Study of Phase Shifting Mechanism of Inertance Tube at Low Temperatures

B. Wang[1], L.Y. Wang[1], J.K. Zhu[1], J. Chen[1], Z.P. Li[1], Z.H. Gan[1,2], L.M. Qiu[1,2]

[1]Cryogenics Lab, Institute of Refrigeration and Cryogenics, Zhejiang University, Hangzhou, China, 310027, P.R. China

[2]The State Key Laboratory of Clean Energy Utilization, Zhejiang University, Hangzhou 310027, P.R. China

ABSTRACT

The inertance tube is widely used as phase shifter at ambient temperature in high frequency pulse tube cryocoolers, but it is difficult to obtain a comparatively large phase shift when there is a smaller acoustic power at the inertance tube inlet. However, inertance tube at cryogenic temperature, so called cold inertance tube, can provide a solution to this difficulty. Based on the transmission line theory by introducing real-gas properties of helium, the operating pressure, pressure ratio and reservoir volume, the performance of inertance tube at different temperatures are studied. The results indicate that the cold inertance tube can provide a better phase angle even with a smaller acoustic power. The phase angle increases with the decrease of temperature and pressure ratio. For the convenience of design, the charts of inertance tube working at different temperatures and dimensions are also provided in this paper.

INTRODUCTION

Pulse tube cryocoolers (PTCs) with no moving parts at the cold end have been studied intensively for many years for their advantages such as compact structure, high reliability and long life. Radebaugh's[1,2] researches indicate the losses in regenerator are minimized when the amplitude of the mass flow is minimized for a given acoustic power, which requires that the mass flow and pressure are in phase near the midpoint of the regenerator. To achieve the optimal phase relation in a PTC requires that the mass flow lags the pressure by about 30° at the cold end of the regenerator. The orifice[3], double inlet valve[4] and inertance tube[5,6] are the three main phase shifters of PTCs. Among the three phase shifters, the inertance tubes, which rely on the inertia of the oscillating working gas in a long thin tube to produce a proper phase shift that the pressure oscillation leads the mass flow, make it possible to keep the pressure and mass flow in phase at the midpoint of the regenerator. The inertance tubes can also avoid the DC flow, making the performance of cryocoolers more stable, and it is now widely used in the high frequency PTCs.

In ideal cases, the cooling power at the cold end of the pulse tube is equal to the acoustic power at the hot end of the pulse tube[7], so inertance tubes can provide enough phase at large cooling powers for high frequency PTCs[8-10]. However, it is difficult for an inertance tube to produce a comparatively large phase shift in small cooling power PTCs due to the small acoustic power at the inlet of the inertance tube.

Cryocoolers 17, edited by S.D. Miller and R.G. Ross, Jr.

The inertia of the inertance tube increases with the density of the working gas, which enhances the phase shift of the inertance tube. An inertance tube working at low temperatures, a so called cold inertance tube, can provide a solution to the difficulty of a small phase shift at small cooling powers for high frequency PTCs[11-14]. In this paper, a calculation analysis method based on the transmission line theory and including real-gas properties for simulating a cold inertance tube is introduced. The influences of operating pressure, pressure ratio and reservoir volume on the performance of inertance tubes at different temperatures are studied. The results indicate that the cold inertance tube can provide a larger phase angle even with a smaller acoustic power. The phase angle increases with the decrease of the temperature and pressure ratio. For the convenience of design, the charts of inertance tube working at different temperatures and dimensions are also provided.

THEORETICAL ANALYSES

Transmission Line Theory

Based on the fluid network theory[15], a transmission line theory model is adopted to calculate the phase shift of inertance tubes. The reliability of this model has been experimentally verified by Radebaugh[16,17].

