Losses and Related Design Tradeoffs in Floating Piston Expanders

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ABSTRACT

Piston expanders are widely used in cryogenic machinery, but are plagued by a multitude of thermodynamic losses which reduce their performance. Many of these losses favor different geometries or operating parameters; however, the tradeoffs between such losses are only clear if their effects are considered in aggregate. The matter is further complicated in high-pressure-ratio expanders, where widely-used analyses of fluid flow behavior in the clearance space between the piston and cylinder prove inadequate for modeling purposes.

This work examines the loss-related design tradeoffs in a particular type of piston expander known as a floating piston expander (FPE). Such expanders use differences in gas pressure instead of mechanical linkages to control piston motion, and are being incorporated into a Collins-type cryocooler capable of providing 20 and 100 watts of cooling at 25 and 100 Kelvin, respectively. The tradeoffs are examined using a cyclic-steady-state expander model which considers heat transfers in the cold working volume, irreversible "blow-in" and "blow-out" through cold-end valves resulting from imperfect expansion and recompression, irreversible mixing, and the effects of the clearance space (or "appendix gap") between the piston and cylinder. For the latter, a newly-developed model is used to solve for the gap mass flows and heat transfers; this model is well suited to the high pressure ratios and low operating frequencies of the expanders in question.

The model predicts several interesting (and often unexpected) results which should be of interest to expander designers; intentionally increasing blow-in and blow-out, for example, can improve expander performance by decreasing the required stroke and diameter and thereby reducing appendix gap losses. The results also demonstrate the cross-coupling between design variables and suggest strategies for mitigating the impact of various manufacturing constraints.

INTRODUCTION

Floating piston expanders (FPEs) are a novel type of cryogenic expander which use controlled differences in gas pressure to control the motion of a piston instead of mechanical linkages. Such expanders are a key component of recent Collins-type cryocooler designs explored by MIT and AMTI, including a single-load cryocooler intended to provide 2 watts of cooling at 10 Kelvin and a two-load cryocooler intended to provide a simultaneous 20 and 100 watts of

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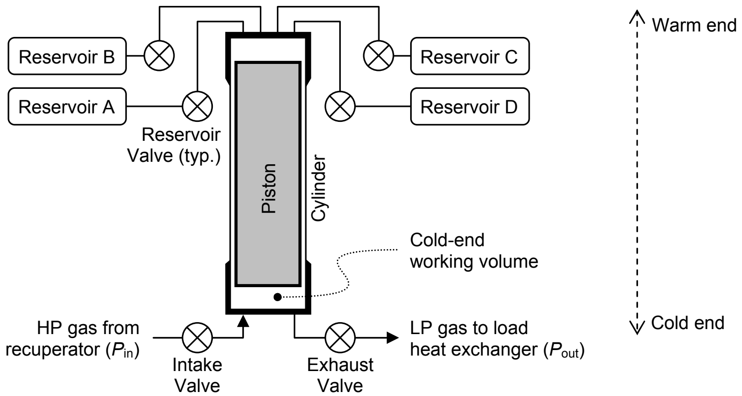

Figure 1. Simplified schematic of a floating piston expander (FPE). Using computer-controlled valves at both ends of the expander, the piston's motion can be controlled to expand cold-end fluid.

cooling at 25 and 100 Kelvin, respectively.[1,2]

A schematic of an FPE is shown in Figure 1. At the cold end, custom-designed electronic "smart" valves allow high-pressure gas to enter the expander and low-pressure gas to exit following expansion. At the warm end, a series of reservoirs (each with its own computer-controlled valve) contain gas at four pressures spanning the inlet and exhaust pressures of the expander; that is, $P_{in} > P_A > P_B > P_C > P_D > P_{out}$, where P_A is the pressure in reservoir A and so on. Nominal operation of the expander proceeds as follows, beginning with all valves closed and the pressure in both the cold- and warm-end working volumes slightly below P_A:

1. **Blow-in:** The cold-end intake valve opens and gas rushes into the expander, moving the piston upward (and compressing gas in the warm-end working volume) until the pressure in both working volumes is close to P_{in}.

2. **Intake:** The valve to reservoir A then opens and the piston moves upward a prescribed distance as high-pressure gas is admitted to the cold volume.

3. **Expansion (×4):** The cold-end intake valve closes and the piston continues to move upward as gas flows into reservoir A; this continues until the pressure in the working volumes decreases to slightly above P_A. The valve to reservoir A then closes and the valve to reservoir B opens to continue the expansion, and so on. Nominally, the piston reaches the top of the cylinder when the pressure is just above P_D. The valve to reservoir D then closes.

