Performance of Thermoelectric Coolers with Boundary Resistance for Different Optimization Criteria

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ABSTRACT

The cooling capacity, efficiency and cooling flux for a single stage thermoelectric cooler, as part of a multistage cooler, is analyzed in this study. The Lagrange Multiplier method is used to develop nonlinear algebraic equations for the optimum condition for different objective functions. The analysis includes the effect of heat leak, thermal boundary resistances with thermal reservoirs and the electric contact resistance at the boundary of the thermoelectric element. The effect of thermal contact resistance and thermal spreading resistance are included in the thermal boundary resistance. In addition to important thermoelectric properties, e.g., the effect of the electric and thermal contact resistances, the thickness of the thermoelectric material and the area aspect ratio of thermoelectric elements on the performance of the thermoelectric cooler at the optimum condition is presented and discussed. The results are presented in terms of non-dimensional variables convenient for system design analysis and performance evaluation.

INTRODUCTION

Thermoelectric refrigerators possess great advantages for cooling of space-based infrared detectors because they are solid state devices having no moving parts and are miniature, highly reliable, and easy to integrate into the system. There have been many applications of the thermoelectric effect in both cooling and power generation[1,2]. The development of thermoelectric refrigerators for application at cryogenic temperatures is hampered by the fact that thermoelectric materials for application at low temperatures are not available and challenges exist in improving their cooling capacity and efficiency. In this study we consider a single stage thermoelectric refrigerator and investigate its optimization including the effect of important losses on the optimum performance of the refrigerator (see Fig. 1). We use the Lagrange Multiplier optimization method to find the effect of thermal boundary resistance, electric contact resistance, heat leak, and the geometry of thermoelectric elements on the performance of the refrigerator. In addition, different optimization criteria are used to find the performance of the refrigerator at the optimum conditions. The optimization is performed with respect to cooling capacity and Coefficient of Performance (COP) and the effects of important parameters on the cooling flux at the optimum condition are presented. It is important to find the thermodynamic bounds for the

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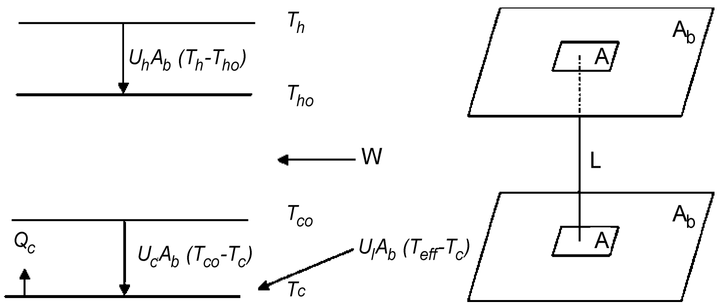

Figure 1. Schematic of thermoelectric refrigerator and the basic parameters used in the model.

performance of thermoelectric refrigerators and how the optimum performance changes with variation in important design parameters.

MODELING AND OPTIMIZATION OF THERMOELECTRIC REFRIGERATORS

The thermodynamic model of thermoelectric (TE) refrigerators using the average thermal and electrical properties of the TE couples in terms of electric current has been previously developed[3-6]. The optimization of a thermoelectric refrigerator including the effect of thermal boundary resistance at the hot side of refrigerators has been reported[7,8]. The energy balance at the cold side and hot side of thermoelectric refrigerators including the heat leak, the thermal boundary resistances at the hot and cold sides, and the effect of electric contact resistance of the TE element can be written as:

$$\dot{Q}_c = 2N[\alpha I T_c - 0.5 I^2 \rho_{eff} L/A - kA(T_h - T_c)/L] - U_l A_b (T_{eff} - T_c) \tag{1}$$

$$2N[\alpha I T_c - 0.5 I^2 \rho_{eff} L/A - kA(T_h - T_c)/L] - U_l A_b (T_{eff} - T_c) - U_c A_b (T_{co} - T_c) = 0 \tag{2}$$

$$2N[\alpha I T_h + 0.5 I^2 \rho_{eff} L/A - kA(T_h - T_c)/L] - U_h A_b (T_h - T_{ho}) = 0 \tag{3}$$

