Reduction of Self Induced Vibration in Rotary Stirling Cycle Coolers

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ABSTRACT

Cryocooler self induced vibration is a major consideration in the design of IR imaging systems, especially when mounted on stabilized platforms with gimbals. The coolers in these types of applications induce forces and moments that interfere with proper stabilization of the camera leading to image blurring and loss of pointing accuracy. This phenomenon is known as "jitter" and presents a challenge to the motion control system engineer. To overcome this problem system designers often elect to avoid the use of rotary Stirling machines and instead select linear split Stirling or pulse tube coolers. Thus, they trade off size, input power, and reliability for image quality. This paper deals with the problem of a rotary cooler's self-induced vibration and ways to eliminate or minimize its affect on imaging system performance. In this paper we will point out the vibration sources in rotary coolers and methods to reduce their magnitude. Also, we will present actual test data of a cooler with dynamically balanced crack mechanism and an external torque vibration absorber, and compare it to a standard cooler.

INTRODUCTION

First, we introduce a simplified analytical model which quantifies the effect of cooler self-induced moment vibration on system performance in terms of image stability as a function of disturbance frequency and amplitude, and of the stabilized platform mass moment of inertia. This modeling builds on the fundamental dynamics of moving mechanisms.[1,2] A method for measuring cooler self-induced vibration, both angular and linear accelerations, will be presented next. We will then identify cooler sources of vibration and analytically model and quantify them. For example, a well known source of vibration in rotary cooling engines is the crank mechanism and the reciprocating components such as the compressor and expander pistons. A second example is the motor torque output as it responds to the pressure wave loads. This torque ripple, when directly coupled to the stabilized motion control motors, can be very harmful and result in so called image jitter. Also, we will introduce passive and active methods for reducing vibration. The passive approach is based on dynamic balancing techniques, while vibration absorption is based on wave cancellation by artificially generating an equal and opposite vibration to the wave source. Also, a new multispectral self tuning vibration absorber will be discussed.

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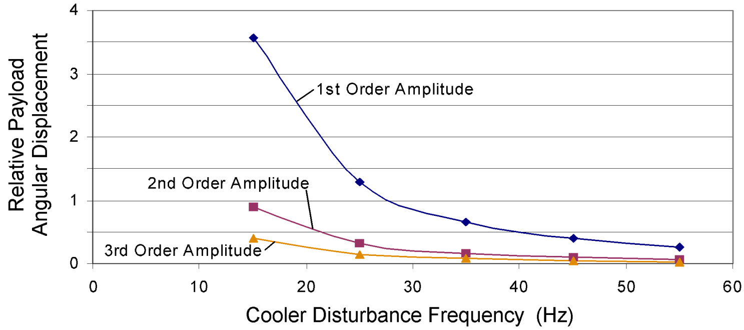


Figure 1. Angular jitter versus frequency of a stabilized mass as a response to cooler torsion vibration.

COOLER VIBRATION EFFECT ON JITTER

The response of the system to vibration input from the cooler is governed by the following equation:

$$\theta_m = \frac{A_m}{I_p \times \omega_n^2} \tag{1}$$

where:

- A_m Cooler self induced moment amplitude.
- I_p Stabilized mass moment of inertia
- ω_n Cooler induced moment frequency
- θ_m Platform angular displacement

As can be seen in Equation (1) and Figure 1, system jitter is directly proportional to the cryocooler vibration amplitude and inversely proportional to the square of the disturbance frequency. In general, coolers operating at higher frequencies are the preferred choice. Also, stabilized systems with a higher mass moment of inertia are less sensitive to cooler vibration.

ROTARY COOLER CRANK MECHANISM DYNAMIC MODEL

The majority of rotary Stirling cycle coolers operate using a two-axes crank mechanism with a 90 degree mechanical phase shift as shown in Figure 2. This mechanism generates a harmonic displacement profile resulting in significant linear accelerations of the components involved, espe-