4. **Blow-out:** The cold-end exhaust valve opens and gas rushes out of the expander until the pressure is close to P_{out}. Since the warm volume contains a small amount of gas due to manufacturing and operating tolerances, the piston moves downward during blow-out as this gas expands in response to the sudden pressure drop.

5. **Exhaust:** The valve to reservoir D then opens and the piston moves downward a prescribed distance as cooled, low-pressure gas is exhausted from the cold volume.

6. **Recompression (×4):** The cold-end exhaust valve closes and the piston continues to move downward as gas flows out of reservoir D; this continues until the pressure increases to slightly below P_D. The valve to reservoir D then closes and the valve to reservoir C opens, and so on. Nominally, the piston bottom reaches the bottom of the cylinder when the pressure is just below P_A. The valve to reservoir A then closes and the cycle repeats.

Ideally, all of these processes would be adiabatic and reversible; however, the blow-in and blow-out processes are by nature irreversible, and other losses also reduce the FPE's performance and give rise to a variety of design tradeoffs. In order to develop an expander model suitable for exploring these design tradeoffs, it is beneficial to first catalog the various loss mechanisms in actual FPEs.

EXPANDER LOSS MECHANISMS

There are at least eleven distinct loss mechanisms that have been identified in floating piston expanders. In what follows, these losses are discussed in more detail along with possible strategies to model them.

Conduction Heat Transfer

One of the simplest losses in an FPE is one-dimensional axial conduction from the warm end of the expander to the cold end of the expander (by means of the cylinder and hollow floating piston, which will likely be made of steel and phenolic, respectively). This heat transfer reduces the amount of effective cooling provided by the expander. Modeling the loss is not straightforward, however, due to the presence of other heat transfers in the radial clearance or "appendix gap" between the piston and cylinder; conduction heat transfer is therefore modeled together with the shuttle heat transfer and gas enthalpy transfer mechanisms.

Shuttle Heat Transfer

In addition to the axial conduction mechanism described above, thermal energy is also physically "shuttled" from the warm end to the cold end by the reciprocating motion of the floating piston. When the piston is closer to the warm end, the temperature of the surrounding cylinder is everywhere warmer than the piston walls, and heat is transferred to the piston through the thin layer of gas in the appendix gap. The piston then moves to the cold end of the expander (carrying with it the thermal energy from the heat transfer) where heat is again transferred across the gap nearer to the cold end, this time from the piston into the cylinder walls.

While multiple analyses of the shuttle heat transfer loss have appeared in the literature in the past 50 years, one of the most useful was presented by Chang and Baik, who treated the piston and cylinder as semi-infinite solids with pure conduction through the intermediate gas.[3] Assuming a linear axial temperature profile and sinusoidal piston motion yielded a clean analytical result (reproduced here in a slightly modified form):

$$\dot{Q}_{\text{shuttle}} = -\left(\frac{T_{\text{warm}} - T_{\text{cold}}}{L_{\text{pis}}}\right)\frac{\pi D S^2 k_{\text{gap}}}{8\delta_{\text{gap}}}\frac{b_{\text{pis}} + b_{\text{cyl}} + 1}{\left(b_{\text{pis}} + b_{\text{cyl}}\right)^2 + \left(b_{\text{pis}} + b_{\text{cyl}} + 1\right)^2}, \tag{1}$$

where T_{warm} and T_{cold} are the mean temperatures of the warm end and cold end, respectively, L_{pis} is the piston length, D is the expander diameter, S is the stroke length of the piston, k_{gap} is the thermal conductivity of the fluid in the appendix gap (which Chang and Baik assumed to be uniform), δ_{gap} is the width of the appendix gap, and b_{pis} and b_{cyl} are Biot numbers with a characteristic length equal to the thermal penetration depth into the solids (where k_{pis} or k_{cyl} and α_{pis} or α_{pis} are the thermal conductivities and thermal diffusivities of the piston or cylinder, respectively, and ω is the angular frequency of operation of the expander):

$$b_{\text{pis}} = \frac{\left(k_{\text{gap}}/\delta_{\text{gap}}\right)}{k_{\text{pis}}}\sqrt{\frac{\alpha_{\text{pis}}}{2\omega}}, \quad b_{\text{cyl}} = \frac{\left(k_{\text{gap}}/\delta_{\text{gap}}\right)}{k_{\text{cyl}}}\sqrt{\frac{\alpha_{\text{cyl}}}{2\omega}}. \tag{2}$$

Notably, the shuttle heat transfer is proportional to the *square* of the stroke length.