where the parameters in the equations are defined as:
A= cross sectional area of each TE element
A_b=area of the base for each element
I = electric current
k = thermal conductivity of the TE couple
L= the length of the TE element
N = number of TE elements
$Q_c = U_c(T_{co} - T_c)$ = cooling capacity of the TE refrigerator
T_c = temperature of the cold side of the TE couple
T_{co} = temperature of the cold reservoir
T_{eff} = effective temperature of the source of heat leak to the cold side
T_h = temperature of the hot side of the TE couple
T_{ho} = temperature of the hot reservoir
U_c = average thermal conductance at the cold side including the thermal spreader
U_l =thermal conductance for the heat leak to the cold side
U_h = average thermal conductance at the hot side including the thermal spreader
α = Seebeck coefficient of the TE couple
$\rho_{eff} = \rho + 2\rho_c/L$=effective electric resistivity of the TE couple

ρ = electric resistivity of the TE couple
ρ_c = electric contact resistance

The electric input power to the TE refrigerator can be written as

$$W_c = 2N[I^2\rho_{eff}L/A + \alpha I(T_h - T_c)] \tag{4}$$

The cooling capacity of the refrigerator can in general be considered a function of the electric current I, the cold side temperature T_c and the hot side temperature T_h as given by Eq. (1). The cooling capacity of the refrigerator given by Eq. (1) can be optimized by using the method of Lagrange Multipliers with the energy balance Eqs. (2) and (3) as the constraints. For the given thermoelectric properties of TE elements and values of thermal conductance at the hot and cold sides as well as the geometry of the TE elements, the following five equations give the optimum cooling capacity of the thermoelectric refrigerator.

$$-\lambda_1(T_{hn} - I_n / Z_{eff}T_{ho}A_n) + (1-\lambda_2)(T_{cn} - I_n / Z_{eff}T_{ho}A_n) \tag{5}$$

$$\lambda_1(A_n + U_{hn} - I_n) + (1+\lambda_2)A_n = 0 \tag{6}$$

$$I_n + A_n + U_{\ln} - \lambda_1 A_n - \lambda_2(I_n + A_nU_{cn} - U_{\ln}) = 0 \tag{7}$$

$$I_nT_{hn} + 0.5I_n^2 / Z_{eff}T_{ho}A_n - A_n(T_{hn} - T_{cn}) - U_{hn}(T_{hn} - 1) = 0 \tag{8}$$

$$I_nT_{cn} - 0.5I_n^2 / Z_{eff}T_{ho}A_n - A_n(T_{hn} - T_{cn}) - U_{cn}(T_{co}/T_{ho} - T_{cn}) - U_{\ln}(T_{eff}/T_{ho} - T_{cn}) = 0 \tag{9}$$

The non-dimensional parameters used in the equations are defined by:
$A_n = A/A_b$, normalized area of TE element
$In = \alpha IL/(A_bk)$, normalized electric current
$Qcn = Q_cL/2NA_bT_{ho}k$, normalized cooling capacity
$Q_{hn} = Q_hL/(2NA_bT_{ho}k)$, normalized rate of heat transfer at the hot side
$T_{cn} = T_c/T_{ho}$, normalized temperature of the cold side
$T_{hn} = T_h/T_{ho}$, normalized temperature of the hot side
$U_{hn} = U_hL/k$, normalized thermal conductance at the hot side
$U_{cn} = U_cL/k$, normalized thermal conductance at the cold side
$U_{ln} = U_lL/k$, normalized thermal conductance for the heat leak
$Z_{eff} = \alpha^2L/[k(\rho L + 2\rho_c)]$, figure of merit including the electric contact resistance.

The parameters λ_1 and λ_2 are the Lagrange Multipliers. Eqs. (5) to (9) give the values of unknowns parameters λ_1, λ_2, T_{cn}, T_{hn} and I_n for the optimum cooling capacity of the refrigerator.

