Exergy-based Performance Estimation of Multistage Cryocoolers with Variable Mid-Stage Cooling Loads

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ABSTRACT

Based on a new Figure of Merit (FOM) for regenerative and recuperative heat exchangers developed and analyzed previously, this study considers a method to estimate the performance of multistage cryocoolers under a variable condition at each stage. Using exergy analysis, the FOM for the heat exchangers used in multistage cryocoolers is extended to include variable load conditions using separation of exergy destruction due to heat transfer and fluid flow. Therefore, the effect of intermediate cooling load and load temperature at the mid-stages under off-design conditions can be estimated. The effect of the irreversibility of the expansion process is taken into account using an exergetic efficiency for expansion convenient for estimating overall performance of multistage cryocoolers with only a few free parameters. The result of the model developed in this study based on exergy analysis is compared to the performance of cryocoolers reported in the literature.

INTRODUCTION

The development of high efficiency cryocoolers based on regenerative and recuperative thermodynamic cycles very much depend on the development of high efficiency regenerative and recuperative heat exchangers. In many applications of heat exchangers in cryogenic refrigerators no cooling would be provided if the effectiveness of the heat exchanger were less than 90 percent[1]. Two major sources of irreversibility in heat exchangers are due to fluid friction and heat transfer[2]. Exergy analysis is a powerful method to quantify these irreversibilities and the rational efficiency of heat exchangers has been defined based on exergy analysis[3]. Figure of Merit (FOM) for the heat exchanger defined based on exergy analysis helps to quantify the effect of design changes on its efficiency as a component in relation to the efficiency of the system. Recently we proposed a Figure of Merit (FOM) for heat exchangers with application to cryogenic refrigerators that represents the important effect of the heat exchangers as a component in overall efficiency of the refrigerator[4]. The analysis of the recuperative and regenerative heat exchangers with application to the design of cryocoolers have shown that a compromise between fluid friction and heat transfer irreversibilities results in a shallow optimum for the FOM of the heat exchangers[5,6]. Another major source of irreversibility in cryocoolers is due to the expansion process. This irreversibility occurs as a result of the process of expansion converting the pressure component of exergy to the thermal component in the refrigerator[7,8].

Estimation of the performance of a particular cryocooler can be based on the correlations of existing cryocoolers that have been reported by several authors, to just name a few[9-11]. In this study we would like to use the FOM of heat exchangers to estimate the performance of multistage regenerative and recuperative cryocoolers using a minimum number of free parameters. The parameters selected are the compressor efficiency, the FOM of the multistage heat exchangers and the expansion efficiency of the expansion processes. The goal is to estimate the performance of single and multistage cryocoolers with variable cooling load at the mid-stage in terms of the estimates of exergetic efficiency of important processes of the components of the cryocoolers. In addition, the effect of heat rejection temperature on the performance of the cryocooler can be estimated.

Exergy-based model of multistage cryogenic refrigerators

The schematic of n-th heat exchanger in a multistage crycooler with basic parameters of the heat exchanger representing its interactions with the inlet and exit streams is shown in Figure 1. The heat exchanger represents a component of multistage cryocooler with the inlet hot and cold streams at any temperature. The temperature of the inlet hot stream of the first stage is assumed to be the temperature of the environment and the inlet temperature of the cold stream of the last heat exchanger is assumed to be the temperature of the last cold reservoir. At the mid-stage between two heat exchangers two interactions with the reservoir are possible. The reservoir can cool the stream resulting in input exergy to the multistage cryocooler. This input exergy is provided by an auxiliary cooler. It should be pointed out that the thermal exergy associated with heat transfer is opposite to the direction of the heat transfer when the reservoir temperature is lower than the temperature of the environment. The mid-stage reservoir can also provide the required cooling load where the thermal exergy is taken from the multistage cryocooler reducing exergy input to the heat exchanger down stream. When a mid-stage load between two heat exchangers exists, an expansion process is assumed that reduces the temperature of the stream below the temperature of the mid-stage reservoir providing the desired cooling load. It is assumed in this study that the heat exchange between a stream and the reservoir is ideal and the reservoir has infinite thermal capacity so that the temperature of the exit stream is equal to the temperature of the reservoir. The temperature of inlet hot stream at the first stage is assumed to be the environmental temperature and the cold stream temperature of the last stage is assumed to be the temperature of the cold reservoir.