Gas Enthalpy Transfer and Combined Gap Losses

While the analysis by Chang and Baik is sufficient for predicting the shuttle heat transfer when the gas in the appendix gap behaves as a pure conduction resistance, large pressure fluctuations in actual expanders force gas into and out of the gap. These flows and pressure fluctuations can give rise to temperature gradients which drive additional heat transfers between the gas and the piston. Additionally, the moving gas contributes an enthalpy transfer of its own, a loss referred to in the literature as the gas enthalpy transfer or "pumping" loss.[a]

The gas enthalpy transfer (GET) mechanism is conceptually distinct from the shuttle heat transfer but it is difficult to separate the two mathematically due to their multiple interactions. Furthermore, the literature examining this loss is relatively sparse, and none of the existing analyses seemed well-suited to the conditions in a high-pressure-ratio cryogenic FPE. A new analysis of the shuttle, conduction, and GET losses was therefore carried out; the details are reported separately in the present proceedings, but the end result combines local analytical estimates with numerical integration and a one-dimensional shooting method to accommodate arbitrary gap temperature profiles and temperature-dependent material and fluid properties.[4] The GET model provides three outputs for use in the expander model:

1. The total heat transfer from shuttle, conduction, and gas enthalpy transfer combined,
2. A ratio dm/dP relating the mass flow into the gap to pressure changes in the cold end (neglecting phase differences which are expected to be small when GET is significant), and
3. The average temperature difference between fluid exiting the gap (during the expansion and blow-out processes where P is decreasing) and the cold-end walls.

Cyclic Heat Transfer

Even if shuttle, conduction, and GET losses could be entirely eliminated, heat transfers between the working fluid and the walls of the cold-end working volume can still reduce the performance of expanders or other gas-handling equipment. The loss is greatest at intermediate operating speeds where the expander is neither adiabatic nor isothermal, yielding entropy-generating heat transfers that decrease the efficiency of the expander.

Since it was first identified in the 1940s, many studies of this effect have appeared in the literature under the names of cyclic heat transfer, hysteresis heat transfer, compression-driven heat transfer, transient heat transfer, gas spring heat transfer, and gas spring hysteresis loss. One of the highlights is an analytical study presented by Lee in 1983; beginning from a one-dimensional model of transient heat transfer between parallel walls, Lee derived a complex heat transfer coefficient governing cold-end heat transfer as a function of operating parameters.[5] The complex heat transfer coefficient determines both the magnitude and the phase of the cold-end heat transfer, which is generally out-of-phase with the temperature difference due to pressure work on the gas near the walls. Lee's result may be presented as a correlation between a complex Nusselt number and the oscillating Peclet number:

$$\mathrm{Nu} = \sqrt{2\mathrm{Pe}_\omega}\,\frac{(1+i)\tanh z}{1-(\tanh z)/z}, \quad \text{where} \quad z \equiv (1+i)\sqrt{\mathrm{Pe}_\omega/8} \quad \text{and} \quad \mathrm{Pe}_\omega = \omega D_h^2/(2\alpha). \tag{3}$$

Here, D_h is the mean hydraulic diameter of the cold-end working volume, α is the mean thermal diffusivity of the cold-end gas, and i is the square root of -1.

While Lee's work included no experimental data to validate equation (3), Kornhauser[6] conducted a range of experiments and suggested that a modified correlation was more appropriate at Peclet numbers greater than ~100:

$$\mathrm{Nu} = (1+i)\,0.56\mathrm{Pe}_\omega^{0.69}. \tag{4}$$

For the expander model in this work, this modified correlation was used at high Peclet numbers and smoothly blended into the asymptotes of Lee's analytical correlation at low Peclet numbers.

[a] The term "pumping loss" is more typically used to describe flow losses in heat exchangers or engine valves and should therefore be used with caution. The term "gas enthalpy transfer" is preferred as it has no such ambiguity.

This is similar to the approach used by Wang to model heat transfer in a Stirling machine, though Wang used a fixed cutoff of $\mathrm{Pe}_{\omega} = 100$ to choose which correlation would be used.[7]

For most adiabatic expanders, the cyclic heat transfer loss may be mitigated by minimizing the surface area per unit swept volume in the cold working volume, which implies a piston stroke approximately equal to the expander diameter ($S/D \approx 1$). This design heuristic was used in at least one earlier prototype of the floating piston expander.

Intake Mixing

As a result of heat transfer into and within the cold-end working volume, the gas remaining in the volume following recompression is unlikely to be at the same temperature as the gas entering during the blow-in and intake strokes. The resulting temperature difference yields additional entropy generation as the gases mix. Note that the intake mixing loss is conceptually similar to the well-known adiabatic losses in Stirling machinery.

Intake mixing is implicitly taken into account by the numerical expander cycle simulation discussed later in this paper; a separate correlation is therefore not necessary to model this loss.

