Numerical Simulation of 4K GM Cryocooler

M. Y. Xu, T. Morie

Technology Research Center, Sumitomo Heavy Industries, Ltd.
Tokyo 188-8585 Japan

ABSTRACT

A numerical simulation program has been developed to predict the theoretical cooling capacity and losses in regenerators, heat exchangers, etc.

The clearance between the cylinder and the displacer is modeled as a pulse tube in a pulse tube cryocooler. Usually, a slip seal is installed at the warm end of the first stage. Therefore, the clearance is calculated as a pulse tube in a basic pulse tube cryocooler. On the other hand, there is no seal on the warm end of the second stage. Therefore, the clearance is calculated as a pulse tube in a basic double inlet pulse tube cryocooler. The spiral groove on the second stage displacer is also calculated as a pulse tube in a basic double inlet pulse tube cryocooler. The cooling effect in the pulse tube can be calculated by assuming that the pulse tube is adiabatic. In other words, there is no heat exchange between gas and the walls of the cylinder and the displacer. Since the volume of both the clearance and the spiral groove is so small that the cooling effect in the pulse tube is small.

The simulation results of the pressure and volume variation, and the P-V power in the second stage expansion volume are compared with the measurement results obtained from an SHI 4K GM cryocooler. The simulation results are consistent with the measurement results.

INTRODUCTION

Most of the simulation methods related to a two-stage Gifford-McMahon (GM) cryocooler are used to calculate the performance of the second stage by giving the operating parameters of the first stage as a boundary conditions[1, 2]. In the simulation program for a 4K GM cryocooler, only the real gas property effect, the pressure drop loss and the incomplete heat exchange loss in the regenerator and heat exchange are considered. A simulation method to calculate both the first and second stage, simultaneously, is reported by Xu, et al.[3] In addition, except for the shuttle loss and the radiation loss, all other losses, including the loss caused by the real gas property, the regenerator loss, the pumping loss, the heat conduction loss, and the leak loss through the clearance and the spiral groove, can be calculated using the simulation program, simultaneously. Meanwhile, the cooling effect generated by the expansion of the gas in the clearance and the spiral groove can also be calculated using this simulation program. The NIST helium database for real gas properties of helium is incorporated within the simulation program. The simulation results are compared with the results measured in an SHI conventional 1W 4K GM cryocooler.

The details of the simulation method, the comparison of the simulation to the measurement results, and the analysis of the simulation results will be reported in this paper.

SIMULATION

Physical Model

As shown in Figure 1, a physical model of a two-stage GM cryocooler has been designed for calculating both the first stage and the second stage, simultaneously. Gas flow is supplied from and returned to the compressor through a rotary valve. The first stage regenerator is packed with #150 phosphorous bronze screens. The clearance between the displacer and the cylinder at the cold end is modeled as a heat exchanger at both the first and the second stage. The clearance between the first stage displacer and the cylinder is calculated as a pulse tube of a basic pulse tube cryocooler. The clearance between the second stage displacer and the cylinder, and the spiral groove are calculated as a pulse tube of a basic double inlet pulse tube cryocooler.

In order to simplify the problem, the following assumptions are made,

1. The radial temperature graduations are negligible,
2. In the axial direction, the thermal conduction loss through helium gas is negligible,
3. The temperature of the heat exchanger wall is constant.

Simulation Method

The P-V power, the real gas effect, the regenerator loss, the pumping loss, the pulse tube cooling effect and the thermal conduction loss can be calculated using the simulation program.

It is difficult to distinguish the real gas effect from the regenerator loss because both of them come out as a part of the enthalpy flow at the cold end of the regenerator. In order to clarify, the real gas effect is estimated by the method suggested by Xu, et. al.[4] Then, the regenerator loss at the second stage can be estimated by the enthalpy flow, excluding the real gas effect. For the first stage, when calculating the regenerator loss, the enthalpy flow entering the second stage regenerator should also be excluded from the enthalpy flow at the cold end of the first stage regenerator.