We define the figure of merit (FOM) of the n-th heat exchanger by[4]

$$FOM_n = \frac{\dot{E}_{hn,out} - \dot{E}_{cn,in}}{\dot{E}_{hn,in} - \dot{E}_{cn,out}} = \frac{\dot{E}_{netn,hot}}{\dot{E}_{netn,cold}} \tag{1}$$

where $\dot{E}_{netn,hot}$ and $\dot{E}_{netn,cold}$ are the net exergy rate at the hot and cold sides of the n-th heat exchanger, respectively

Given the inlet temperatures of the stream, the effectiveness of the heat exchanger and assuming a balanced heat exchanger, the exit temperatures of the heat exchangers can be determined. For example, assuming streams are ideal gas the FOM for the n-th heat exchanger can be written as[4]

$$FOM_n = \frac{(T_{hn,out} - T_{cn,in}) - T_o \ln(T_{hn,out}/T_{cn,in}) + T_o(1-1/\gamma)\ln(\mathrm{Pr}_n)}{(T_{hn,in} - T_{cn,out}) - T_o \ln(T_{hn,in}/T_{cn,out}) + T_o(1-1/\gamma)\ln[\frac{\mathrm{Pr}_n/(1-\Delta P_{hn}/P_{hn,in})}{(1-\Delta P_{cn}/P_{cn,in})}]} \tag{2}$$

where To is the temperature of the environment, γ is the specific heat ratio, Pr_n is the pressure ratio, and ΔP_{hn} and ΔP_{cn} are the pressure drop for the hot and cold streams, respectively. Using the

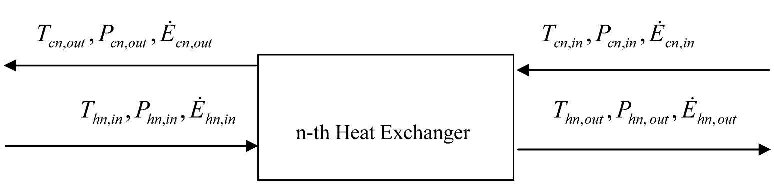

Figure 1. Schematic of heat exchanger with parameters used in the model.

definition of heat exchanger ineffectiveness λ=1−ε with λ << ε and $\Delta P_{hn} \approx \Delta P_{cn} = \Delta P_n$, Eq. (2) can be simplified,

$$FOM_n = \frac{[(T_{hn,in} - T_{cn,in}) - T_o(T_{hn,in} / T_{cn,in} - 1)]\lambda_n + T_o(1 - 1/\gamma)\ln[\Pr_n - \Delta P_n / P_n]}{[(T_{hn,in} - T_{cn,out}) - T_o(1 - T_{cn,in} / T_{hn,in})]\lambda_n + T_o(1 - 1/\gamma)\ln(\Pr_n)} \tag{3}$$

where ΔP_n is the pressure drop and P_n is the average pressure in the n-th heat exchanger, respectively. Based on the above equation we propose the following equation with two correlation parameters for general estimation of the *FOM* of the heat exchangers for application to cryogenic refrigerators.

$$FOM_n = \frac{a_n[(T_{hn,in} - T_{cn,in}) - T_o(T_{hn,in} / T_{cn,in} - 1)]\lambda_n + \eta_{pn}}{b_n[(T_{hn,in} - T_{cn,out}) - T_o(1 - T_{cn,in} / T_{hn,in})]\lambda_n + 1} \tag{4}$$

Where a_n and b_n are correlation parameters and η_{pn} is the exergetic efficiency of pressure component of exergy for the n-th heat exchanger. The exergetic efficiency of the pressure component of a heat exchanger is defined by the pressure component of exergy at the cold side divided by the pressure component of exergy at the hot side of the heat exchanger. It should be pointed out that a compromise between the heat exchanger effectiveness and exergetic efficiency of the pressure component exists in the design of heat exchanger. Therefore, a well-known challenge exists in designing a heat exchanger for the cryogenic refrigerators where both the effectiveness and the efficiency of the pressure component of the exergy are maximized. Comparing equations (3) and (4) one can see that for application to the recuperative heat exchanger, the correlation coefficient $a_n = b_n$ and exergetic efficiency of the pressure component is related to the pressure drop in the heat exchanger. For an ideal heat exchanger Eq. (4) gives $\lambda_n = 0$, $\eta_{pn} = 1$ and $FOM_n = 1$. For application to recuperative heat exchangers it can be shown, using the ideal gas equation of state, that the correlation coefficient is the thermal capacity flow rate per unit exergy flow rate of the pressure component at the hot side of the heat exchanger. Therefore, for the purpose of estimating the performance of heat exchangers, we simplify Eq.(4) using one free correlation parameter $a_n = b_n$ and write Eq.(4) as

$$FOM_n = \frac{[(T_{hn,in} - T_{cn,in}) - T_o(T_{hn,in} / T_{cn,in} - 1)]\lambda_n + \eta_{Pn} e_{pn}}{[(T_{hn,in} - T_{cn,out}) - T_o(1 - T_{cn,in} / T_{hn,in})]\lambda_n + e_{pn}} \tag{5}$$