Figure 1 shows the schematic of an inertance tube with a length of L and a reservoir. According to the transmission line theory model, the complex impedance of a terminated transmission line of length L is given by:

$$Z(x,D) = Z_0(D)\frac{Z_r + Z_0(D)\tanh\left[(\zeta(D)(L-x)\right]}{Z_0(D) + Z_r\tanh\left[(\zeta(D)(L-x)\right]} \tag{1}$$

Where $Z_0(D)$ is the complex characteristic impedance of inertance tube, $\zeta(D)$ is the complex propagation function or wave number of the inertance tube, and Z_r is the impedance of the reservoir, and the expressions for them are:

$$Z_0(D) = \sqrt{\frac{r(D) + i\omega l(D)}{i\omega c(D)}} \tag{2}$$

$$\zeta(D) = \sqrt{[r(D) + i\omega l(D)]\cdot i\omega c(D)} \tag{3}$$

The resistance per unit length is: $$r(D) = (2/\pi)\frac{32 fr\rho U}{\pi^2 D^5} \tag{4}$$

The inertance per unit length is: $$l(D) = \frac{4\rho}{\pi D^2} \tag{5}$$

The compliance per unit length is: $$c(D) = \frac{\pi D^2}{4\kappa\rho RT} \tag{6}$$

The compliance of reservoir is: $$C_r = \frac{V_r}{\kappa\rho RT} \tag{7}$$

The impedance of reservoir is: $$Z_r = \frac{1}{i\omega C_r} \tag{8}$$

Where D is the diameter of the inertance tube, U is the amplitude of the average volume flow rate of the inertance tube, k is the adiabatic exponent. As helium can be regarded as an ideal gas, k is equal

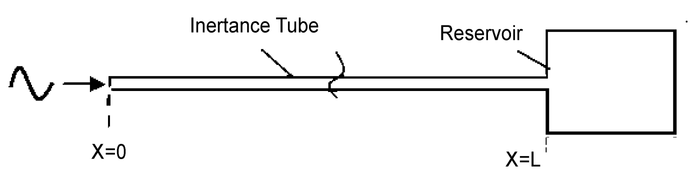

Figure 1. Schematic of inertance tube and reservoir

to the specific heat ratio, which is 1.667, but the actual process of the gas in the tube lies between isothermal and adiabatic process, so we can take k to have any value between 1 and 1.667. R is the gas constant, T is the temperature; ρ is the density of gas while *fr* is the Fanning friction factor, V_r is the volume of reservoir. The incorporation of the friction factor in Eq. (4) allows the use of this model in both the laminar flow and turbulent flow regions.

Acoustic power at the inlet of inertance tube is expressed as:

$$W = \frac{1}{2} p_A U_A \cos\theta = \frac{p_A^{\ 2} \cos\theta}{2\left|Z(0,D)\right|} \tag{9}$$

Where p_A and U_A are the amplitude of pressure and the amplitude of volume flow rate at the inlet of inertance tube respectively, and q is the phase angle that the volume flow rate lags the pressure at the inlet of the inertance tube, and

$$\theta = \frac{180^\circ}{\pi} \arg Z(0,D) \tag{10}$$

Temperature Dependence of Phase Shift of Inertance Tube

Based on the transmission line theory above, a typical operating condition is chosen and the temperature dependence of the impedance amplitude, and the phase angle at the inlet of the inertance tube with different lengths are calculated, the results are shown in Figure 2.

Figure 2 shows the phase angle of the inertance tube with a length of 1 m increases significantly as the temperature decreases, and according to Eq. (7). The temperature decrease makes the impedance amplitude $|Z(0,D)|$ increase and the cos θ decrease, and the acoustic power at the inlet of the inertance tube also decreases, which means a large phase shift can be obtained by a cold inertance tube even with a smaller acoustic power at the inlet of the inertance tube. However, not all cold inertance tubes have the ability of enlarging the phase angle and increasing the acoustic power at low temperatures. The phase angle produced by the inertance tube with a length of 4 m, as shown in Figure 2, decreases as the temperature decreases, and it reverses at temperature below 200K, which will worsen the pulse tube cryocooler performance. It is crucial to investigate the influence of geometric parameters and operating parameters on the cold inertance tube performance and then decide whether the conditions of the cold inertance tube could be used to improve the PTCs performance.

Equation Modifications at Non-Ideal Gas Region

Figure 3 shows the temperature dependence of the compressibility factor of helium-4. Helium-4 shows an obvious departure from ideal gas at temperature below 20K. The ideal gas equation is not applicable any more at this temperature region. Therefore modifications are required at non-ideal gas region, and the compressibility factor z is introduced to modify the gas equation, and the expression of $p=z\rho RT$ is obtained.