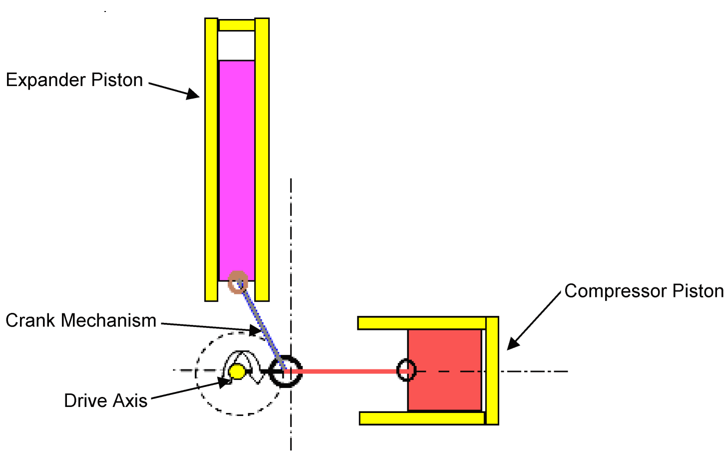


Figure 2. Rotary Stirling cycle cooler crank drive mechanism.

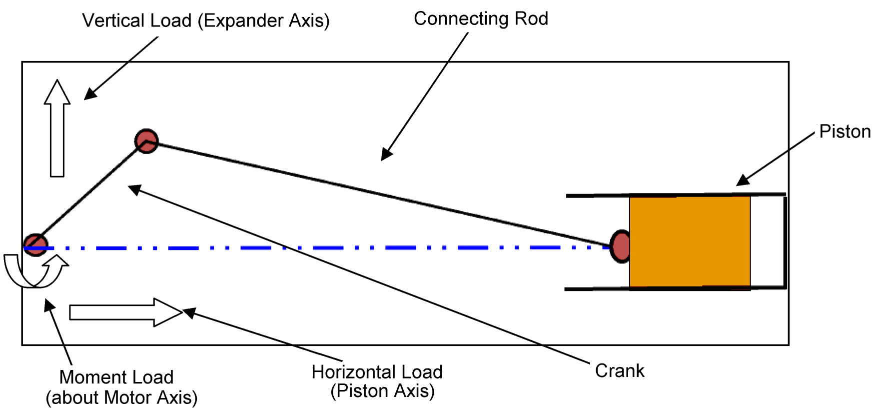


Figure 3. Typical crank mechanism of a rotary Stirling cycle engine.

cially the expander and compressor pistons. As illustrated in Figure 3, these accelerations result in external inertial dynamic loads operating on the cooler and causing movements of the cooler as a rigid body, and thus directly affect all mechanical member attached to it. These vibration loads are modeled and analyzed in the following section. Our goal is to determine their levels in terms of amplitude and frequencies as a function of the crank geometry and mass. We will then show that it is possible to significantly cancel them by passive means alone.

Compressor Crank Inertial Dynamic Load Analysis

The loads and moments at the crank axis are governed by the following equations:

$$F_{v-comp} = (M_P + M_{ROT}) \times r \times \omega^2 \cos(\omega t) + M_P \times r^2 \times \omega^2 \cos(2\omega t)/l \tag{2}$$

$$F_{h-comp} = M_{ROT} \times r \times \omega^2 \sin(\omega t) \tag{3}$$

$$M_{comp} = .5 \times M_p \times \omega^2 r^2 ((r \times \sin(\omega t))/l) - \sin(2\omega t) - (3 \times r \times .5 \sin(3\omega t))/l \tag{4}$$

where,

$F_{v\text{-}comp}$ Force along compressor piston axis
$F_{h\text{-}comp}$ Force normal compressor piston axis
M_p Piston mass (including weighted porting of the connecting rod mass)
M_{rot} Crank mass (including weighted portion of the connecting rod mass)
r Crank Radius (eccentricity)
ω Crank angular velocity
l Connecting rod length
M_{comp} Moment load about crank axis

Similarly, the loads and moments at the crank of the expander axis are governed by the following equations:

$$F_{v-\exp} = (M_P + M_{ROT}) \times r \times \omega^2 \cos(\omega t) + M_P \times r^2 \times \omega^2 \cos(2\omega t)/l \tag{5}$$

$$F_{h-\exp} = M_{ROT} \times r \times \omega^2 \sin(\omega t) \tag{6}$$

$$M_{\exp} = .5 \times M_p \times \omega^2 r^2 ((r \times \sin(\omega t))/l) - \sin(2\omega t) - (3 \times r \times .5 \sin(3\omega t))/l \tag{7}$$

where:

$F_{v\text{-}exp}$ Force along expander piston axis
$F_{h\text{-}exp}$ Force normal to expander piston
$M_{\exp}$ Moment load about crank axis.