Where e_{pn} is the exergy flow rate per unit of heat capacity rate and has the unit of temperature. For regenerative heat exchangers in general at least two correlation components should be used. These heat exchangers require integration over period of oscillation to calculate the exergy flow in and out of the heat exchanger. In addition the volume of the heat exchanger and the phase shift between the mass flow and pressure plays a role in defining the figure of merit. Extensive numerical calculations of the regenerative and recuperative heat exchangers in application to well-designed cryogenic refrigerators have shown that the FOM is of the order of 0.7. For a recent high performance recuperator it has been reported that the FOM is of the order 0.85.[12]

Another important source of irreversibility in cryocoolers is the exergy destruction in an expansion process. This process converts pressure exergy into thermal exergy for the purpose of cooling the reservoir. Intermediate cooling is necessary for some cryocooler applications. Exergy efficiency of expansion and throttling processes have been analyzed previously[7,8]. An exergy balance for the processes connected to the expansion process such as heat transfer with the n-th reservoir can be written in general as

$$E_{n,in} = E_{n,out} + E_{Qn} + I_{expn} \tag{6}$$

Where E_{Qn} is the magnitude of the thermal exergy leaving the n-th cold reservoir. The irreversibility of expansion, I_{expn}, used in Eq. (6) includes other irreversible processes related to the expansion process such as heat transfer with the reservoir. We define the expansion efficiency using the following relation

$$I_{expn} = (1 - \eta_{expn})(E_{pn,in} - E_{pn,out}) \tag{7}$$

Using Eqs. (6) and (7), the rate of delivered thermal exergy to the n-th reservoir can be written as

$$E_{Qn} = (E_{n,in} - E_{n,out}) - (1 - \eta_{expn})(E_{pn,in} - E_{pn,out}) \tag{8}$$

The first and last parentheses on the right hand side of the above equation represent the net rate of total exergy and net rate of pressure exergy at the boundary of the system containing the expansion process. This system includes the expansion process and the heat transfer process with the n-th reservoir located between the n-th and (n+1)-th heat exchangers. For oscillating regenerative processes the quantities are time dependent and should be integrated over a period of oscillation.

RESULTS AND DISCUSSION

In order to show the application of the method, we consider the most important components of a single-stage cryocooler and try to estimate its performance using only a few free parameters. The exergy input to the cryocooler is in the form of electric power and it is partially destroyed in the compressor and aftercooler while it is converted to pressure component of exergy with a very small thermal exergy. We assume the aftercooler is highly effective and the thermal exergy exiting it is zero. The effect of the compressor and aftercooler is taken into account defining an exergetic efficiency for the combined compressor and aftercooler. The input exergy to the heat exchanger is partially destroyed by fluid friction and heat transfer and exits the cold side of the heat exchanger. The efficiency of the heat exchanger is given by *FOM* defined in this study and is estimated by Eq. (5). Higher order methods can be used to quantify the effect of the heat exchanger. The other two parameters needed for estimation of the FOM of the heat exchanger are its effectiveness and the efficiency of the pressure component as given in Eq. (5) with $1 \geq \eta_p \geq FOM$. The other major process that affects the performance of cryocoolers is the expansion process converting the available pressure exergy at the exit of the heat exchanger to thermal exergy so that heat can be extracted from the cold reservoir. Using the definition of the exergy efficiency of the processes developed in this study, the rate of exergy delivered for a single-stage cryocooler, dropping the subscript *n* in the equations, can be written as