Blow-In and Blow-Out

As previously mentioned, the blow-in and blow-out processes in the expander are by nature irreversible. The amounts of blow-in and blow-out depend on the pressure difference across the reservoir valves (a non-zero difference is required for the piston to move) and the pressures P_{A} and P_{D} of the warm-end reservoirs.

The mechanism by which stable pressures are established in the warm-end reservoirs was recently examined by Hogan, who also proposed a means of controlling these pressures by changing the expander control algorithm to deliberately cause blow-by flows past the floating piston.[8] For the present work, however, the pressure defects driving the blow-in and blow-out processes were considered an input to the expander model. For simplicity, the same pressure defect was assumed for both blow-in and blow-out processes; this was specified as a dimensionless factor f_{bio} defined as the ratio of the pressure defect to the pressure difference between the expander inlet and outlet:

$$f_{\mathrm{bio}} \equiv \Delta P_{\text{blow-in}} \big/ \left(P_{\mathrm{in}} - P_{\mathrm{out}} \right) \quad \text{or} \quad \Delta P_{\text{blow-out}} \big/ \left(P_{\mathrm{in}} - P_{\mathrm{out}} \right). \tag{5}$$

Based on Hogan's simulations, f_{bio} factors in the range of 0.05–0.1 seem attainable for high-pressure-ratio expanders similar to the ones explored in this work.

Additionally, the presence of valves, flow passages, and piston inertia necessitates an additional clearance or "dead" space at either end of the piston's stroke (not to be confused with the appendix gap), and these clearance spaces also aggravate the blow-in and blow-out processes. In the present work, the clearances are modeled by assuming that the piston does not get closer than a distance t_{clear} to the bottom or top of the expander.

Other Losses

Other possible FPE loss mechanisms exist in addition to the six described above that were not incorporated into the current version of the cycle model. The first of these, piston blow-by, was already mentioned as part of Hogan's work on reservoir pressures. Such flows result from a combination of imperfect piston seals and pressure differences across the piston; the latter in turn may be caused by frictional forces, gravitational forces in vertically-oriented expanders, piston inertia, and contact forces between the piston and either end of the expander. Blow-by flows are of particular concern if the warm end of the expander is permitted to interact directly with the cryocooler's compressor as this could yield net mass flows from the warm to the cold end that impose an additional heat load on the expander. While this was the case in previous versions of the FPE, Hogan's studies suggest that a sealed-warm-end expander with no flows to or from the compressor (e.g., Figure 1) is indeed achievable.

Another source of loss is the dissipation inherent in the warm-end valves and reservoirs, though this does not directly affect the expander efficiency (the typical definition of expander efficiency considers only with how much the cold-end gas is cooled, not what happens to the energy removed from the gas). The loss nevertheless affects the efficiency of the cryocooler, as it represents a loss of potential work. While it is reasonable to imagine that some of this work could be recovered and used instead of being dissipated, no mechanism has yet been proposed to do so which retains the simplicity and robustness of the current design.

In addition to the inherent valve losses in the warm end, there will also be some dissipation in the cold-end valves. This loss was not included in the present expander model, but is expected to scale with the total mass flow rate through the expander (which in turn is inversely proportional to the cryocooler efficiency). Nominally, cold-end valves should be designed to limit such losses to an acceptable amount.

Finally, there exists a potential for an "appendix gap hysteresis" loss within the appendix gap which seems conceptually similar to the cold-end cyclic heat transfer loss. An analysis never appears in the open literature, however. The appendix gap model developed for the present work does not explicitly include this loss, though some of its effects may have been implicit in the analysis approach used.

EXPANDER MODELING

To accommodate all of the interactions between the losses described above a numerical solution approach was chosen for the expander model. The approach was based around a single shooting method: guesses were first chosen for a set of state and process variables, governing ODEs for each process in the cycle were numerically integrated in sequence (beginning with the blow-in process and ending with the recompression process), and the guesses were adjusted to satisfy thermodynamic and cyclic-steady-state constraints.

A number of simplifying assumptions were made in deriving the governing equations of the processes comprising the expander cycle:

1. The working fluid was assumed to be an ideal gas with constants R and c_P;
2. The cold-end fluid was assumed to be uniformly mixed at all points in time;
3. Friction, piston inertia, and piston body forces (e.g., due to gravity) were all neglected yielding identical pressures in the warm end and cold end working volumes;
4. The walls of the cold volume (including the face of the piston) were assumed to have large heat capacity and thermal effusivity and so remain at a constant temperature over one cycle;
5. Kinetic and gravitational terms were neglected in the governing equations; and
6. The rates of change of cold end pressure and volume (dP/dt and dV/dt, respectively) were approximated as known constants in the governing equations.

