Modeling Transient Behavior of an Integral Rotary Cryocooler

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ABSTRACT

This paper describes an investigation of the transient behavior of an integral rotary cryocooler, capable of relatively fast cool-down from ambient to about 50 K. A method was developed for evaluating the cooling power at different cold-end temperatures from a simple cool-down and subsequent warm-up test of the cryocooler. A series of experiments was conducted, in which different amounts of excess thermal mass were added to the cold end. For each of them, the cold-end temperature was measured as a function of time during cool-down/warm-up with no heat load. A transient heat transfer model developed under a separate study was applied to consider the effects of the cooling power produced by the cryocooler and that of the heat inlet (gain) from the ambient on the cool-down time. The heat gain factor was calculated from warm-up data. Using the same model with cool-down data enables a determination of both the gross and net cooling power as functions of time, and more importantly as functions of the cold end temperature.

LIST OF SYMBOLS

A	Regenerator cross-section area	[m^2]
c_g	Specific heat of regenerator matrix	[J/kg-K]
c_m	Specific heat of excess thermal mass at cold end	[J/kg-K]
COP	Coefficient of Performance	[-]
k_g	Thermal conductivity of regenerator matrix	[W/m-K]
L	Regenerator length	[m]
m_m	Mass of excess thermal mass at cold end	[kg]
Q, Q'	Gross and net cooling power	[W]
r	Ratio of heat capacities, excess mass to regenerator, Eq. (4)	[-]
t	time	[sec]
T	Temperature	[K]
T_0	Temperature at warm end	[K]
T_H, T_L	Upper and lower limits of temperature range in cool-down and warm-up	[K]
T_m	Temperature at cold end	[K]
u	Dimensionless heat gain factor, Eq. (4)	[-]
U	Heat gain coefficient	[W/K]
x	Axial coordinate	[m]

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INTRODUCTION

Cryocoolers employed for tactical applications are characterized by several performance parameters. In addition to their operation at steady state, cooldown rates are important with varying excess mass at the cold end (representing the payload - Dewar and IR detector), especially where considerations such as mission readiness are relevant. Another important performance parameter is the cooling power of the cryocooler as a function of the cold end temperature.

In an earlier study[1], a method was developed for evaluating these performance parameters from a simple cool-down and subsequent warm-up test of the cryocooler. The tests[1] were performed on a fairly small (2 W at 85 K) pulse tube cryocooler operating at 120 Hz with an average pressure of 3.5 MPa, capable of relatively fast cool-down to about 60 K. A series of cool-down/warm-up experiments were conducted with several different amounts of excess thermal mass at the cold end. A physical model was developed that was validated against the data and was able to predict the cool-down time at no-load with any amount of excess mass at the cold end. The same model also predicts other important operating parameters of the cryocooler, such as the net and gross cooling powers at different cold-end temperatures.

In the present study, the same method was applied to an integral rotary cryocooler, K508 by Ricor Cryogenic and Vacuum Systems[2]. Figure 1 describes the cryocooler and some of the parts employed in the tests. Five sets of cool-down/warm-up tests were conducted with 0, 2, 4, 6, and 8 gram copper masses added at the cold end, as explained in the following section. The data once again validated the model assumptions and allowed for the determination of the cryocooler performance at different cold-end temperatures.