Since the angle between the expander and compressor is 90 degrees, the total load on the crank shaft is a simple arithmetical sum for each axis and is as follows:

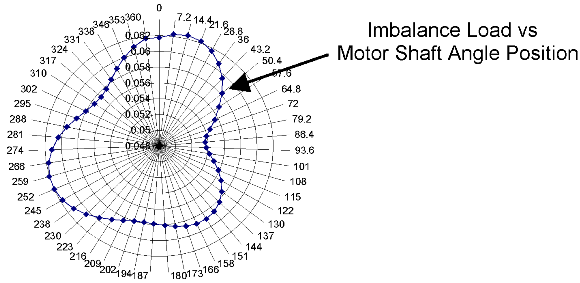


Figure 4. Polar plot of typical crank mechanism imbalance load versus motor shaft angular position.

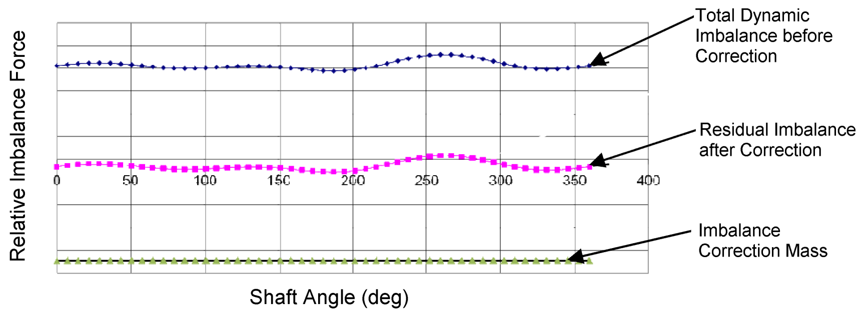


Figure 5. Typical crank mechanism imbalance force versus motor shaft angular position with and without correction mass.

Total crank horizontal load: $$F_{h-crank} = F_{v-\exp} + F_{h-comp} \tag{11}$$

Total crank vertical load: $$F_{v-crank} = F_{h-\exp} + F_{v-comp} \tag{12}$$

Total moment about crank axis: $$M_{crank} = M_{comp} + M_{\exp} \tag{13}$$

Note that in the special case where the expander and compressor are identical in geometry and mass, the combined resultant load magnitude will be independent of crank angle and can be totally cancelled by a simple dynamic balancing method.

In most instances this is not the case, and a complete correction is not possible. To determine the amount of imbalance and its angle relative to compressor axis rotation, we need to calculate the resultant load using the following expression:

Total crank load: $$F_{crank-resul\tan t} = \left(F^2{}_{v-crank} + F^2{}_{h-crank}\right)^{.5} \tag{14}$$

To calculate the angle relative to the compressor axis rotation vs. time, we use the following expression for the imbalance angular position vs. time:

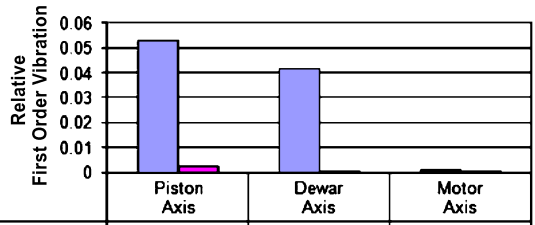
$$\alpha = \arcsin(F_{h-crank} / F_{crank-resul\tan t}) \tag{15}$$

An actual case has been analyzed and modeled, and the results are shown in Figures 4, 5 and 6.

Crank Balancing Analysis and Results Summary

As shown in Figure 6, balancing the crank mechanism can result in a close to 90% reduction in linear vibration loads at the fundamental frequency (first order) of the cooler running speed. Also, we find that reduction of the second and third order vibration is much more difficult using passive means, i.e. using the "one per revolution" dynamic balancing approach. The second and third orders of vibration are usually much smaller in amplitude and less harmful, as noted earlier.

The moment vibration, which results from inertia of the crank, was calculated and found to be very small. It can be neglected for all practical purposes, especially when it comes to micro-coolers.