Most of these assumptions are straightforward except for assumption 6, which was used to replace the time dependence in the governing equations and allow pressure or volume to be used as the independent variables during integration; this is advantageous since it permits the cycle to be solved without knowing a-priori how long each of the processes will take.

Further details of the solution approach have been omitted here for brevity, but are available in a thesis by Segado along with MATLAB source code of the expander model.[9]

TEST CASES

The expander model described above was used to simulate the performance of a variety of expander designs. The tests described in the present work focused on the performance of two specific high-pressure-ratio (HPR) expander configurations: one providing 20 W of cooling at 25 K (suitable for zero-boil-off storage of liquid hydrogen), and the other providing 100 W of cooling at 100 K (suitable for zero-boil-off storage of liquid oxygen). While the model should

Table 1. Parameter values used in the present study. The first six rows contain the free design parameters; for each simulation, one of these was chosen pseudo-randomly and assigned a value in the corresponding range from a log-uniform probability distribution while the others were each pseudo-randomly assigned a discrete value. In this way, the design space was explored with a six-dimensional hypergrid. The last five rows contain fixed parameters.

		Discrete values					
Variable	Range	VL	Low	Nom.	High	VH	Units
f	0.25–4.0		0.3	1.0	3.0		[cycles/s]
f_{bio}	0.005–0.25		0.01	0.05	0.2		[]
L_{pis}	0.25–2.0		0.25	0.5	1.0	2.0	[m]
δ_{gap}	0.01–1.0	0.01	0.03	0.1	0.3	1.0	[mm]
t_{clear}/D	0.005–0.1		0.005	0.02	0.1		[]
S/D	0.05–2.0	0.1	0.2	0.5	1.0		[]
T_{warm}				300			[K]
t_{pis}/D				0.03			[]
t_{cyl}/D				0.01			[]
$\mathrm{mat}_{\mathrm{pis}}$				G10			–
$\mathrm{mat}_{\mathrm{cyl}}$				304 steel			–

enable the analysis of many other configurations, these were based on a modified Collins cryocooler prototype nearing completion at AMTI.

A six-dimensional hypergrid with pseudo-random sampling was used to explore the design space. For each simulation, five of the design parameters were pseudo-randomly assigned values from a set of discrete options (defining a line in six-dimensional space), while the remaining free parameter was pseudo-randomly assigned a value along the resulting line from a continuous log-uniform probability distribution. The choice of which parameter was continuous was also randomized. This permitted a simple form of parallel computing: multiple instances of the code were run on multiple processing cores or even multiple computers, and the data was later harvested and combined into a larger dataset. Each instance of the code was started at a different time; since the pseudo-random number generator was seeded from the system timer, this made it unlikely that two instances would generate identical data.

The design variables used in this study are listed in Table 1 along with their continuous ranges and discrete values. Note that the discrete variables are approximately evenly distributed on a logarithmic scale. Table 1 also lists the fixed parameters common to all the simulations.

RESULTS AND DISCUSSION

About 310,000 valid data points were generated by the HPR expander simulations. As a discussion of every feature in this dataset would be quite lengthy, only first-order interactions between design variables will be discussed (i.e., the impact of simultaneously varying any two design variables while the remaining four are constant at the nominal values listed in Table 1). In all results, expander efficiency is defined as the ratio of the actual enthalpy removed from the working fluid to the enthalpy that would be removed in an adiabatic, reversible expander with identical inlet conditions.

Efficiency vs. Stroke-to-Diameter Ratio

The discussion of cyclic heat transfer loss suggests an optimum S/D of around one. Simulations, however, suggest optimum S/D ratios in the range of 0.2 to 0.5 (typified by Figure 2). This discrepancy appears to stem from the shuttle heat transfer loss, which scales with the square of the stroke length and therefore favors lower S/D ratios.

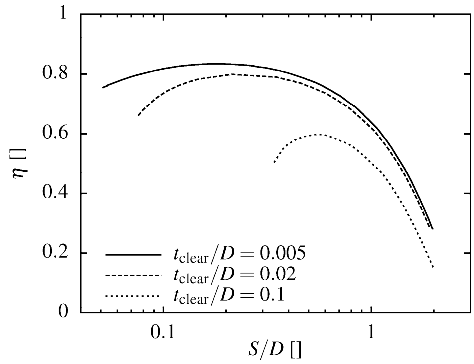

Figure 2. Efficiency vs. S/D for a range of clearance fractions in a 25 K HPR expander. The results typify one of the key conclusions in this work: the presence of shuttle heat transfers tends to yield lower optimum S/D ratios (around 0.2 to 0.5) than those expected from a cyclic heat transfer analysis alone (around 1.0).