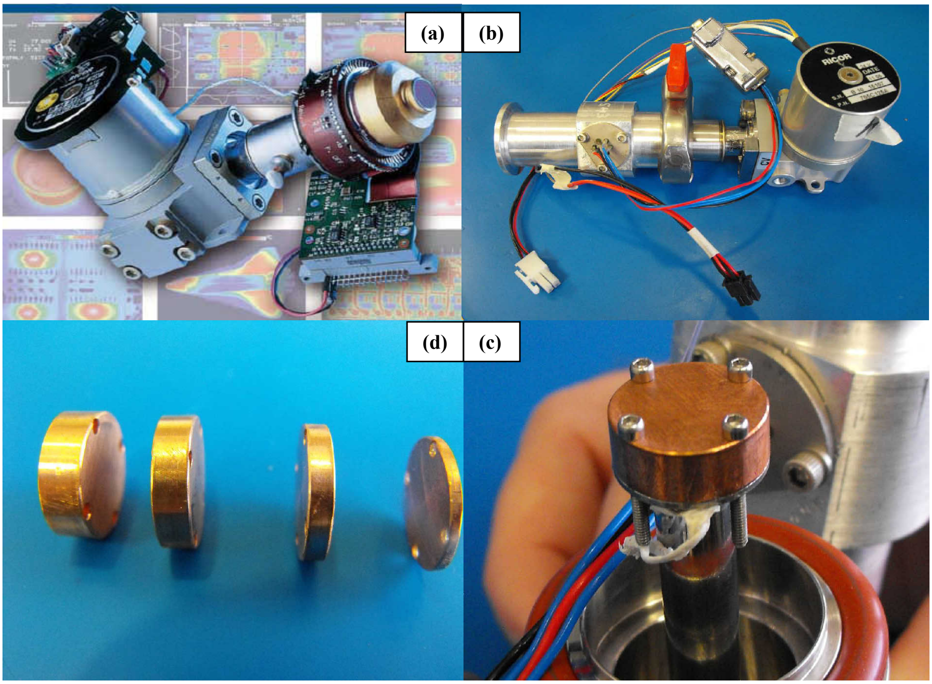

Figure 1: Integral rotary cryocooler: (a) K508 cryocooler with Dewar attached to its cold finger, including IR detector with associated electronics (b) K508 cryocooler with simulation Dewar (c) Thermal mass bolted to cold end of cold finger (d) Four copper masses employed in the tests

EXPERIMENTAL METHOD AND PROCEDURE

The experimental procedure was as follows: With different amounts of added mass at the cold end, the cryocooler was started with no load and the cold end temperature measured as a function of time during cool-down. The ambient temperature was monitored as well. After the minimum temperature was reached, the compressor was turned off and the same temperatures were monitored during warm-up. Figure 2 shows a typical cool-down/warm-up plot of cold end temperature as a function of time. What may look like a continuous curve is actually a succession of many points: data were taken every 2 seconds.

Tests with different amounts of excess thermal mass did not always start at the same warm temperature, nor ended at the same minimum temperature. In order to compare the results on a uniform basis, a temperature interval indicated in Figure 2 by T_H and T_L was selected and used in the same form to reduce the data from all tests.

TRANSIENT HEAT TRANSFER MODEL

Figure 3 describes schematically a model of the system under investigation. It shows a "lean" (no excess mass) cryocooler with an added mass at the cold end. The temperature T in the cryocooler varies with time and with the axial coordinate x between the warm and the cold end; the temperature of the added mass T_m is assumed uniform and varies only with time. Heat is removed from the cold end at a rate Q corresponding to the gross cooling power of the device. Q will be determined from the experimental data. Heat gain at the warm end due to the temperature gradient from the ambient at T_0 is assumed proportional to the temperature difference between the two ends, with a heat gain coefficient U. This assumption remains to be validated.