The method of Lagrange Multipliers can also be used to obtain the condition for the optimum COP. The equations developed based on the method in terms of non-dimensional parameters can be written as

$$\left(T_{cn} - \frac{I_n}{A_nZ_{eff}T_{ho}}\right)W_n - \left(\frac{2I_n}{A_nZ_{eff}T_{ho}} + T_{hn} - T_{cn}\right)Q_{cn} - \lambda_3\left(T_{hn} - \frac{I_n}{A_nZ_{eff}T_{ho}}\right)W_n^2 - \lambda_4\left(T_{cn} - \frac{I_n}{A_nZ_{eff}T_{ho}}\right)W_n^2 = 0 \tag{10}$$

$$(I_n + A_n + U_{\ln})W_n + I_n Q_{cn} - \lambda_3 A_n W_n^2 - \lambda_4 (I_n + A_n + U_{\ln} + U_{cn})W_n^2 = 0 \quad (11)$$

$$A_n W_n + I_n Q_{cn} - \lambda_3 (A_n - I_n + U_{hn})W_n^2 - \lambda_4 W_n^2 = 0 \quad (12)$$

$$Q_{cn} = I_n T_{cn} - 0.5 I_n^2 / Z_{eff} T_{ho} A_n - A_n (T_{hn} - T_{cn}) - U_{\ln}(Z_{eff} / T_{ho} - T_{cn}) = 0 \quad (13)$$

$$W_n = I_n^2 / Z_{eff} T_{ho} A_n + I_n (T_{hn} - T_{cn}) \quad (14)$$

Eqs. (8) to (14) are seven equations and seven unknowns for W_n, Q_{cn}, I_n, T_{cn}, T_{hn}, l_3 and l_4 for the optimum COP of the thermoelectric refrigerator given other normalized system parameters.

RESULTS AND DISCUSSION

Cooling capacity**,** the coefficient of performance (COP) and the cooling flux are important quantities for the performance evaluation of thermoelectric refrigerators. The optimum cooling capacity can be obtained from five simultaneous nonlinear equations given by the Eqs. (5) to (9). The equations can be solved for l_1, l_2, T_{cn}, T_{hn} and I_n for the optimum cooling capacity with other normalized parameters assumed to be given. In this study we concentrate on the effect of thermal boundary resistances at the hot side of the cooler, the length of the thermoelectric element and electric contact resistance on the optimum performance of the refrigerator. The effect of thermal boundary resistance at the hot side of the cooler is especially important for refrigerators designed for space applications where the heat must be rejected through the radiator to space. The thermal resistances can also be due to thermal contact resistance and thermal spreading resistance. Figure 2 shows the optimum cooling capacity and corresponding normalized electric current as a function of the normalized thermal conductance at the hot side of refrigerator for different values of the temperature of the cold reservoir. The length of the thermoelectric element is fixed in these calculations at L=0.0001m. In all calculations the important thermoelectric properties are assumed to be fixed at electric resistivity of $\rho=10^{-5}$ Ωm, Seebeck coefficient α=0.0002 V/K and thermal conductivity of 1.5 W/mK resulting in the non-dimensional figure of merit of ZT_{ho}=0.8. In addition, the temperature of the hot reservoir is fixed at T_{ho}=300 K in this study. Other parameters of interest used in the calculations are given in the figure. The significant effect of thermal conductance at the hot side of the refrigerator on the optimum cooling capacity is evident in the figure. The figure shows that no cooling is provided if the thermal conductance at the hot side is not reasonably high. It should be pointed out that in this study the effect of the thermal spreader is modeled as an effective increase of the thermal conductance calculated for the base area A_b. Figure 3 shows the same

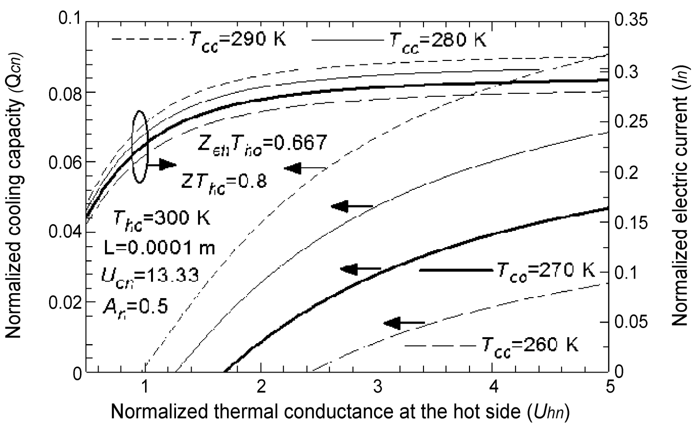

Figure 2. Optimum cooling capacity and corresponding electric current as a function of thermal conductance for different temperatures of the cold reservoir.