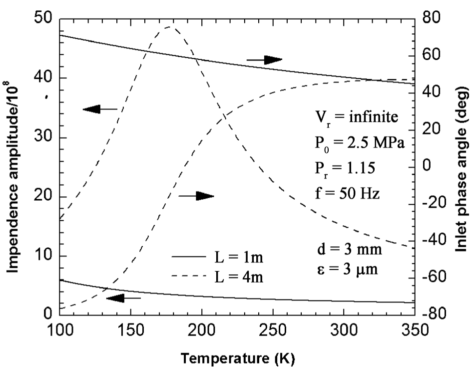

Figure 2. Temperature dependence of impedance amplitude and phase angle at inlet of inertance tubes with different lengths

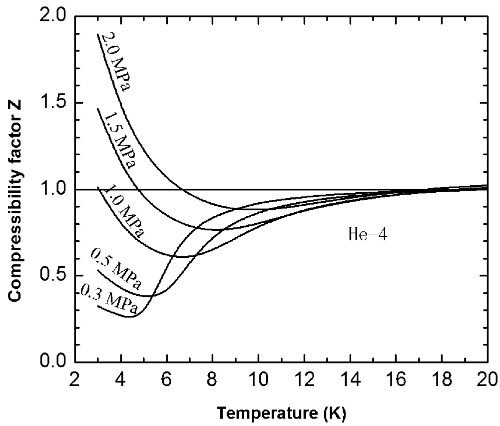

Figure 3. Temperature dependence of the compressibility factor

When considering the non-ideal gas effect, Eq. (6) and Eq. (7) become:

$$c(D) = \frac{\pi D^2}{4\kappa z\rho RT} \tag{11}$$

$$C_r = \frac{V_r}{\kappa z\rho RT} \tag{12}$$

For an ideal gas, the adiabatic exponent κ equals the specific heat ratio, but when the temperature decreases below 20K, the specific heat ratio cannot be used any more, and the adiabatic process can be viewed as an isentropic process, so the adiabatic exponent can be replaced by isentropic exponent in the calculation. As shown in Figure 4[18], the isentropic exponent is different from the specific heat ratio when temperature is below 20K; therefore the isentropic exponent of the real gas is used in the calculation of fluid compliance in adiabatic processes. When the process is considered as the isothermal process, κ=1 is also suitable in non-ideal gas region.

The flow pattern in inertance tubes is turbulent flow at ambient temperature[19]. The density of helium-4 increases and the viscosity decreases as the temperature decreases. According to the expression for the Reynolds number, $\mathrm{Re} = \frac{\rho|\dot{u}|D}{A\mu}$, Re increases significantly as the temperature decreases, so the flow pattern is still turbulent flow, and Eq. (4) can still be used.

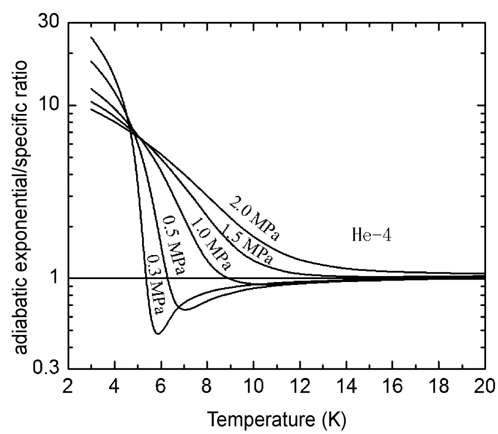

Figure 4. Temperature dependence of the ratio of isentropic exponent and specific heat ratio [18]

From the expressions for the viscous boundary layer $\delta_v = \sqrt{\frac{2\mu}{\omega\rho}}$ and thermal boundary layer $\delta_k = \sqrt{\frac{2\lambda}{\omega\rho C_p}}$, $\delta\varpi\ \alpha\nu\delta\ \delta\kappa$ decrease as the temperature decreases. The decrease of δ_k means the actual flow section area in the tube is close to inner area of the tube, and the inertance can be expressed as Eq. (5). The decrease of δ_k indicates the heat transfer between fluid and tube wall decreases in the oscillation process, and the flow process tends to be adiabatic, Eq. (11) is used and κ takes the value of isentropic exponent.