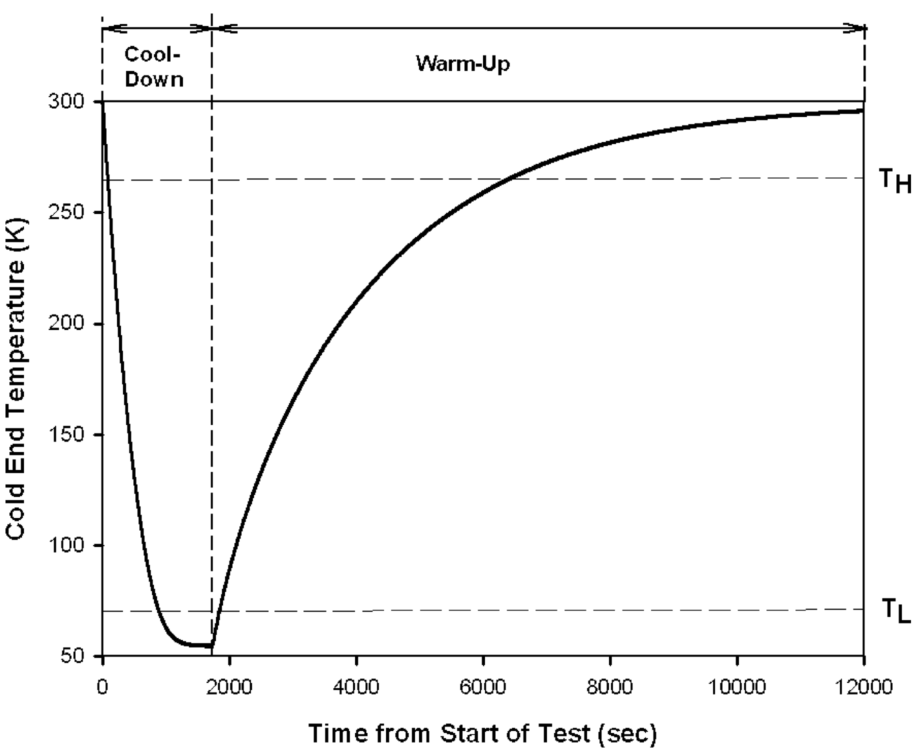

Figure 2: Typical cool-down/warm-up plot of cold end temperature as a function of time

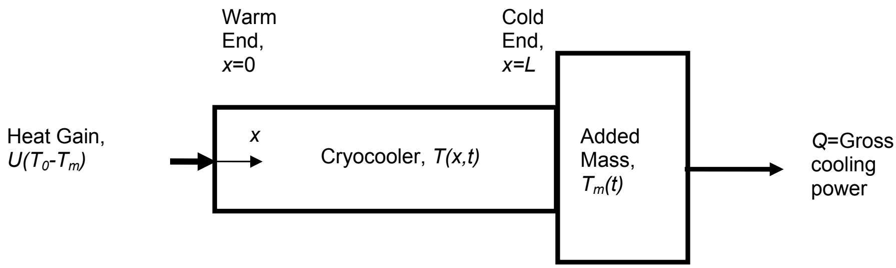

Figure 3: Model of cryocooler with added mass undergoing transient temperature change

The cold end temperature T_m may be determined from a heat balance for the entire assembly:

$$\rho_g c_g A \frac{d}{dt}\int_o^L T dx + m_m c_m \frac{dT_m}{dt} = U(T_0 - T_m) - Q \tag{1}$$

where m_m is the excess mass at the cold end, c_m is its specific heat and U is the heat gain coefficient. The primary component of the cryocooler where temperature variations exist is the regenerator. Under steady-state operation this temperature is known to vary linearly with x[3]; the situation is, however, quite different under transient conditions.

Neglecting the convective effect of the gas, the temperature distribution in the regenerator is governed by the Fourier heat conduction equation. This equation was solved for the present configuration, using an integral method, to determine the temperature distribution T(x). The solution is rather lengthy, and its details may be found in Grossman[1], with the following result:

$$T(x,t) = T_0 + [T_m(t) - T_0][u(x/L) + (1-u)(x/L)^2] \tag{2}$$

Substituting *T(x,t)* from (2) into (1) and performing the integration yields:

$$Q = -\frac{(6r + u + 2)}{6} \rho_g c_g AL \frac{dT_m}{dt} + U(T_0 - T_m) \tag{3}$$

where

$$u = UL/k_g A \qquad r = m_m c_m / \rho_g c_g AL \tag{4}$$

Here A is the cross-sectional area of the regenerator and L is its length. ρ_g , c_g , k_g are the density, specific heat and thermal conductivity of the regenerator matrix, respectively (taking into consideration the porosity and axial contact resistance). These quantities were assumed constant in the calculations conducted here, although it is realized that the specific heat and thermal conductivity can vary significantly over the temperature range of a typical regenerator. A more accurate but more complicated analysis should take into consideration this temperature dependence.