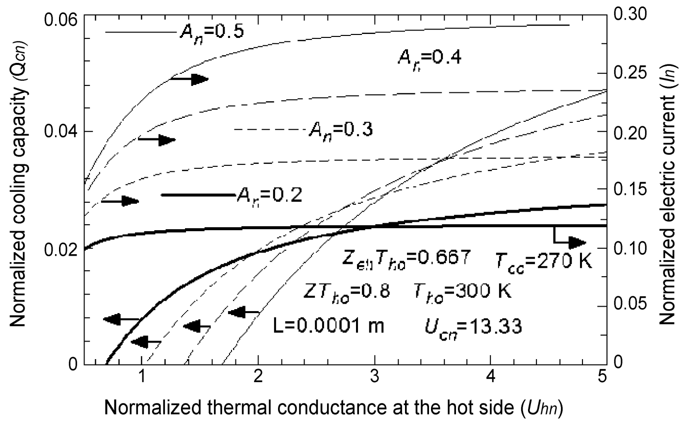

Figure 3. Optimum cooling capacity and corresponding electric current as a function of thermal conductance for different values of normalized area of thermoelectric elements.

results as in Figure 2 using the normalized area of TE elements A_n as a parameter. The normalized area has influence on the applied electric current at the optimum cooling capacity. The electric current increases when the normalized area of TE elements increases. As expected, the optimum cooling capacity increases as the thermal conductance at the hot side increase but the rate of increase depends on the value of the normalized area of TE elements. It should be pointed out that as the cooling capacity increases the effect of higher thermal conductance at the hot side of refrigerator becomes more important and plays an important role on the performance of the refrigerator.

Figure 4 shows the effect of normalized thermal conductance at the hot side on optimum cooling capacity and corresponding normalized electric current with electric contact resistance as a parameter. In this figure the length of the thermoelectric element is fixed at L=0.0002 m. The effect of electric contact resistance on the performance of the refrigerator is significant. This is due to the fact that the effective figure of merit of TE couples is reduced as the electric contact resistance is increased. The effect also depends on the length of the thermoelectric element. For large electric contact resistances no cooling is provided unless thermal conductance at the hot side is increased to high values. Figure 5 shows the effect of the length of thermoelectric elements on the optimum normalized cooling capacity and corresponding cooling flux with the electric contact resistance as

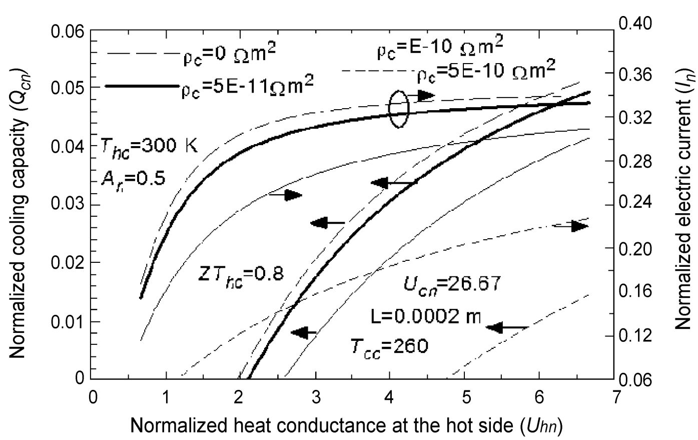

Figure 4. Optimum cooling capacity and corresponding electric current as a function of thermal conductance for different values of electric contact resistance.