The location of the optimum was found to be most dependent on t_{clear}/D (the "clearance fraction") and the appendix gap width. Large clearance fractions yield a larger amount of recompressed "dead" mass which does not contribute to cooling the load, favoring a higher S/D ratio with a higher ratio of exhausted mass to recompressed mass. Wide appendix gaps also increase the amount of dead mass and additionally decrease the importance of shuttle heat transfer; both these effects yield higher optimum S/D ratios.

Similarly, longer pistons both reduce the importance of shuttle heat transfer and result in larger appendix gap volumes, though the resulting increase in the optimum S/D was not as pronounced as for the clearance fraction and gap width. The optimum was also larger with higher operating frequencies as these reduce the swept volume needed to achieve a given mass flow rate and thereby reduce the importance of shuttle heat transfers.

The blow-in and blow-out fractions had very little impact on the optimum S/D, though higher values of f_{bio} did yield broader optima (that is, they reduced the impact of suboptimal designs).

Efficiency vs. Appendix Gap Width

The results above suggest that appendix gap heat phenomena play an important role in determining the efficiency of FPEs; this appears to be particularly true for short piston lengths as shown in Figure 3. Such a result is unsurprising since short pistons have higher average temperature gradients and therefore it makes sense that they may increase the importance of shuttle and gas enthalpy transfers.

The combination of shuttle, GET, and possibly other gap-related losses typically gives rise to an optimum gap width. This optimum shifts with piston length (Figure 3); note that appropri-

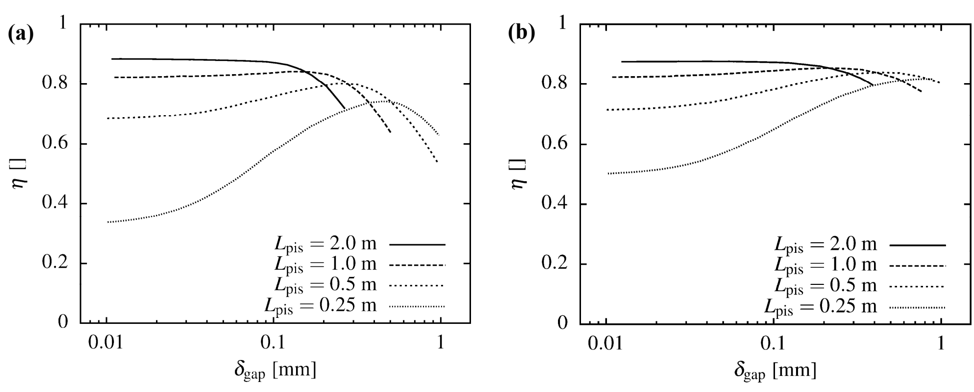

Figure 3. Efficiency vs. appendix gap width for a range of piston lengths in (a) 25 K and (b) 100 K HPR expanders. The optima result from the tradeoff between shuttle heat transfers (mitigated by wide gaps) and gas enthalpy transfer losses (mitigated by narrow gaps). Note that using the optimum appendix gap width can largely mitigate the efficiency impact of shorter pistons, especially in the 100 K expander.

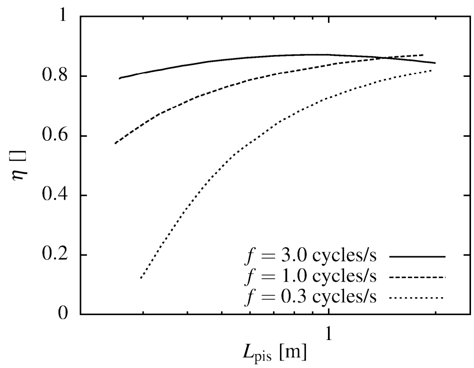

Figure 4. Efficiency vs. piston length for a range of operating frequencies in a 25 K HPR expander. Longer pistons typically yield more efficient expanders, though exceptions to this trend do exist at high frequencies as shown. The figure similarly demonstrates that intermediate frequencies can become optimal for long pistons.

ately wide gaps can mitigate much of the detriment of using shorter pistons, though this assumes that the assumptions inherent in the appendix gap heat transfer model still hold in such cases.

Similar behavior is also visible for other design variables. Lower frequencies favor wider gaps, as these require larger swept volumes to achieve suitable mass flow rates and therefore suffer more from the effects of stroke-dependent shuttle heat transfers. Higher S/D ratios also favor wider gaps due to the shuttle's dependence on S^2. Both the clearance faction and the blow-in and blow-out factors, however, had comparatively little effect on the optimum gap width.