	Piston Axis	Dewar Axis	Motor Axis
Standard Crank	0.05302	0.041795	0.0007301
Balanced Crank	0.002586	0.0005384	0.0001303

Figure 6. Quantitative comparison of first order self-induced vibration before and after applying imbalance correction mass.

Cooler Motor Induced Moment Vibration Analysis

As a result of the pressure wave, the cooler drive motor rotor is subjected to moment harmonic loads resulting in equal but opposite torsion loads reacting on the motor housing. These torsion loads subject the mounting surface and the platform drive motors to moment vibration. When these moments are large, it causes the entire system to vibrate, resulting in audible noise or image jitter, depending on the application. In this section we will model the load and the motor reaction, and determine the moment vibration loads in terms of frequency and amplitude.

The motor motion is governed by the following differential equation:

$$\ddot{\theta} I_m + C_m \theta + e_c \pi D_p^2 \times .25\cos(\theta) \times \left(P_1 \sin(\theta) + P_2 \cos(2\theta) + P_3 \sin(3\theta)\right) + T_{bf} + T_w + T_{hyst} = K_n \dot{\theta} + T_{stall} \quad (16)$$

where the various key terms are defined visually in Figure 7, and are listed below:

θ Angular displacement of the motor shaft
I_m Motor rotor and flywheel mass moment of inertia
C_m Viscous damping
e_c Crank eccentricity
D_p Compression piston diameter
P_1 Pressure wave first harmonic amplitude
P_2 Pressure wave second harmonic amplitude
P_3 Pressure wave third harmonic amplitude
T_{bf} Flywheel drag torque
T_{hys} Motor hysteresis drag
K_n Motor speed/torque curve slope
T_{stall} Motor stall toque at max input current.

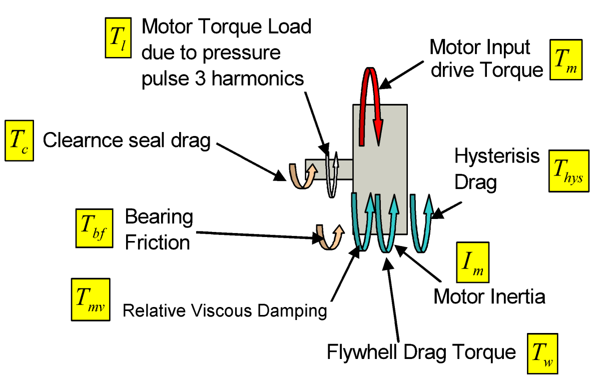


Figure 7. Definition of terms in cooler crank mechanism direct-drive motor load analysis model.

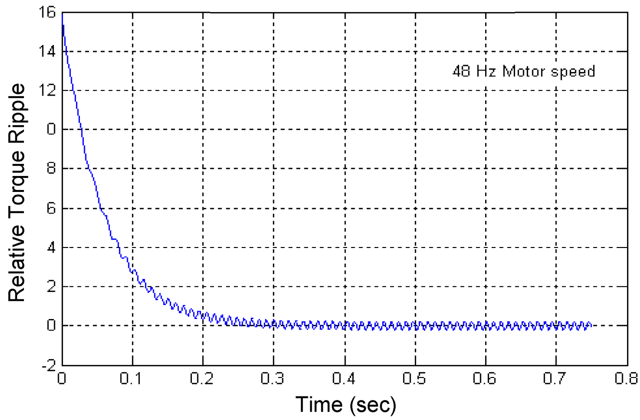


Figure 8. Predicted torque ripple vs time from cooler crank mechanism direct drive motor load analysis model.

To further simplify the above equation we define the following expressions;

$$A = \frac{C_m}{I_m} - \frac{K_n}{I_m} \tag{17}$$

$$B = e_c \pi D_p^2 \times .25 \times \frac{1}{I_m}\left(P_1 + P_2 + P_3\right) \tag{18}$$

$$C = \frac{T_{bf} + T_w + T_{hys}}{I_m} \tag{19}$$

$$K_6 = \frac{T_{stall}}{I_m} \tag{20}$$

Equation (16) in simplified form,

$$\ddot{\theta} + A\dot{\theta} + B\cos(\theta) \times \sin(\theta) + C - K_6 = 0 \tag{21}$$

Equation (21) written in MATLAB format,

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \begin{bmatrix} \dot{Z}_2 \\ K_6 - C - A \times Z_2 - B \times \cos(Z_1) \times \sin(Z_1) \end{bmatrix}$$

The transient solution for the motor torque output vs. time is shown in Figure 8. Note that the torque ripple at steady state—as expanded in Figure 9—is what the system is actually subjected to. This torque can only be cancelled by applying an equal and opposite torque at the same frequency. A method for doing so will be presented next. Also, note that the torsion vibration that is induced by the cooler on the environment is always at a frequency that is double the cooler running speed. In this specific case, the cooler speed is 48 Hz, and the torque ripple is 96 Hz.