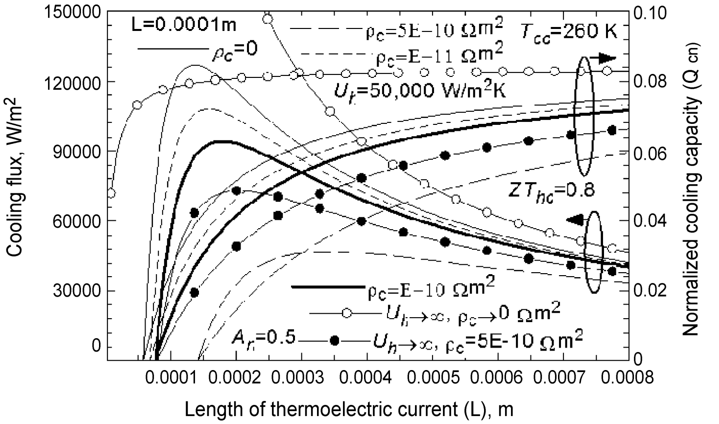

Figure 5. Optimum cooling capacity and corresponding cooling flux as a function of the length of thermoelectric elements for different values of electric contact resistance and thermal conductance.

a parameter. Most of the results shown in the figure are for a fixed value of U_h =50,000 W/m²K for the thermal conductance at the hot side assuming an effective thermal spreader exists for the heat rejection. These are shown by curves without open or solid circles in the figure. The cooling flux at the optimum cooling capacity has a maximum at a particular length of the thermoelectric element. The cooling flux increases as the electric contact resistance is reduced reaching a maximum value of more than 125,000 W/m² assuming no electric contact resistance. The value of the cooling flux is limited by the value of thermal conductance used in these calculations. As the value of thermal conductance at the hot side increases, the maximum cooling flux increases as well. In fact, for infinite thermal conductance at the hot side and zero electric contact resistance, the cooling heat flux approaches infinity. To show the effect, the results for two cases for very large values of thermal conductance at the hot side and two values of electric contact resistances of ρ_c = 0 Wm^{-1} and ρ_c = 5E-10 Wm^{-1} are given in the figure. These are shown by the curves with open circles and solid circles, respectively.

Figure 6 shows the optimum Coefficient of Performance (COP) and corresponding normalized electric current as a function of thermal conductance with electric contact resistance as a parameter. Electric contact resistance has significant effect on the optimum COP especially at high values of electric contact resistance. The effect of electric contact resistance on normalized electric current at the optimum COP is not significant especially at high values of normalized thermal

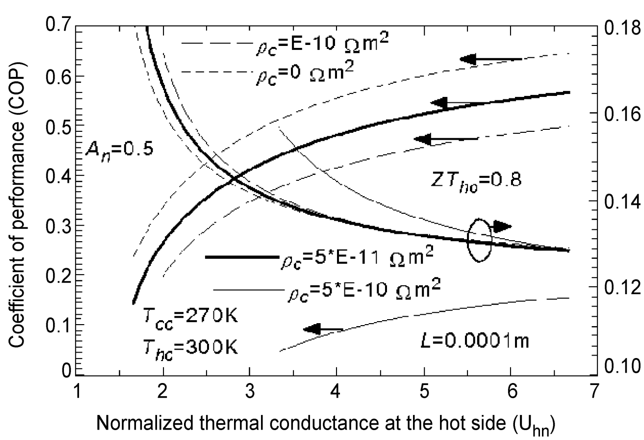

Figure 6. Optimum coefficient of performance and corresponding electric current as a function thermal conductance for different values of electric contact resistance.

conductance. It is interesting to compare the results of optimization given by Eqs. (8) to (14) to analytical solutions obtained for an ideal case of no electric contact resistance as well as zero thermal resistance at the cold and hot sides of the refrigerator. The bound for the optimum COP is obtained to be 0.83 for the parameters used in these calculations. This value compares favorably with the values obtained in Figure 6 for high values of thermal conductance at the hot side and zero electric contact resistance. It should be pointed out that a high value of thermal conductance at the cold side is used in Figure 6. The values obtained for the normalized current at the optimum COP match the results given in Figure 6 for the similar condition. It should be pointed out that the relation for the normalized electric current at the optimum bound under ideal conditions is given by $ZT_{ho}A_n(1-T_{co}/T_{ho})/(\sqrt{1+Z(T_{ho}+T_{co})/2}-1$[5]. To include the effect of electric contact resistance, thermal conductance at the hot side and the cold side as well as the heat leak on the optimum COP, Eqs. (8) to (14) developed in this study must be solved. Figure 7 shows the optimum COP and the corresponding cooling flux as a function of the length of the thermoelectric element for a given value of U_h=50,000 W/m^{2}K for four values of electric contact resistance. The cooling flux at the optimum COP shows a maximum for a given length of thermoelectric element. This maximum is a strong function of the value of electric contact resistance. When the values of electric contact resistance are small their effect on the optimum COP are not significant and show a saturation as the length of thermoelectric element is increased. A clear compromise between the optimum COP and the corresponding cooling flux is seen in the figure. In general, significant compromise between the cooling capacity and efficiency exists in thermoelectric refrigerators reported previously[8].