Two or more stages are needed in high frequency PTCs to achieve low temperatures, and the cooling power of high frequency PTCs is usually very small, thus the acoustic power at the inlet of inertance tube is also small. From the above discussion, the first stage cooling power can be used to cool down the reservoir and inertance tube, which can provide a larger phase angle and improve the performance of PTCs[11-14]. As the first stage temperature does not change with time at steady state, the process should be regarded as an isothermal process when Eq. (12) is used to calculate the compliance of reservoir, and k takes the value of 1.

The above modifications of the equation in the non-ideal region are also suitable for the ideal gas region, and a joint model can be obtained by combining Eq. (9) and Eq. (10). The model can be used to calculate the phase angle and the inlet acoustic power of the inertance tube at different temperatures. An inertance tube model has been established based on LabVIEW, and the influences of temperature, pressure, pressure ratio on the performance of inertance tube have been calculated.

DESIGN AND CALCULATION OF COLD INERTANCE TUBES

As high frequency PTCs working at low temperatures are very sensitive to frequency, the frequency and internal surface roughness of inertance tube ε are set to be 30 Hz and 3 μm, respectively in order to make the discussion easier.

Effect of Temperature on the Phase Shift of Inertance Tube

Figures 5~7 show the relation between the inlet acoustic power and phase angle produced by inertance tubes with different lengths at different temperatures (300K, 80K, and 20K). The charging pressure is 1.0 MPa, and the pressure ratio is 1.15. The longitudinal coordinate is the phase angle that mass flow lags pressure oscillation. From the figures, the phase angle is smaller than 60° (which makes the performance of regenerator best) when the inlet acoustic power is below 10W at 300K, and the largest phase angle of the inertance tube at the same inlet acoustic power increases as the temperature decreases, especially at small inlet acoustic power condition. The benefit of cold inertance tubes is obvious. For example, the largest phase angle of an inertance tube at 300K with an inlet acoustic power 0.5W is just 6° (as shown in Fig 5); the result at 80K is 28° (as shown in Fig 5);

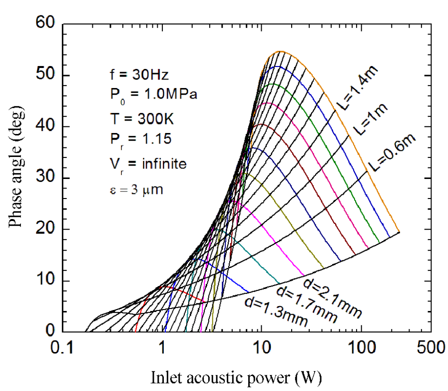

Figure 5. Chart of inertance tubes at 300K and Pr = 1.15

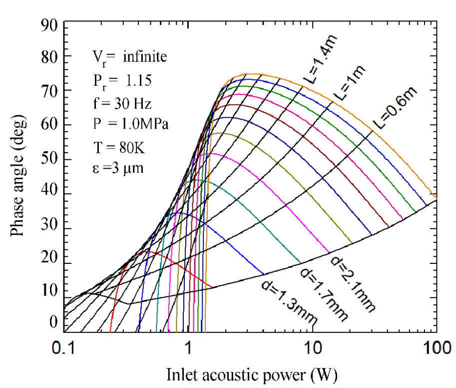

Figure 6. Chart of inertance tubes at 80K and Pr = 1.15

while the phase angle at 20K is 84° (as shown in fig 5), which is larger than 60°, and gives the best performance.

When the inlet acoustic power and the required phase angle are known, it is convenient to get the diameter and the length of the inertance tube according to the results presented in Figs. 5~7.

Effect of Pressure Ratio on the Phase Shift of Inertance Tube

Figures 7~9 represent the phase shift of inertance tubes with different pressure ratios (1.10, 1.15, and 1.20) respectively at 20K and 1.0 MPa. The figures indicate that the phase angle of inertance tubes decreases as the pressure ratio increases, but the inlet acoustic power increases as the pressure ratio increases, and the phase angle decreases slowly as the diameter of the inertance tube increases while the inlet acoustic power increases rapidly, which means an increase of pressure ratio will have the opposite effect on improving the phase shift of inertance tubes. However, the PTCs performance is enhanced as the pressure ratio increases, so the influence of the pressure ratio should be taken into account, and the design of inertance tubes should be carried out on the basis of ensuring the PTCs performance. The pressure ratio at the inlet of inertance tubes should be determined first, and it is better to set the actual inlet pressure ratio at the design point.