Rotary Cooler Torsion Vibration Absorber Modeling and Analysis

As previously noted, in order to cancel or significantly reduce torsion or moment vibration, we have to generate equal and opposite vibration at the same disturbance frequency. This can be done by passive means such as a spring-mass system made of a flywheel and a set of torsion springs. There are different more sophisticated "active" approaches to solving this problem and they include:

- Cooler motor vibration electronically inserted into the cooler drive controller equal and opposite the disturbance at double the cooler motor operating frequency
- Applying same technique to the system motion control loop.

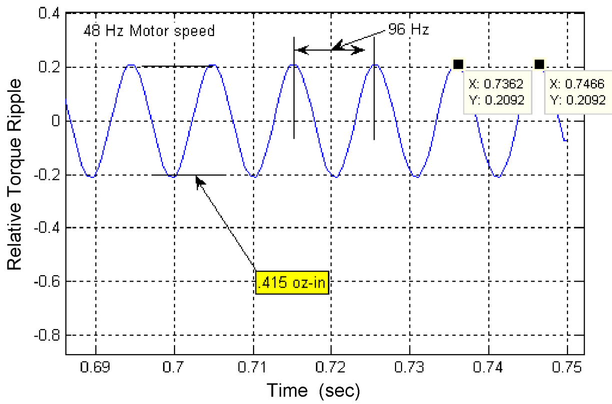


Figure 9. Torque ripple vs time at steady state.

These approaches, or similar, have been presented and published by Ricor, Ltd.,who reported excellent results using this method.[3, 4]

Here, we present a simplified low-cost approach that involves a multispectral, multi-flywheel absorber concept. The absorber system automatically turns on and off the flywheels as the cooler operating speed changes in response to ambient temperature and heat loads. The flywheels must be turned off to avoid "constructive" combinations of cooler/absorber torsion vibration modes.

VIBRATION ABSORBER MODEL AND MOTION ANALYSIS

The torsion vibration absorber is essentially a single spring-mass system with damping, and it reacts to external excitation accordingly. The external disturbance, in this case, is the cooler housing, which we analyzed and quantified previously. Figure 10 provides a schematic of the concept and its key parameters.

Analysis

The general absorber flywheel equation (absorber basic dynamic equation) is:

$$I_d \ddot{\theta}_d - K(\theta_m - \theta_d) - C\left(\dot{\theta}_m - \dot{\theta}_d\right) = 0 \qquad (22)$$

while the cooler torsion vibration-absorber excitation is given by

$$T_m = A_m \times sin(\omega_m t) \qquad (23)$$

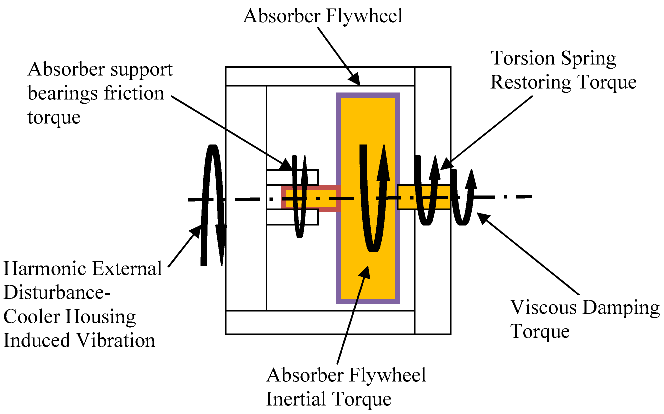


Figure 10. Absorber loading model.