It is also difficult to distinguish the pulse tube cooling effect from the pumping loss and the heat conduction loss because all of them come out as a part of the enthalpy flow at the cold end

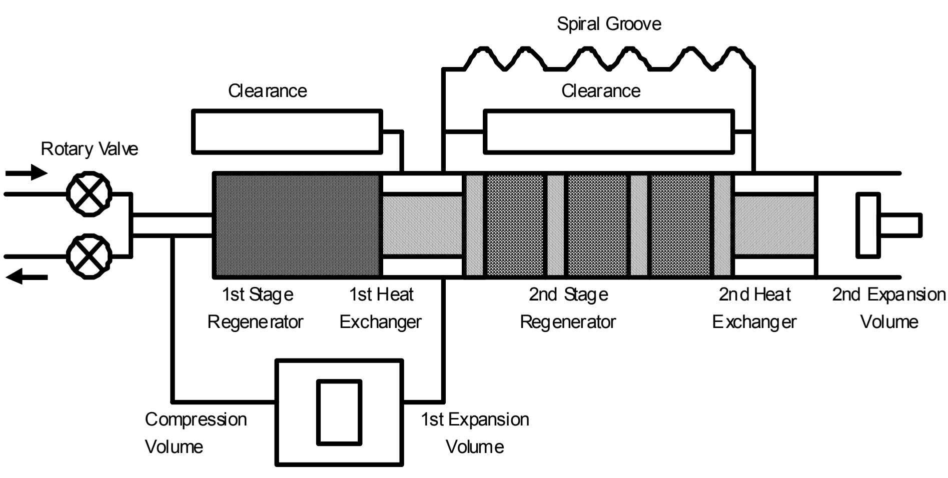

Figure 1. Schematic diagram of a two-stage GM cryocooler.

of the clearance and spiral groove. If the heat conduction loss is neglected, while the heat exchange between gas and the walls of the cylinder and displacer is considered, the pumping loss combined with the pulse tube cooling effect will be equal to the enthalpy flow at the cold end. Furthermore, if the heat exchange between gas and the walls of the cylinder and displacer is neglected, the pulse tube cooling effect will be equal to the enthalpy flow at the cold end.

The heat conduction loss through the regenerator material and the wall of the cylinder and displacer is considered. The shuttle loss is calculated separately using the method suggested by Nishio, et al[5] and the radiation loss is also calculated separately.

Basic Equations

In the simulation model, the two-stage G-M cryocooler is divided into many elements. Based on the above assumptions, the state in each element can be calculated by solving the continuity equation, the energy conservation equation of gas flow, the momentum conservation equation and the state equation of gas.

The continuity equation and the energy conservation equation of gas flow are,

$$\frac{dm_i}{dt} = \dot{m}_i - \dot{m}_{i+1} \tag{1}$$

$$\frac{\partial(m_i h_i)}{\partial t} + (\dot{m}h)_{f_{i+1}} - (\dot{m}h)_{f_i} + \alpha_i A_i (T_i - T_{wi}) - V_i \frac{dP_i}{dt} = 0 \tag{2}$$

where α is the heat exchanger coefficient.

The momentum conservation equation is,

$$\frac{\partial P_i}{\partial x} = -f_r \frac{1}{d_h} \frac{\rho u_i^2}{2} \frac{u_i}{|u_i|} \tag{3}$$

where f_r is the friction coefficient, ρ is the density and d_h is the hydraulic diameter.

The energy conservation equation of the regenerator material and the wall of the displacer and cylinder is,

$$V_{wi} c_{wi} \frac{\partial T_{wi}}{\partial t} = -\lambda_{wi} A_{wi} \frac{\partial^2 T_{wi}}{\partial x^2} dx + \alpha_i A_i (T_i - T_{wi}) \tag{4}$$

where c_w is the specific heat capacity of material and λ_w is the heat conductivity of material.

The displacement of the first stage displacer is,

$$x_{e1} = S_{ce1} + 0.5 \times S_{e1} \times \left\{1 - \cos\left[(\theta - \varphi_{e1}) \times \frac{\pi}{180}\right]\right\} \tag{5}$$

where S_{ce1} is the length of the dead space and S_{e1} is the stroke of the first stage displacer, φ_{e1} is the phase angle of the displacer motion to the start angle of supply process,

The length variation of the compression volume is,

$$x_{b1} = S_{cb1} + 0.5 \times S_{e1} \times \left\{1 + \cos\left[(\theta - \varphi_{e1}) \times \frac{\pi}{180}\right]\right\} \tag{6}$$

where S_{cb1} is the length of the dead space of the compression volume.