Based on the discussion above, a systematic comparison of the phase shift limit of inertance tubes at different pressure ratios and temperatures is carried out, and shown in Fig. 9. Figure 10 shows the phase shift limit at a given pressure ratio increases significantly when the temperature decreases; but the phase shift limit at a given temperature decreases significantly as the pressure ratio increases. The phase shift limits at 20K and 80K with a pressure ratio of 1.3 are almost equal to that of 80K and 300K with a pressure ratio of 1.5 respectively, indicating that the decrease of phase angle due to the increase of pressure ratio from 1.3 to 1.5, which is almost equal to the increase of phase angle along with the decrease of temperature from 80K to 20K. This also indicates that pressure ratio and temperature have a close relationship with inertance tube phase shifting, so the effects of the two parameters should be considered in the design of inertance tube to make sure the optimal phase angle is smaller than the phase shift limit of inertance tube.

Effect of Reservoir Volume on the Phase Shift of Inertance Tube

The reservoir of a cold inertance tube is also placed at low temperature. The gas-supply and the difficulty of keeping the working pressure steady are directly determined by the volume of the reservoir. A larger reservoir needs more gas-supply and makes it more difficult to stabilize the working pressure, so it is essential to investigate the effect of reservoir volume on the phase shift ability of inertance tubes. Under the condition of satisfying the requirement of phase shift, the volume of the reservoir should be as small as possible to make the structure more compact.

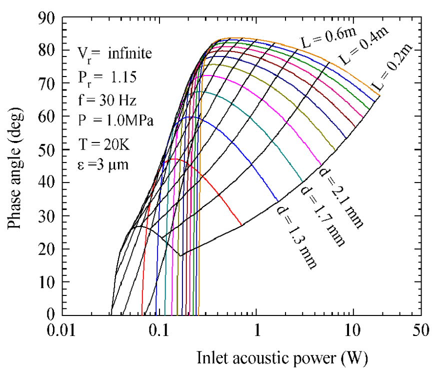

Figure 7. Chart of inertance tubes for 20K and Pr = 1.15

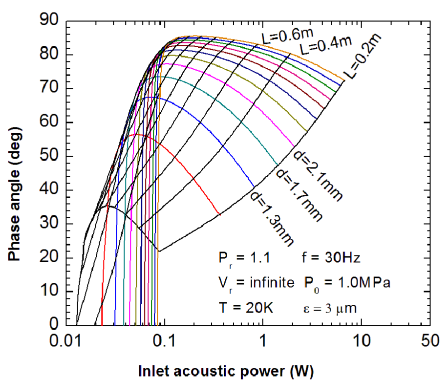

Figure 8. Chart of inertance tubes for 20K and Pr = 1.10

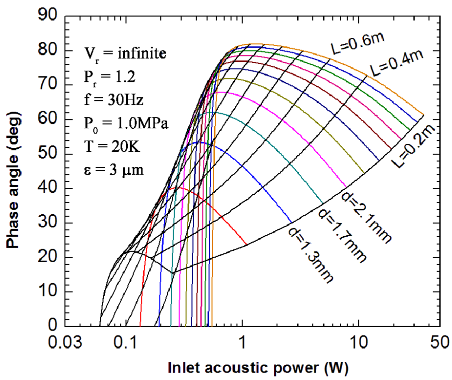

Figure 9. Chart of inertance tubes for 20K and Pr = 1.20

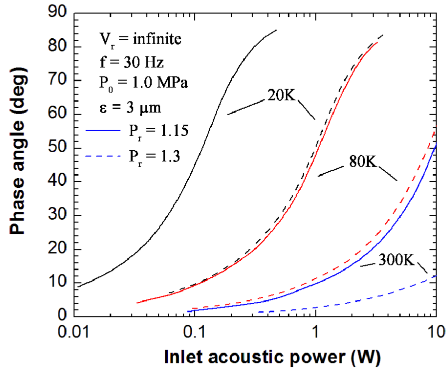

Figure 10. Phase shift limit of inertance tubes for different temperature and pressure ratio