This gives the cooler/absorber housing velocity

$$\dot{\theta} = -\frac{A_m}{I_{cooler} \times \omega_m} \cos(\omega_m t) \tag{24}$$

which is derived from the cooler/absorber housing angular displacement

$$\theta_m = -\frac{A_m}{I_{cooler} \times \omega_m^2} \times sin(\omega_m t) \tag{25}$$

where:

I_d	Absorber mass moment of inertia about bearing axis
$\ddot{\theta}$	Absorber angular acceleration
K	Absorber torsion spring constant
θ_m	Cooler/absorber housing angular displacement
C	Absorber flywheel viscous damping coefficient
T_m	Cooler/absorber housing induced torsion vibration (has been calculated previously)
A_m	Cooler vibration torsion vibration amplitude
ω_m	Cooler induce torque ripple frequency
I_{cooler}	Cooler mass moment of inertia about absorber axis

We substitute Equations 23, 24, and 25 into Eq. (22), and to further simplify we define the following expressions as system constants:

$$k1 = \frac{A_m}{I_{cooler}} \qquad k2 = -\frac{A_m}{I_{cooler} \times \omega_m} \qquad k3 = -\frac{A_m}{I_{cooler} \times \omega^2{}_m} \qquad k4 = K \times k3 / I_d$$

$$k5 = C \times k2 / I_d \qquad k6 = \frac{K}{I_d} \qquad k7 = \frac{C}{I_d}$$

The simplified result is the differential dynamic equation of the absorber:

$$\ddot{\theta}_d - k_4(\sin(\omega_m t)) + \frac{K}{I_d} \times \theta_d - k_5(\cos(\omega_m t)) + \frac{C}{I_d} \times \dot{\theta}_d = 0 \tag{26}$$

We convert to MATLAB solution format using the following definitions: $Z_1 = \theta_d$, $\dot{Z}_1 = \dot{\theta}_d$, $\dot{Z}_2 = \ddot{\theta}_d$, *and* $Z_2 = \dot{\theta}_d$; thus,

$$k_4(\sin(\omega_m t)) - k_6 \times Z_1 + k_5(\cos(\omega_m t)) - k_7 \times Z_2 = \ddot{\theta}_d \tag{27}$$

And, in matrix form Equation (27) is as follows:

$$\begin{pmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{pmatrix} = \begin{pmatrix} Z_2 \\ k_4(\sin(\omega_m t)) - k_6 \times Z_1 + k_5(\cos(\omega_m t)) - k_7 \times Z_2 \end{pmatrix} \tag{28}$$

The solution to equation (28) in terms of absorber flywheel acceleration, velocity, and displacements allows us to calculate absorber reaction torque using the following expressions:

$$T_D = K(\theta_m - \theta_d) + C\left(\dot{\theta}_m - \dot{\theta}_d\right) \tag{29}$$

where T_D is the absorber reaction torque.

Next calculated, is the sum of the cooler induced torque T_m and the absorber reaction torque T_d:

$$T_b = T_m + T_d \tag{30}$$

where T_b is the residual vibration torque induced or "exported" to the system.

Figure 11 presents a representative plot showing the theoretical results of the torque vibration on a system with an absorber.

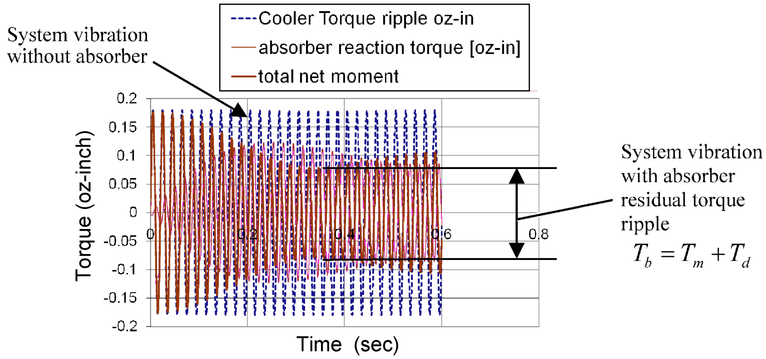


Figure 11. Analytical prediction of torque vibration with and without a vibration absorber.

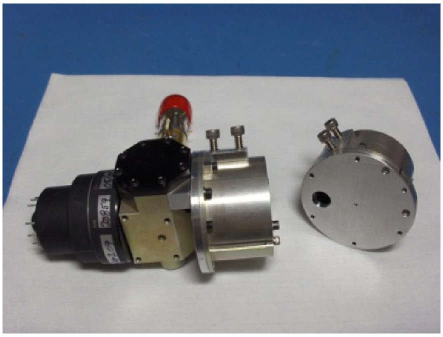


Figure 12. FLIR cooler with dual flywheel torque ripple vibration absorber.