The displacement of the second stage displacer is,

$$x_{e2} = S_{ce2} + 0.5 \times S_{e2} \times \left\{1 - \cos\left[(\theta - \varphi_{e2}) \times \frac{\pi}{180}\right]\right\} \tag{7}$$

where S_{ce2} is the length of the dead space, S_{e2} is the stroke of the second stage displacer, φ_{e2} is the delay angle of the displacer motion to the start angle of supply process. Since the first stage displacer and the second stage displacer are connected together, S_{e1} is equal to S_{e2} and φ_{e1} is equal to φ_{e2}.

The mass flux through the rotary valve can be calculated by the formula for a nozzle with a correction factor,

$$\dot{m}_2 = \begin{cases} \mu_1 A_0 \sqrt{\dfrac{2k}{k-1} \dfrac{P_1^2}{RT_1} \left[\left(\dfrac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\dfrac{P_2}{P_1}\right)^{\frac{k+1}{k}} \right]} & P_1 > P_2 \\ -\mu_2 A_1 \sqrt{\dfrac{2k}{k-1} \dfrac{P_2^2}{RT_2} \left[\left(\dfrac{P_1}{P_2}\right)^{\frac{2}{k}} - \left(\dfrac{P_1}{P_2}\right)^{\frac{k+1}{k}} \right]} & P_1 < P_2 \end{cases} \tag{8}$$

where μ_1 and μ_2 are the correction factor, $\mu_1 = 0.95$ and $\mu_2 = 0.85$, A_0, A_1 are the cross-section area of the valve, P_1, P_2 are the pressure before and after the valve, R is the gas constant, k is the specific heat capacity ratio.

COMPARISON WITH EXPERIMENT

In order to verify the simulation program, measurements have been performed on an SHI RDK-408D2 two-stage GM cryocooler. The cold-head is operated at 1.0Hz and the compressor is operated at 50 Hz. For a typical calculation, the high and low pressures are 2.25MPa and 0.76 MPa, respectively. The inner diameters of the first and second stage cylinder are 82 mm and 35 mm, respectively. The stroke of the displacer is 25 mm. The first stage regenerator is filled with #150 phosphorous bronze screens. The second stage regenerator is filled with lead and $HoCu_2$ spheres.

The pressure and volume variation (Fig. 2), and the P-V power in the second stage expansion volume (Fig. 3) are measured. The heat load on the second stage is 1.0 W. As shown in Figures 2 and 3, the simulation results are consistent with the measurement results.

1. The high and low pressures are assumed to be constant,
2. The cross-section area variation of the rotary valve opening is taken to be linear,
3. The valve opening is calculated as a nozzle.

ANALYSIS OF SIMIULATION RESULTS

Cooling Capacity and Losses

Table 1 shows the simulation results for the P-V power, the cooling capacity and the losses. The temperatures are assumed to be 40 K at the first stage and 4.2 K at the second stage. The compressor is operated at 50 Hz and the cold head is operated at 1.0 Hz. As shown in Table 1, the P-V power is 90.1 W at the first stage and 19.33 W at the second stage.

The cooling capacity after considering the real gas effect is 89.3 W and 3.54 W. The cooling capacity after considering real gas effect at the second stage is much lower than the P-V power because the properties of helium at the second stage temperature are far from an ideal gas. A large amount of extra enthalpy flow enters into the second stage expansion volume from the second stage regenerator, which reduces the cooling capacity significantly. Since the real gas effect is dependent on the properties of working fluid, it is difficult to reduce this kind of loss. One possible way to reduce this loss is to change the working fluid from helium-4 to helium-3[4].