Efficiency vs. Piston Length

As expected, longer pistons typically yielded more efficient expander designs, though a few exceptions did exist in extreme cases. The first of these was visible in Figure 3: progressively shorter pistons become optimal for gaps wider than about 0.1 mm, presumably because this mitigates the added dead volume of the wider gaps without excessively increasing the shuttle loss. (Note that the effect on the GET loss is unclear since the mass flow rates should be smaller but the temperature gradient should be higher.)

A more subtle exception is visible in Figure 4 which shows that, even with a nominal gap width, intermediate piston lengths become optimal for sufficiently high operating frequencies. High frequencies permit the use of smaller working volumes for a given mass flow rate, and the resulting decrease in S^2 reduces the importance of shuttle enough to shift the optimum toward shorter pistons with higher temperature gradients but lower-volume appendix gaps.

The remaining design variables primarily affected the sensitivity of efficiency to piston length in a manner resembling Figure 4, with the highest sensitivities occurring for large S/D ratios, large clearance fractions, and small blow-in/blow-out factors. All of these increase the required stroke and thus the importance of shuttle, so it stands to reason that they would also increase the importance of piston length. As with Figure 4, an optimum piston length below 2 m did appear for the lowest S/D and the highest f_{bio}, though these optima were barely apparent, very close to 2 m, and had a lower efficiency than could be obtained with intermediate S/D and f_{bio}.

Efficiency vs. Clearance Fraction

Unlike the other variables examined, higher clearance fractions were unconditionally detrimental to expander performance in the cases examined here. The sensitivity of efficiency to variations in clearance fractions is reduced by higher frequencies, higher blow-in/blow-out factors, longer pistons, and larger S/D ratios. The appendix gap width appeared to have little effect on the sensitivity to clearance fraction, though simulations with widest gap (1 mm) failed to converge above a clearance fraction of $t_{\mathrm{clear}}/\mathrm{D} \approx 0.025$.

Efficiency vs. Blow-In and Blow-Out Factors

One notable result mentioned above is that increasing the blow-in and blow-out factor can *improve* the performance of an expander. This is especially true at large clearance fractions as

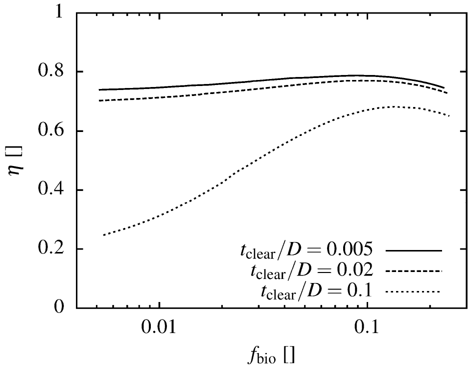

Figure 5. Efficiency vs. blow-in and blow-out factor for a range of clearance fractions in a 25 K HPR expander. Allowing more irreversible blow-in and blow-out permits the use of shorter strokes with a corresponding reduction in shuttle losses and thereby increases overall efficiency. This is particularly true when large clearance fractions increase the stroke length required for recompression, with the resulting increase in shuttle compounded by a "multiplier" effect: longer strokes increase shuttle heat transfer, requiring even longer strokes to compensate for the lost cooling.

shown in Figure 5; for expanders with the largest clearance examined the choice of f_{bio} can make the difference between an expander efficiency of 25% (minimum blow-in and blow-out) and one of 68% (optimum blow-in and blow-out).

A straightforward physical explanation exists for this effect. While blow-in is itself irreversible, recompressing to a lower pressure requires less piston stroke and therefore reduces the shuttle heat transfer losses. Similarly, a shorter and therefore incomplete expansion stroke yields less reversible cooling and more blow-out, but is outweighed by a larger reduction in shuttle losses.

The effect above, and presumably many of those discussed earlier, is compounded by a shuttle-related "multiplier" effect: lower values of f_{bio} yield expanders with longer strokes and correspondingly larger shuttle losses, which in turn requires expanders with longer strokes and so on. This multiplier effect is only apparent in regions of the design space where shuttle dominates the expander's performance, e.g., when large clearance fractions yield high strokes as in Figure 5.

The optimum value of f_{bio} is a function of the other design variables, though with varying degrees of sensitivity. As seen in Figure 5, higher clearance fractions favor more blow-in and blow-out, but only marginally. Slightly larger shifts occur with narrower appendix gaps and larger S/D ratios, both of which increase the optimum f_{bio}. The biggest shifts, however, occur for different frequencies and piston lengths. At a frequency of 3.0 Hz or a piston length of 2.0 m the optimum f_{bio} is around 0.05, while at the opposite extreme ($f = 0.3$ Hz or $L_{\mathrm{pis}} = 0.25$) the optimum increases to $f_{\mathrm{bio}} \approx 0.2$. The optimum also becomes more pronounced at lower f and L_{pis}, though not as much as with lower t_{clear}/D.