Figure 13. FLIR mc-3 cooler with angular accelerometer in line with motor axis.

VIBRATION TORQUE ABSORBER TEST RESULTS

A few prototypes were built and tested using a vibration spectrum analyzer, computer, angular accelerometer, and cooler drive controller. We used the MC-3 cooler manufactured by FLIR Systems Inc., Boston facility shown in Figure 12.[5] The cooler was suspended in the air by a soft rubber cord ~3.0 feet long as shown in Figure 13. The cooler rpm was carefully controlled during the test, and the results are shown in Figure 14.

The cooler was equipped with a dual flywheel absorber designed to operate at two different frequencies, and could be turned on and off during the test. When the flywheels were both off, the absorber acted as a rigid body or "dead weight."

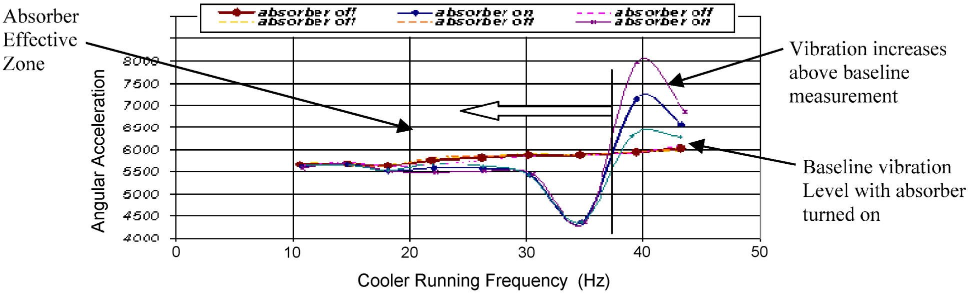


Figure 14. Vibration absorber test results with single flywheel and no deactivation feature. Note the increase of vibration level above 35 Hz.

Table 1. Multispectral absorber peak results at 14 and 20 Hz cooler frequency.

Absorber Natural Freq [HZ]	Cooler Operating Freq [Hz]	Vibration Reduction at Operating Freq
35	20	85%
25	14	73%

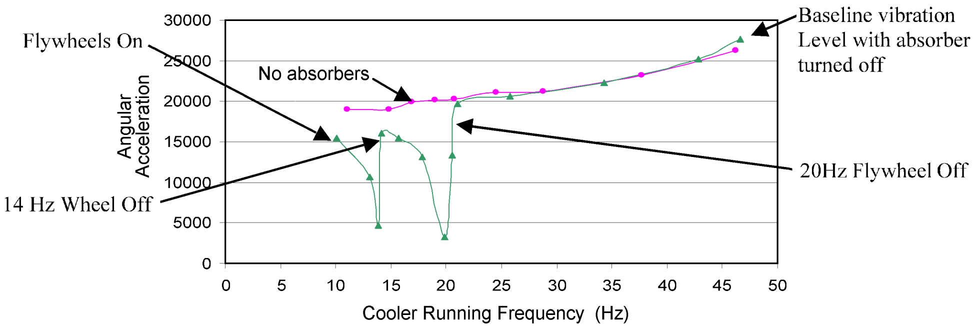


Figure 15. Multispectral absorber with flywheel activation, note the lack of a vibration spike above the baseline measurement.

RESULTS SUMMARY

A significant reduction in cooler self induced torque vibration can be achieved using a mechanical multispectral absorber. The results show a reduction of 80% average at the peak frequency as shown in Table 1. Also, it is clearly shown that the vibration reduction curve (Figure 15) can be further improved by adding more flywheels.

CONCLUSIONS

A rotary Stirling cycle cooler's self-induce vibration can be significantly reduced, and more specifically:

- Linear self induced vibration resulting from crank mechanism and motor can be almost totally eliminated at the first order of cooler speed by proper dynamic balancing techniques.
- Moment/torque vibration can only be reduced by angular vibration absorbers (active or passive); one means is multiple flywheels with auto switching.
- With a stabilized platform, the effect of second and third order vibration on system performance is not as significant as that of first order.

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