HEAT GAIN COEFFICIENT FROM WARM-UP DATA

The observed behavior of the cold-end temperature as a function of time during warm-up is depicted in the typical plot of Figure 2. During warm-up the gross cooling power Q is zero; Eq. (3) may therefore be solved with Q=0 to give:

$$\ln\left(\frac{T_0 - T_m}{T_0 - T_L}\right) = -\frac{6u}{(6r + u + 2)}\left(\frac{k_g}{\rho_g c_g L^2}\right)t \tag{5}$$

where the warm-up initial condition $T_m(t=0)=T_L$ has been applied. Thus, if our model assumption of the heat gain being proportional to the temperature difference between the warm and cold ends (Figure 3) is correct, a logarithmic plot of the dimensionless cold end temperature $(T_0\text{-}T_m)/(T_0\text{-}T_L)$ vs. time should yield a straight line. The slope of that line should enable us to obtain the heat gain coefficient U.

Figure 4 provides such plots from the five tests. As evident, all the plots are quite close to perfect straight lines. This validates the assumption in the model of heat gain being proportional to the temperature difference between the warm and cold ends. Table 1 lists the values of U calculated from warm-up data for the five tests. As evident, despite the widely varying values of added mass and corresponding heat capacity ratio r, the heat gain coefficient U in all five tests is quite similar, with an average value of 1.6818E-03 W/K and a standard deviation of 1.4205E-04 W/K. Note again that during cool-down, the temperature distribution along the regenerator is not linear, unlike in steady state.

COOLING POWER FROM COOL-DOWN DATA

Having calculated the heat gain coefficients from warm-up data, it is now possible to use the heat transfer model to find the cooling power. The gross cooling power is, from Equation (3):

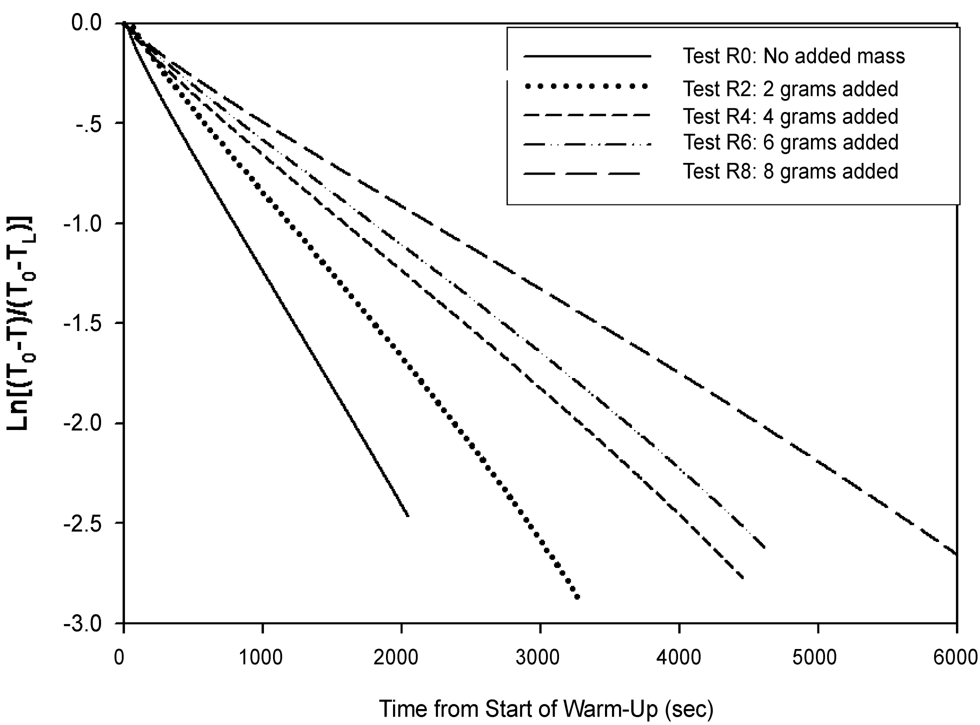

Figure 4: Logarithmic plot of dimensionless cold-end temperature as a function of time during warm-up tests