Efficiency vs. Operating Frequency

Lastly, higher frequencies typically yielded more efficient expander designs, with a few exceptions in extreme cases. One such exception occurs toward the upper right of Figure 4 which shows that intermediate frequencies become optimal for long pistons. Appendix gaps wider than about 0.1 mm also shift the optimum frequency lower, much in the same way that wide appendix gaps yielded a shorter optimum piston (Figure 3). Both shifts involve an increase in appendix gap volume, suggest either gas enthalpy transfers or added dead volume as a likely cause.

The remaining variables primarily affect the sensitivity of efficiency to frequency. As with piston length, the highest sensitivities to frequency occurred with large S/D ratios, large clearance fractions, and small blow-in/blow-out factors. The reasoning is likely the same as for piston length: operating frequency affects the swept volume required to achieve a given mass flow rate and therefore can affect the stroke, thus changing the importance of shuttle heat transfers losses (which appear to dominate except with wider gaps and longer pistons as explained above).

CONCLUSIONS

The results presented above may be distilled into three key conclusions. First, the $S/D \approx 1$ heuristic used in the past should be adjusted when shuttle heat transfer losses are significant. As shuttle losses scale with the stroke length squared, a stroke-to-diameter ratio of 0.2–0.5 appears to be more suitable for typical conditions in the expanders examined, even with large clearance fractions (e.g., t_{clear}/D = 0.1). One interesting direction for further study would be to examine this behavior across a wider range of expander operating temperature and cooling power.

Second, blow-in and blow-out were found to be quite beneficial to expander performance. While these processes are themselves irreversible, they permit the use of shorter stroke lengths which in turn reduce shuttle heat transfers losses. In addition to yielding higher expander efficiency, such designs also impose less stringent requirements on the stable warm-end reservoir pressures since it is no longer necessary for P_A and P_D to settle as close to P_{high} and P_{low}, respectively. However, as large pressure defects have the potential to accelerate the piston to high velocities, piston dynamics should also be considered in potential high-f_{bio} expander designs.

Third, choosing too wide or narrow an appendix gap can significantly impact expander efficiency, especially for shorter pistons; while this is not an original conclusion, it does underscore the importance of having a suitably-accurate model of the processes in the appendix gap in order to make such tradeoffs.

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REFERENCES

1. Hannon, C.L., Gerstmann, J., Traum, M.J., Brisson, J.G., and Smith Jr., J.L., “Development of a Medium-Scale Collins-Type 10 K Cryocooler,” *Cryocoolers 12*, Kluwer Academic/Plenum Publishers, New York (2003), pp. 587–594.
2. Segado, M.A., Hannon, C.L., and Brisson, J.G., “Collins Cryocooler Design for Zero-Boil Storage of Liquid Hydrogen and Oxygen in Space,” *Advances in Cryogenic Engineering 55*, American Institute of Physics, Melville (2010), pp. 1377–1384.
3. Chang, H.-M. and Baik, J.H., “An Exact Expression for Shuttle Heat Transfer,” *Advances in Cryogenic Engineering 41*, Plenum Press, New York (1996), pp. 1535–1542.
4. Segado, M.A. and Brisson, J.G., “Appendix Gap Losses with Pressure-Driven Mass Flows,” present proceedings.
5. Lee, K.P., “A Simplistic Model of Cyclic Heat Transfer Phenomena in Closed Spaces,” *Proceedings of the 18th Intersociety Energy Conversion Engineering Conference*, American Institute of Chemical Engineers, New York (1983), pp. 720–723.
6. Kornhauser, A.A. and Smith Jr., J.L., “Application of a Complex Nusselt Number to Heat Transfer During Compression and Expansion,” *ASME Journal of Heat Transfer*, Vol. 116, No. 3 (1994), pp. 536–542.
7. Wang, A.C., *Evaluation of Gas Spring Hysteresis Losses in Stirling Cryocoolers*, Master’s thesis, Massachusetts Institute of Technology, Cambridge (1989).
8. Hogan, J., *Development of a Floating Piston Expander Control Algorithm for a Collins-Type Cryocooler*, Master’s thesis, Massachusetts Institute of Technology, Cambridge (2012).
9. Segado, M.A., *Analysis and Mitigation of Key Losses in a Multi-Stage 25–100 K Cryocooler*, Master’s thesis, Massachusetts Institute of Technology, Cambridge (publication expected 2012).