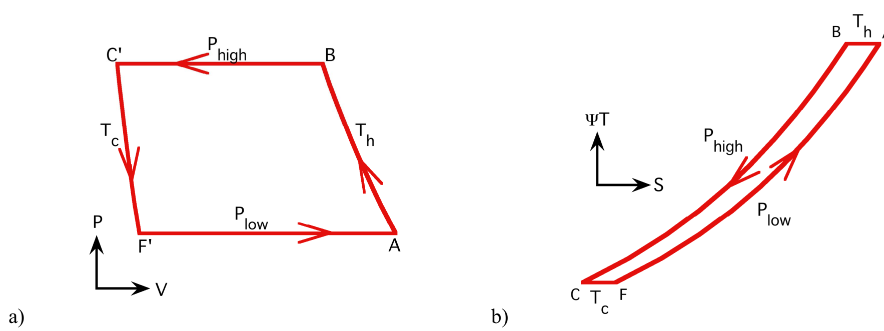

Figure 7. 2-D representation of the pulse tube cycle for an OPTR in a) P-V and b) ΨT-S space. The dimensionless quantity Ψ is discussed in the text.

this omission is justified. The function of the pulse tube, orifice, and reservoir is to control the phase shift between the mass flow and pressure at the cold heat exchanger. They are not otherwise important to the thermodynamics of an idealized system. The adiabatic expansion in the pulse tube was treated in the same manor as the near adiabatic expansion piston expansion is treated in a Stirling refrigerator. The principal thermodynamic components are those included in Fig. 7: the compressor, regenerator, and the cold heat exchanger. The pulse tube, hot heat exchanger, and orifice form a phase shifter, as does the cold piston (or displacer) in a Stirling refrigerator.

The dotted lines used to represent P, V, T, and S in the pulse tube do not reflect the actual motion of gas elements there. The ideal pulse tube is adiabatic; i.e., the entropy of a gas element within the pulse tube is constant. This is shown by the gas element *path* (m-n) in Fig. 8. Extending this over the entire pulse tube produces the envelope (M-N-O-P). There is an apparent discontinuity between this path and the paths at the heat exchangers (F-G) and (J-K). Actually, there is no discontinuity. Rather, there is a transition region near the ends of the pulse tube. Most of the heat transfer occurs within these transitions and not in the heat exchangers.[3] This is an irreversible process and not readily presented on P-V and T-S diagrams. With this limitation, presenting the OPTR cycle as an Ericsson cycle is a reasonable idealization.

SUMMARY

Generating P-V and T-S diagrams involves a certain level of idealization (simplification) of a thermodynamic cycle. This is usually straightforward in recuperative machines. The simplification allows gas elements to traverse all of the components fundamental to the thermodynamics. In regenerative machines, the idealization involves eliminating all of the void volumes and using non-physical piston motions. While this removes some of the loss mechanisms, it preserves the fundamental features in machines such as the Stirling cooler and results in P-V and T-S diagrams that have been useful tools in introductory texts.[4,5] If void volumes were included, then individual gas volumes may not traverse all of the important components. This difficulty can be overcome by using the envelope of the motions of all gas elements to define the P-V and T-S diagrams. In Pulse Tube coolers, not all of the void volumes can be eliminated, as the pulse tube is a large void volume and a critical component. Using this approach and introducing a normalized mass flow parameter, Ψ; the P-V-Ψ and T-S-Ψ diagrams for an Orifice Pulse Tube refrigerator were developed. These diagrams required simplification of the compression and expansion heat transfer at the ends of the pulse tube. A similar simplification is used (but often unstated) for Stirling refrigerators. When reduced to 2-D, the OPTR diagrams were found to be similar to those for an Ericsson cycle.

Non-realistic approximations need to generate P-V and T-S diagrams for regenerative cycles that look like P-V and T-S diagrams for recuperative cycles. For the Stirling cycle, these diagrams are often used as an introduction to recuperative coolers. The assumptions behind diagrams are not always discussed.[4,5] As such they cannot be used for anything more than

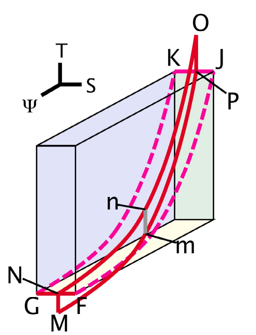

Figure 8. The T-S-Ψ diagram of the pulse tube with its two heat exchangers. Because there is less mass flow at the hot heat exchanger than at the cold heat exchanger, *path* (O-P) is longer than *path* (M-N).[3]

superficial analysis. For the pulse tube cycle, similar simplifications are used. These allow similar superficial analyses. The biggest difficulty with the P-V and T-S diagrams for regenerative machines is the non-physical piston motions required to make the diagrams similar to those of recuperative machines. Therefore, while it is possible to create P-V and T-S diagrams for regenerative cycles, they are not very meaningful, especially without a discussion of the assumptions and simplifications used. The small amplitude approximation coupled with assuming sinusoidal piston motions is a useful simplification for more extensive analyses.[2] The latter approach takes the analysis away from the emphasis of constant thermodynamic processes; e.g., isobars and isochors. Rather the emphasis is on localized dynamic processes.

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