Table 1: Heat Gain Coefficient U calculated from Warm-Up Data

Test	Added Mass (grams)	Minimum No-Load Temperature Reached (K)	Cool Down time (270K - 70K) (sec)	Heat Capacity Ratio: Excess Mass/ Regenerator r	Slope of Warm-Up Data Plot According to Eq. (5) (sec^{-1})	Calculated $u = \frac{UL}{k_g A}$	Calculated U (W/K)
R0	0.0	54.39	265	0.2271	-1.1814E-03	1.4288	1.5091E-03
R2	2.0	54.92	442	0.7723	-8.5943E-04	1.7459	1.8441E-03
R4	4.0	53.56	553	1.1628	-6.0137E-04	1.5158	1.6010E-03
R6	6.0	55.80	697	1.5617	-5.5040E-04	1.7143	1.8107E-03
R8	8.0	54.36	822	1.9554	-4.2960E-04	1.5568	1.6444E-03

Hence, the net cooling power $Q' = Q - U(T_0 - T_m)$ is:

$$Q' = -\frac{(6r + u + 2)}{6} \rho_g c_g AL \frac{dT_m}{dt} \tag{6}$$

Figure 5 describes the cool-down data from all five tests, showing the cold-end temperature as a function of time. It is evident that cool-down rates vary significantly for the different cases, as expected with the varying amount of excess mass at the cold end (Table 1). Figure 6 shows the net cooling power calculated from the cool-down data for the different tests using Equation (6). To calculate Q', the derivative dT_m/dt was determined numerically at each point from the data presented in Figure 5. The variations in Q' with time differ significantly for the different tests (as do the temperatures, Figure 5). However, when plotted against the cold-end temperature, Q' values are quite close to each other for all the tests, over the entire temperature range. The only exception is the test with no added mass, where the resulting Q' deviates upward from the results of the other tests. The reason for this is not fully understood at this time; it may be attributed to different radiation properties (shape factor and emissivity) of the cold end with and without the added mass. Note that while the temperature vs. time data look quite smooth, Q' looks less so, as a result of the numerical derivative evaluation. The closeness to each other of Q' values for the different tests is attributed to the fact that the net cooling power variation with cold end temperature is practically independent of r; the cryocooler delivers the same cooling power regardless of the excess mass. This is of course expected; the fact that our model reduces data from widely different excess mass tests to conform to this pattern lends credibility to the heat transfer model.

The gross cooling power may now be determined from

$$Q = Q' + U(T_0 - T_m) \tag{7}$$

A numerical regression of the Q' calculated from the data of the five tests shown in Figure 6 yields the following empirical correlation (with an R^2 value better than 0.98486):

$$Q' = -9.559 - 0.1889 T_m + 6.103 \times 10^{-4} T_m^2 - 1.932 \times 10^{-5} T_m^{2.5} + 2.501 T_m^{0.5} \tag{8}$$

where Q' is in Watts and T_m is in Kelvin. This correlation is valid within the temperature range 60 K$< T_m <$ 270 K.