The regenerator loss is 23.2 W at the first stage and 1.85 W at the second stage. It should be pointed out that the regenerator loss includes the pressure drop loss, the incomplete heat exchange loss and the heat conduction loss through the regenerator material. The shuttle losses are 7.8 W and 0.16 W and the pumping losses are 0.4 W and 0.12 W. It is apparent that the pumping loss is much smaller than the shuttle loss. If the pumping loss is calculated by the equation suggested in Walker[6], it is about 6.6 W at the first stage. In addition, Walker[6] assumes that all of gas entering from the cold end is warmed to the temperature of the hot end and then flows back to the cold end. In this simulation, the actual heat exchange is calculated. Since the heat exchange between gas and the walls of the cylinder and displacer is poor, gas entering from the cold end is actually only warmed a little higher than the cold end temperature, which results in a small pumping loss.

The pulse tube cooling effect in the clearance can be calculated by assuming that the walls of the cylinder and the displacer are adiabatic, in other words, there is no heat exchange between

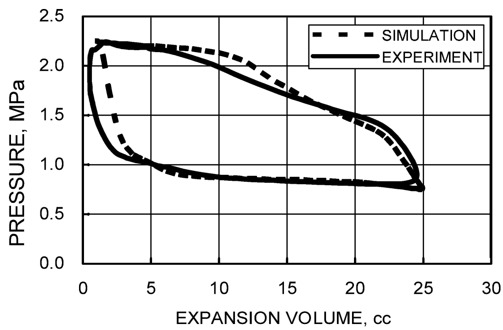

Figure 2. Pressure and volume variation of the second stage.

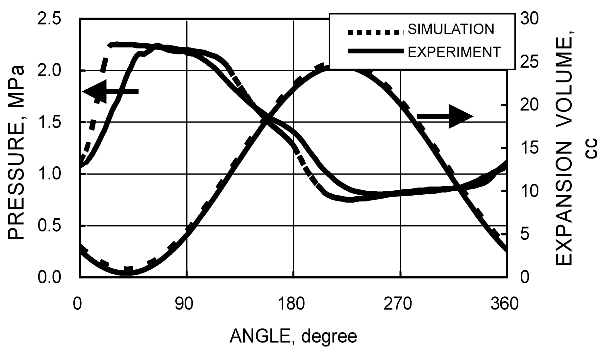

Figure 3. P-V diagram of the second stage.

Table 1. Cooling Capacity and Losses of a Two-stage 4K GM Cryocooler

	1st stage at 40 K (W)	2nd stage at 4.2 K (W)
P-V power	90.1	19.33
Cooling capacity after considering real gas effect	89.3	3.54
Regenerator loss	-23.2	-1.85
Shuttle loss	-7.8	-0.16
Pumping loss	-0.4	-0.12
Pulse tube cooling effect in clearance/spiral groove	0	+0.02
Thermal conduction loss through walls	-5.4	-0.33
Radiation loss	-5.7	0
Net cooling capacity	46.9	1.11

the gas and the walls of the cylinder and the displacer. The cooling effect generated by an adiabatic expansion of the gas in the clearance of the first stage is 0 W and the cooling effect generated by the gas in the clearance and the spiral groove of the second stage is 0.02 W. Since the volumes of both the clearance and the spiral groove are small, the pulse tube cooling effect of the gas in the clearance and spiral groove is small. The actual pulse tube cooling effect will be even smaller if there is no flow smoother at the end of the clearance.

Mass Flow Rate

For half a cycle, the average mass flow rate is 4.90 g/s through the valve, 3.16 g/s at the cold end of the first regenerator and 7.21 g/s at the cold end of the second stage regenerator. Figure 4 shows the dynamic mass flow rate through the valve, at the cold end of the first and second stage regenerators. As shown in Figure 4, during a certain interval, the mass flow rate at the cold end of the second regenerator is larger than the cold end of the first stage regenerator. This behavior is similar to that predicted by Wang, et., al.[2] And also, the mass flow rate at the cold end of the first stage regenerator is larger than that through the valve during some duration. The reason is that even though there is no gas flowing into the cold-head from the valve, gas in the compression volume and the regenerators will flow into the expansion volumes when the displacer moves to the warm end.

Temperature Profile in the First and Second Stage Regenerators

Figure 5 shows the temperature profile in the first and second stage regenerators. In the first stage regenerator, the temperature is almost linear along two-third of the regenerator length, while in the second stage regenerator, the temperature decreases sharply at the warm end and becomes flat at the cold end.