CONCLUSIONS

Using a control thermodynamic model of thermoelectric refrigerators, their cooling performance and their efficiency were studied with emphasis on the effect of the thermal boundary resistance, the electric contact resistance, the ratio of the area of thermoelectric element to the base area, and the length of the thermoelectric elements. The optimization of a single stage thermoelectric refrigerator was performed using the method of the Lagrange Multipliers. Analytical equations are developed to obtain the optimum performance of the thermoelectric refrigerator in terms of non-dimensional parameters convenient for parametric studies and design analysis of the refrigerator. Cooling flux is obtained at the optimum cooling capacity and efficiency. The electric contact resistance and thermal conductance at the hot side have significant influence on the performance of thermoelectric refrigerators. The model has the capability to assess the effect of important system parameters on the optimum condition of thermoelectric refrigerators for parametric studies and design analysis.

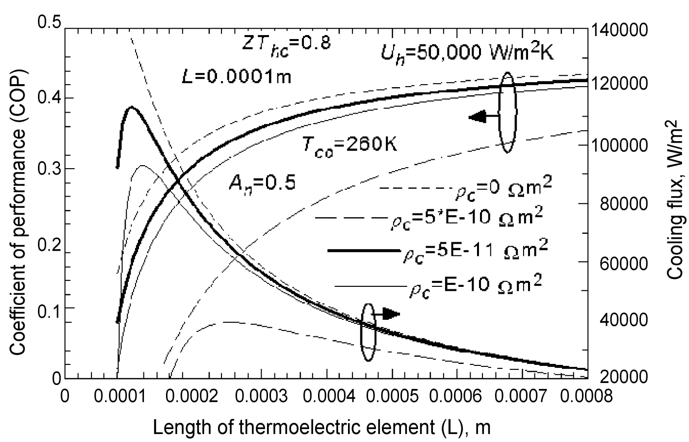

Figure 7. Optimum coefficient of performance and corresponding cooling flux as a function of the length of thermoelectric elements for different values of electric contact resistance.

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REFERENCES

1. Rowe, D.M., *CRC Handbook of Thermoelectric*, CRC, Boca Raton, FL, (1995).
2. DiSalvo, F.J., "Thermoelectric Cooling and Power Generation," *Science*, Vol. 285, (1999), pp. 703-706.
3. Lee, K.H., and Kim, O.J., "Analysis on the Cooling Performance of Thermoelectric Micro-cooler," *International Journal of Heat and Mass Transfer*, Vol. 50, (2007), pp. 1982-1992.
4. Phelan, P.E., Chiriac, V.A., Lee, T.T., "Current and Future Miniature Refrigeration Cooling Technologies for High Power Electronics," *IEEE Transactions on Components and Packaging Technologies*, Vol. 25, (2002), pp. 356-365.
5. Riffat, S.B., Ma, X., "Improving the coefficient of performance of thermoelectric cooling systems: a review," *International Journal of Energy Research*, Vol. 28, (2004), pp. 753-768.
6. Min, G. and Rowe, D.M., "Improved model for calculating the coefficient of performance of a Peltier Module," *Energy Conversion and Management*, Vol. 41, (2000), pp. 163-171.
7. Pettes, A.M., Hodes, M.S., and Goodson, K.F., "Optimized Thermoelectric Refrigerator in the Presence of Thermal Boundary Resistance," *IEEE Transaction on Advanced Packaging*, 32, (2009), pp. 423-430.
8. Razani, A., Fraser, T., and Dodson, C., "A thermodynamic model for the effect of thermal boundary resistance on multistage thermoelectric cryogenic refrigerator," *Adv. in Cryogenic Engineering*, Vol. 57, Amer. Institute of Physics, Melville, NY (2012), pp. 1899-1907.