$$E_Q = W_{comp}\eta_{comp}[FOM - (1 - \eta_{exp})\eta_p] \tag{9}$$

where the *FOM* is given by Eq. (5) for the single-stage heat exchanger where the subscript *n* should be dropped. The magnitude of *FOM* decreases as the cold end temperature *Tc* is reduced. Therefore, a very low value of heat exchanger ineffectiveness is required for positive cooling load from the cold reservoir. The no-load temperature of a cryocooler is the consequence of the bracket in Eq. (9) reaching a value of zero as *FOM* is reduced. This no load temperature is approximately independent of the compressor power as shown in Eq. (9). In addition, because of the definition of cooling exergy given by $E_Q = Q\left(\frac{T_o}{T_c} - 1\right)$, and the variation of the *FOM* with *Tc* as given by Eq. (5), it can be shown that the cooling capacity is approximately a linear function of cold end temperature *Tc* usually observed in developing the load curves for cryocoolers[13]. In addition, the slope of the load curves increases with the compressor power. This can be shown using Eqs. (5) and (9). As the cold end temperature approaches the temperature of the environment the exergy delivered approaches zero and the *FOM* of the heat exchanger approaches the efficiency of the pressure component of exergy η_p. Therefore, the exergetic expansion efficiency η_{exp} should approach zero when the cold end temperature approaches the temperature of environment.

Figure 2 shows the delivered thermal exergy to the cold reservoir as a function of load temperature for the TRW 95 K HEC crycooler characterized in our laboratory[13]. The data was selected as an example to see how the parameters used in the model change when compared with data from the actual cryocooler. In this example the input compressor power is 150 W and the heat rejection temperature is 300 K. In addition, the exergetic compressor efficiency of 0.7 and exergetic efficiency of 0.95 for the pressure component of heat exchanger are selected. The delivered exergy to the cold reservoir shown on the left y-axis is fitted to experimental data. To show the effect of important parameters selected in the model, three values of ineffectiveness of 0.006, 0.008, and 0.01 are selected and the corresponding FOM for the heat exchanger are calculated from Eq. (5) and are given on the right y-axis. For each selected value of ineffectiveness, the corresponding exergetic efficiency of expansion is calculated using Eq. (9) to match the experimental results. The variation of the important free parameters corresponds to the expected physical phenomena they represent. The no-load temperature is approximately 43 K for this cryocooler for the given condition and is mainly the consequence of large drop in the FOM of the regenerator as the load temperature ap-

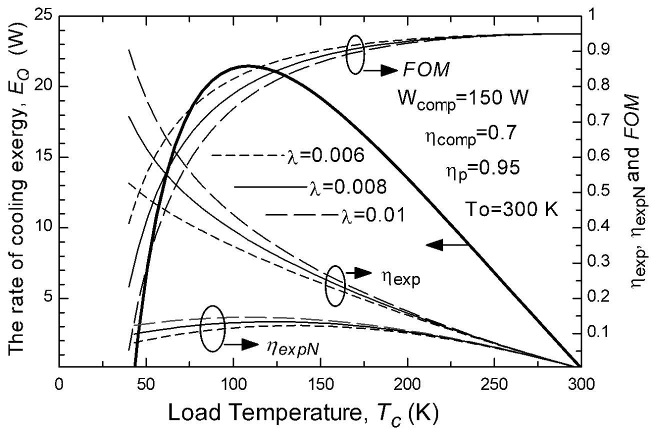

Figure 2. The effect of selected model parameters based on HEC 95 K cryocooler performance.

proaches the no-load temperature. It is interesting to note that the exergetic expansion efficiency, as defined in this study, is reduced when the load temperature increases. This is the consequence of the definition of exergy associated with heat load. The exergetic expansion efficiency can be normalized with respect to *Tc / To* given by $\eta_{expN} = \eta_{exp} T_c / T_o$. The normalized exergetic expansion efficiency is also given on the right y-axis of Figure 2 for comparison. It is interesting to note that η_{expN} shows a shallow maximum near the maximum of the rate of cooling exergy.

The data generated for the case of regenerator ineffectiveness of 0.008 and the corresponding expansion efficiency calculated from the figure are selected for further study. The expansion efficiency corresponding to the selected data is fitted as a function of the cold end temperature and is used to find the effect of the heat rejection temperature on the performance of the cryocooler. Two values of heat rejection temperature of T_o = 330K and T_o =270 K are chosen for comparison to the base temperature of T_o =300K. The results for the rate of exergy delivered to the cold reservoir are given on the left y-axis of Figure 3. The corresponding values of the *FOM* and the selected exergetic efficiency of the expansion process are also given on the right y-axis for comparison. The effect of heat rejection temperature on *FOM* of the heat exchanger is clearly seen. Since the exergetic efficiency of expansion is assumed to be the same for a given cold end temperature, as the heat rejection temperature increases, the exergy delivered to the cold reservoir is reduced.