CONCLUSIONS

A numerical simulation program for a 4K GM cryocooler has been developed to predict the theoretical cooling capacity and losses in regenerators, heat exchangers, etc. The simulation results of the pressure and volume variation, and the P-V power in the second stage expansion volume are consistent with the measurement results.

The cooling capacity after considering real gas effect at the second stage is much lower than the P-V power because a large amount of extra enthalpy flow enters into the second stage expansion volume from the second stage regenerator.

The pumping loss is much smaller than the shuttle loss. The pulse tube cooling effect is very small because the volume of either the clearance or the spiral groove is small.

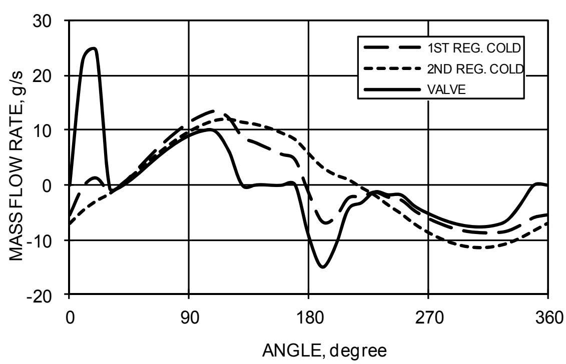

Figure 4. Mass flow rate in a 4K GM cryocooler.

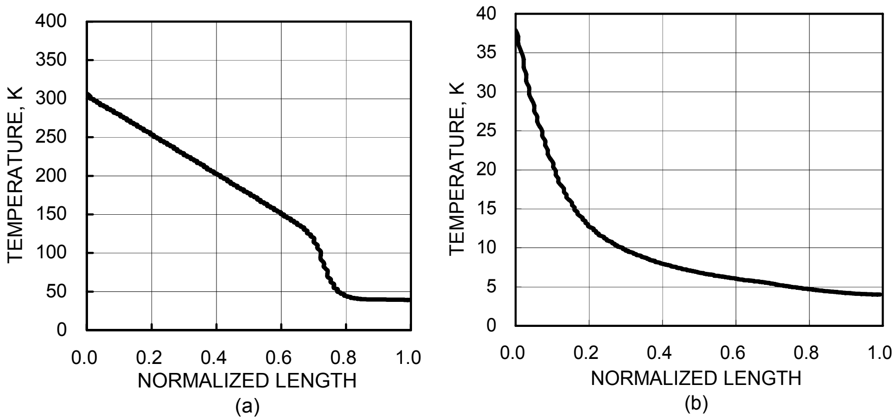

Figure 5. Temperature profile in the regenerators. (a) first stage; (b) second stage.

ACKNOWLEDGEMENT

We thank Yoji Mizuno from Precision Equipment Group of Sumitomo Heavy Industries, Ltd. for providing the measurement data of RDK-408D2 4K GM cryocoolers.

REFERENCES

1. Wu, P., Huang, W., Hu, S.L., Hea, Y.L., Zhang, L., Cheng, G.B., "Dynamic Simulation of Magnetic Material Regenerator and Comparison with Experiment," *Cryogenics*, Vol. 36, Issue 4 (April 1996), pp. 259-261.
2. Wang, C., Häfner, H.U. and Heiden, C., "Performance and Internal Process of a 4K GM Cooler," *Adv. in Cryogenic Engineering*, Vol. 43B, Plenum Publishing Corp., New York (1998), pp. 1775-1782.
3. Xu, M. and Morie, T., "Development of High-efficiency 4K GM Cryocoolers," *Proceeding of the 24th International Cryogenic Engineering Conference and International Cryogenic Material Conference* (2012), (submitted for publication).
4. Xu, M.Y., de Waele, A.T.A.M. and Ju, Y.L., "A pulse tube refrigerator below 2K," *Cryogenics*, Vol. 39, Issue 10 (October 1999), pp. 865-869.
5. Nishio, S., Inada, T. and Nakagome, H., "Shuttle Heat Transfer in Refrigerators," *Journal of the Cryogenic Society of Japan*, Vol.27 No. 6 (1992), pp. 507-512.
6. Walker, G., *Cryocoolers*, Plenum Press, New York (1983), pp. 221.