Effects of reservoir volume on phase angle and inlet acoustic power of inertance tubes for 20K at different operating pressures and pressure ratios are shown in Fig. 11 and Fig. 12. The figures show that the phase angle of inertance tubes becomes larger with the increase of reservoir volume, but the inlet acoustic power of inertance tubes is reduced as the reservoir volume increases. When the reservoir volume is small, an increase in reservoir volume makes the phase shift of inertance tubes improve significantly, but an increase in phase shift of inertance tubes tends to be gentle when the reservoir volume reaches a certain value, which means further increase in the reservoir volume will not improve the phase shift of inertance tubes. The phase shift of an inertance tube with a 100 cm^3 reservoir has no significant difference at different pressures, as shown in Fig. 11, so the reservoir volume is set at 100 cm^3, and it is independent of operating pressure. When the reservoir volume is fixed, the phase angle increases with the operating pressure, and the inlet acoustic power of inertance tubes decreases with the increase of the operating pressure. The change is not significant, and this means the operating pressure has little influence on the phase shift of inertance tubes. Figure 12 shows the pressure ratio has significant effect on the phase shift of an inertance tube, which is in accordance with the previous analyses, but the ideal volume is 100 cm^3, regardless of pressure ratios, which means the choice of reservoir volume can be independent of pressure ratio.

CONCLUSIONS

This paper focuses on the issue that ambient temperature inertance tube cannot provide enough phase shifts in small cooling power high frequency PTCs. This paper investigates the

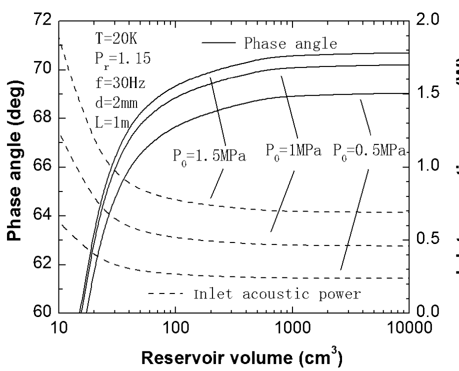

Figure 11. Phase shift limit of inertance tubes for different temperature and pressure ratio

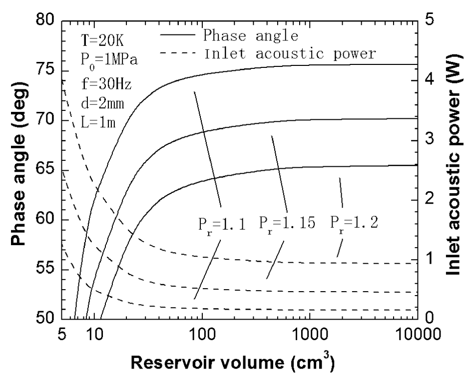

Figure 12. Effect of reservoir volume on phase shift of inlet of inertance tube

mechanism of cold inertance tubes and establishes a calculation method for cold inertance tubes. Effects of different temperatures, pressure and pressure ratio on the phase shift of inertance tubes are discussed. The results show the phase shift of inertance tubes increases as the temperature and the inlet pressure ratio of inertance tubes decreases, and the optimal reservoir volume is independent of the pressure and the pressure ratio. The charts of phase angle and the inlet acoustic power for different temperatures, lengths and diameters are also presented, which can provide guidance for the design of cold inertance tube in small cooling power high frequency PTCs.

ACKNOWLEDGMENT

This work is financially supported by National Natural Science Foundation of China (No.51176165) and partially supported by the Fundamental Research Funds for the Central Universities (2011QNA4008).