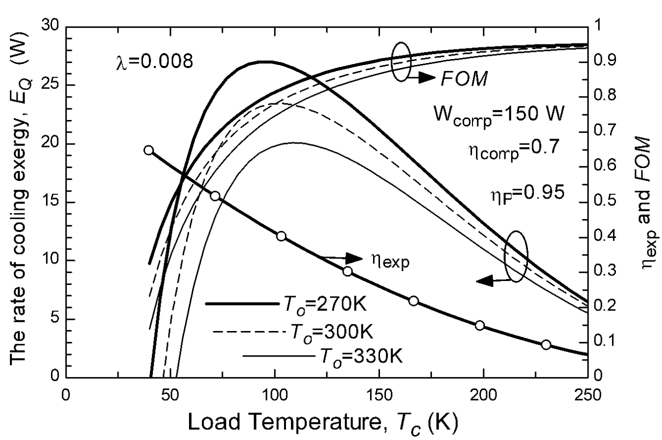

Figure 3. The effect of heat rejection temperature on HEC 95 K cryocooler performance for selected parameters used in the model.

In application of Eq. (8) to multistage cryocoolers with an intermediate cooling load it is more convenient to use Eq. (8) in terms of the interaction of the expansion process with n-th and (n+1)-th heat exchangers. Therefore, the FOM given by Eq. (5) can be used in the exergy balance equations connecting the expansion process with the heat exchangers. We assume that the exit temperature of a stream exiting a cold reservoir is the same as the temperature of the reservoir. Therefore, we can calculate the temperature of all streams from the assumed values of the ineffectiveness of heat exchangers. In order to estimate the exergy going into the (n+1)-th heat exchanger, we introduce a free parameter defined as the ratio of inlet exergy of the (n+1)-th heat exchanger to the exit exergy of the n-th heat exchanger. Therefore Eq. (8) can also be written in terms of the exit of the n-th heat exchanger and the inlet of the (n+1)-th heat exchanger, as

$$E_{Qn} = E_{n,out}(1 - f_n) - (1 - \eta_{expn})(E_{pn,out} - E_{p(n+1),in}) \tag{10}$$

Where f_n is the free parameter. For f_n=1 the exergy at the exit of the expansion process is the same as the exergy at the inlet of the process. Therefore, the exergy delivered to the n-th reservoir would be negative and an auxiliary cooler must be used to maintain the temperature of the reservoir. It should be pointed out that the pressure exergy at the stage n driving the expansion process for intermediate cooling results in lower available pressure exergy that can be used to cool the thermal reservoirs at a lower temperature. We write the last term of Eq. (10) in terms of the difference between the total and the thermal exergy. Therefore, Eq. (10) now becomes.

$$E_{Qn} = \eta_{expn}(1 - f_n)E_{n,out} + (1 - \eta_{expn})(E_{thn,out} - E_{th(n+1),in}) \tag{11}$$

Using Eq. (11) we can write f_n as follows

$$f_n = 1 - \frac{E_{Qn} + (1 - \eta_{expn})(E_{th(n+1),in} - E_{thn,out})}{\eta_{expn} E_{n,out}} \tag{12}$$

Therefore, f_n can be estimated from the given value of E_{Qn}, the estimated value of η_{exp}, and the inlet exergy into the expansion process at the n-th stage. It should be pointed out that the values of the thermal exergy can be calculated from the values of the inlet and exit temperatures of the heat exchangers. The temperatures can be calculated from the values of the effectiveness of heat exchangers; furthermore, is the magnitude of the desired exergy delivered to the n-th cold reservoir. Eq. (12) indicates that even for the ideal expansion process there is a limit for the amount of exergy that can be delivered to the n-th reservoir. Naturally, the delivered exergy cannot be larger than the magnitude of exergy supplied to the n-th expansion process. Eq. (12) can be applied to the (n+1)-th expansion process by replacing the subscript n by n+1. For the last stage of the multistage cryocooler the net exergy driving the expansion process should result in exergy being delivered to the last reservoir.

CONCLUSIONS

A figure of merit based on exergy flow in regenerative and recuperative heat exchangers of cryogenic refrigerators proposed previously is used to develop a model to estimate the performance of multistage cryocoolers with intermediate cooling load. A simple model based on exergy flow for the expansion process is proposed that is convenient for estimation of the performance of multistage cryocoolers. Using exergy analysis, important parameters affecting the performance estimation of multistage cryocoolers with the variable mid-stage cooling load were identified. The model is applied to a single stage cryocooler and the range of free parameters were obtained from comparison to the experimental results. The model is shown to be capable of correctly estimating the effect of the heat rejection temperature on the performance of the crycooler. Efforts are underway to find a simple model to estimate the exergetic efficiency of expansion process for use in the model and the equations developed in this study for estimating the performance of multistage cryocoolers.

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