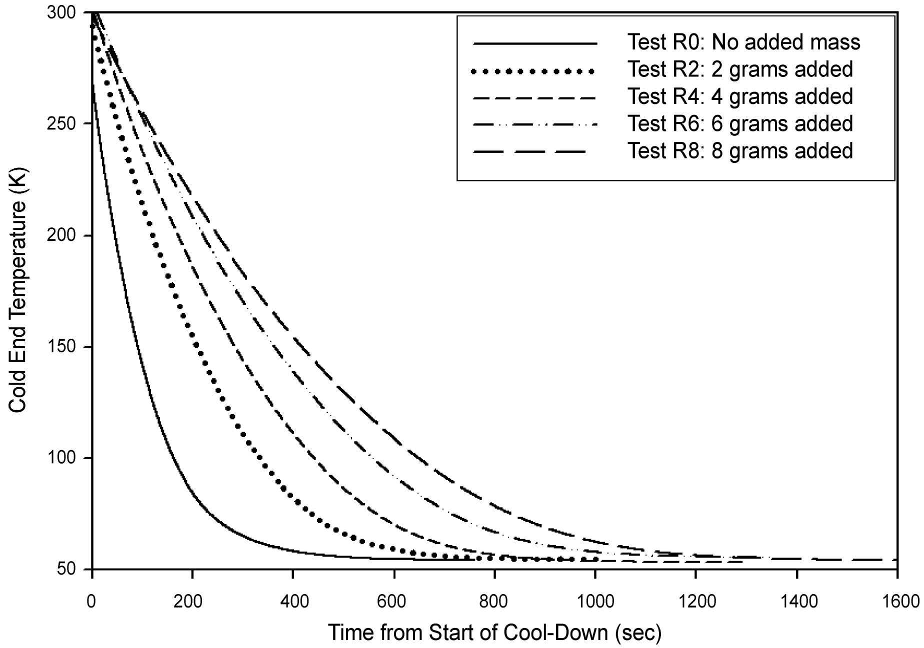

Figure 5: Cool-down data for all five tests (see Table 1 for details)

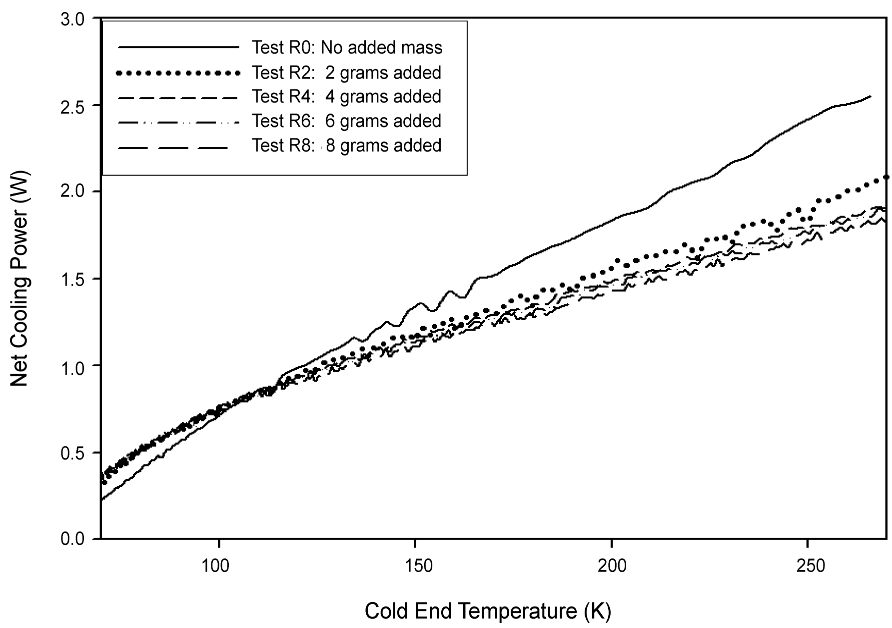

Figure 6: Net cooling power calculated from cool-down data for different tests as a function of cold end temperature