REFERENCES

1. Radebaugh, R, Herrman, S., "Refrigeration efficiency of pulse tube refrigerator," Proceedings of the Second Interagency Meeting on Cryocoolers and the Fourth International Cryocooler Conference, David Taylor Naval Ship Research and Development Center, Bethesda and Annapolis MD, July 1987, pp.119-133.
2. Radebaugh, R., "Development of the pulse tube refrigerator as an efficient and reliable cryocooler," *Proceedings of the Institute of refrigeration, Vol. 96,* London (1999–2000), pp.11–31.
3. Radebaugh, R., Zimmerman, J., Smith, D.R., et al., "A comparison of three types of pulse tube refrigerators-New methods for reaching 60 K," *Advances in Cryogenic Engineering Vol 31*, Plenum Press, Cambridge (1986), pp.779–789.
4. Zhu, S.W., Wu, P.Y., Chen, Z.Q., "Double inlet pulse tube refrigerators: an important improvement," *Cryogenics* vol 30, Issue 6 (June 1990), pp.514-520.
5. Kanao, K., Watanabe, N., Kanazawa, Y., "A miniature pulse tube refrigerator for temperature below 100K," *Cryogenics,* vol 34, Supplement 1 (1994), pp.167-170.
6. Gardner, D.L., Swift, G.W., "Use of inertance tube in orifice pulse tube refrigerators," *Cryogenics*, Vol.37, Issue 2 (1997), pp:117-121.
7. de Waele, A.T.A.M., "Basic Operation of Cryocoolers and Related Thermal Machines," Journal *of Low Temperature Physics*, Vol 164, Numbers 5-6 (2011), pp.179-236.
8. Zia, J.H., "A Commercial Pulse Tube Crycooler with 200W Refrigeration at 80K," *Cryocoolers 13*, Kluwer Academic/Plenum Publishers, New York (2005), pp.165-171.
9. Tanchon, J., Trollier, T., Ravex, A., et al., "Prototyping a Large Capacity High Frequency Pulse Tube Crycooler," *Cryocoolers 14*, ICC Press, Boulder, CO (2007), pp.133-139.
10. Dietrich, M., Thummes, G., "Two-stage high frequency pulse tube cooler for refrigeration at 25 K," *Cryogenics*, Vol.50, Issue 4 (April 2010), pp:281-286.
11. Gan, Z.H., Fan, B.Y., Wu, Y.Z., et al., "A two-stage Stirling-type pulse tube cryocooler with a cold inertance tube," *Cryogenics*, Vol.50, Issue 6-7 (June-July 2010), pp: 426-431.
12. Gan, Z.H., Li, Z.P., Qiu, L.M., et al., "Design and preliminary experimental investigation of a 4 K Stirling type Pulse Tube Cryocooler with precooling," *Journal of Zhejiang Univ Science A*, Vol 10, Number 9 (2009), pp.1277-1284.
13. Qiu, L.M., Cao, Q.,Zhi, X.Q., et al., "A three-stage Stirling pulse tube cryocooler operating below the critical point," *Cryogenics*, Vol.51, Issue 10 (October 2011), pp:609-612.
14. Qiu, L.M., Caoa, Q.,Zhia, X.Q., Hana, L.,Gana, Z.H., Yua, Y.B., Liua, Y., Zhang, X.J, "Operating characteristics of a three-stage Stirling pulse tube cryocooler operating around 5 K," *Cryogenics, Vol. 52, Issues 7-9* (July-September 2012), pp. 382-388.
15. Luo, Z.C., "*Fluid Network Theory*," China Machine Press, Beijing (1988), pp.122-144. (In Chinese).
16. Radebaugh, R., Lewis, M., Luo, E.C., et al. "Inertance tube optimization for pulse tube refrigerators," *Adv. in Cryogenic Engineering*, Vol. 51, Amer. Institute of Physics, Melville, NY (2006), pp.59-67.

17. Vanapalli, S., Lewis, M., Gan, Z., Radebaugh, R., “120 Hz pulse tube cryocooler for fast cooldown to 50 K,” *Applied Physics Letters,* Vol 90, Issue: 7 (2007), pp.072504-3.

18. Lemmon, E.W., McLinden, M.O., Hube,r M.L., “Reference Fluid Thermodynamic and Transport Properties, NIST Standard Reference Database 23 - REFPROP,” Version 7.1. Physical and Chemical Properties Division.2006.

19. Hu, J.Y., Luo, E.C., Dai, W., “Investigation on phase shifters of pulse tube coolers Part 3: Investigation on the inertance tubes with and without reservoirs,” *Cryogenics & Superconductor* 36, 10(2008), pp.9-13.(In Chinese).