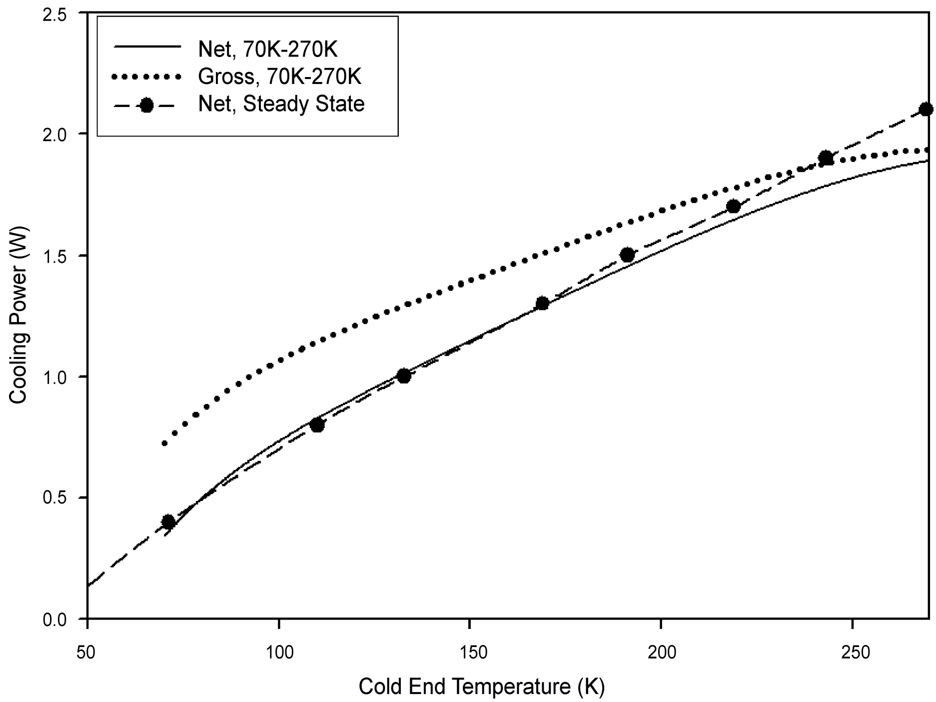

Figure 7: Cooling power as a function of cold end temperature: cool-down and steady-state operation

Figure 7 shows the net and gross cooling powers, plotted as functions of the cold end temperature for the range 70 K - 270 K: *Q'* from the regression in Equation (8) and hence *Q* from Equation (7) with *U* taken as the average for all five tests (1.6818E-03 W/K). As evident, both cooling powers increase with increasing cold end temperature.

COOLING POWER AT COOL-DOWN VS. STEADY-STATE

A series of tests was conducted to determine the net cooling power of the cryocooler under steady state operation. The system was allowed to cool down to its lowest, no-load temperature; then, using a resistor mounted inside the simulation Dewar, increasing amounts of heat were added at the cold end and the corresponding steady-state temperature measured, from the lowest up to ambient temperature. The tests were then repeated in the reverse order – gradually reducing the heat input and measuring the steady state temperature from ambient to no-load. These tests were quite time-consuming as a result of the need to wait for steady state at each temperature. The results are plotted in Figure 7 along with the cooling power under cool-down. The circles indicate the experimental points.

It is evident that the net cooling power at steady-state operation is very close to that measured under cool-down. The difference between the two increases somewhat at the higher temperature range (about 200 K to ambient). Thus, the cool-down/warm-up method provides a good estimate for the net cooling power as a function of cold-end temperature, and is much easier and quicker to measure than by the conventional steady-state method.

CONCLUSIONS

In a series of experiments with an integral rotary cryocooler, the cold-end temperature was measured as a function of time in a complete cool-down and subsequent warm-up cycle, with no load and different quantities of excess mass at the cold end. A transient heat transfer model developed for a system consisting of the “lean” cryocooler with excess thermal mass at the cold end, considers the effects of the cooling power extracted at the cold end and that of the heat gain at the warm end on the cool-down time. The heat gain was assumed proportional to the temperature difference between the warm and cold ends. This assumption was validated against the data from the warm-up tests, and the heat gain factor was calculated and found approximately the same for all experiments. Using the same model with cool-down data enables the determination of both the gross and net cooling powers as functions of time, but more importantly - as functions of the cold end temperature. While the different experiments employed widely varying amounts of excess mass, with varying cool-down rates, the cooling powers were found essentially independent of excess mass.

The net cooling power of the cryocooler under steady-state operation measured in a separate series of tests shows only a small deviation from that under cool-down. The cool-down/warm-up method for evaluating the cooling power of a cryocooler seems simpler than steady-state experiments with a heater simulating load at the cold end. Using the heat transfer model with data from one or two good experiments conducted in the above manner, can yield both the gross and net cooling powers of a cryocooler as functions of the cold end temperature.

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