Matching of Superconducting Motors and Cryocoolers

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ABSTRACT

This paper deals with the matching superconducting motors with cryocoolers. In the steady state the cooling power of the cooler is equal to the sum of the losses. The latter include the AC losses of the magnets, losses via the current leads, power needed to distribute the cooling (e.g. by a cryofan), and the cryostat losses. The AC losses go down if the temperature is reduced, but also the *COP* of the cryocooler goes down. In order to solve this and other conflicting requirements a complete set of expressions is derived which governs the system. This allows the evaluation of the interdependences of the various system parameters such as type of cryocooler, type of flow distribution, working fluid (helium, hydrogen, neon), working pressure, etc.

INTRODUCTION

Electromotors have wide-spread applications. Nowadays the magnetic fields are generated by copper coils. Using superconductors instead of copper may reduce the losses and hence contribute to the energy efficiency. Furthermore superconducting coils give high power density and fast response times. At the company Oswald in Miltenberg, Germany,[1] several superconducting motors are under investigation. One of them, studied in the so-called SUTOR project, is shown in Fig. 1. It involves a torque motor where the windings in the stator are made of high-Tc superconducting tape and the rotor contains an array of permanent magnets. The stator (the outer ring) produces a rotating magnetic field, thus driving the rotor. The superconducting coils are arranged on a diameter of about 60 cm. A typical torque is 20 kNm.

CRYOGENIC CONSIDERATIONS

In principle the superconducting coils can be cooled by a cryogenic fluid such as liquid nitrogen. However, this limits the temperature range to temperatures above the triple point. So-called dry cooling, where the superconductors are cooled by a circulating gas which is cooled by a cryocooler, extends the temperature range. The superconducting coils generate AC losses which should be reduced to the minimum. For this reason the cryostat walls are made of a material such as fiber-glass reinforced epoxy. Iron losses in the cryogenic regions are avoided by putting each superconducting coil in its own little coil cryostat while the iron remains at room temperature.
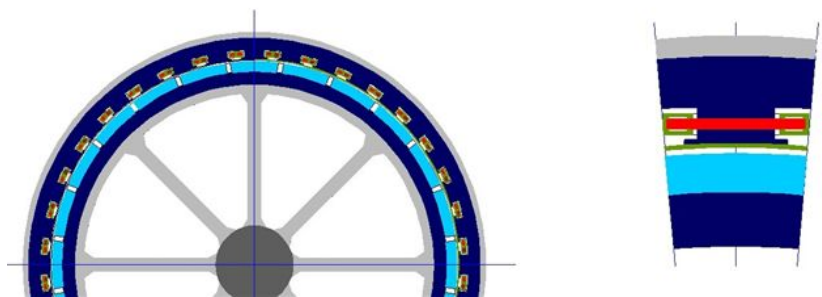
Figure 1. Left: upper half of the SUTOR system. The inner diameter is about 60 cm and the design torque is 20 kNm. Right: section of the motor showing a superconducting coil in its cryostat. Red: superconducting coils, light blue: permanent magnets, dark blue: iron. Only the coils are at low temperatures. The cryocooler and the flow-distribution system are not shown.

The system has four key components: the superconducting coils, with their individual cryostats, a fan, which drives the circulating gas, a cryocooler, and a heat exchanger between the gas and the cryocooler (Fig. 2). The AC losses go up with the temperature, so the temperature should be low. On the other hand the cooling power of the cryocooler also goes up with the temperature, so, from this point of view, the temperature should be high. This is the first conflicting requirement. For the flow channels in the coil cryostats and the heat exchanger there is the usual conflict between long, narrow channels, which give good heat exchange, and wide, short channels which give low flow resistance. This is the second conflicting of requirement. This paper gives a closed set of expressions which allows the optimization of the over-all system parameters.

In the system the pressure and temperature varies with position. However, in a well-designed system, these variations are small with respect to the average pressure and temperature, so we consider these variations only to first order. As the working fluid helium, neon, and hydrogen were considered. It turns out that helium and hydrogen give about the same results and neon gives a slightly worse performance. In this paper we will focus on helium which is considered as an ideal gas
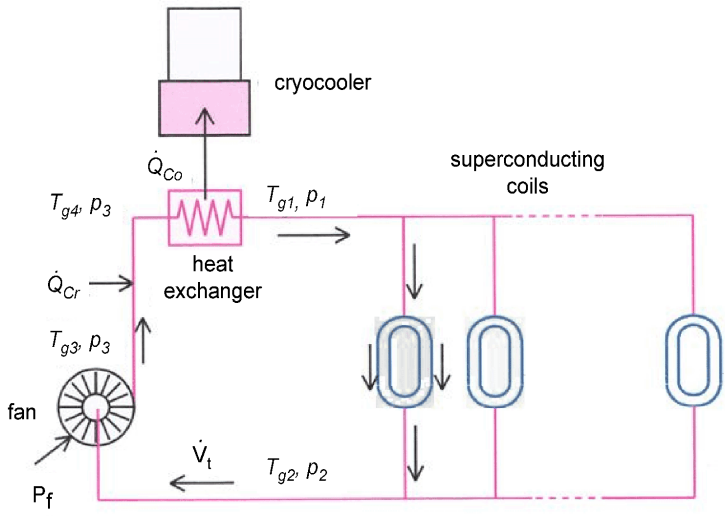

Figure 2. Schematic diagram of the cooling circuit.

At the moment no experimental data about the AC losses $\dot{Q}_{AC}$ as function of the superconductor temperature T_S and the torque M are available for our system, so they are obtained from theoretical models[2,3]. The dependence of the cooling power of the cryocooler $\dot{Q}_{Co}$ on the cryocooler temperature T_{Co} is obtained from the GM-cryocooler specifications of the company Cryomech[4].

IDEAL CASE

In the ideal case the superconductor temperature T_S is equal to the cryocooler temperature T_{Co}. If there are no other losses except the AC loss and the cryostat heat leak $\dot{Q}_{Cr}$ then, in the steady state, $\dot{Q}_{Co} = \dot{Q}_{AC} + \dot{Q}_{Cr}$. An example is given in Fig.3. The blue curve is the cooling-power curve and the red curve gives the AC-loss for a certain torque M. There are two solutions for $\dot{Q}_{Co} = \dot{Q}_{AC} + \dot{Q}_{Cr}$: one at about 70 K, which is unstable, and one at 38 K which is stable. If the torque increases the green point moves up and the red point moves down. The maximum torque is at a temperature of 53 K. Increasing the torque above this value will induce a runaway of the system which has similarities to the quenching of superconducting magnets.

In practical situations the maximum torque is reached at temperatures in the 50 – 55 K range. Using nitrogen as the working fluid[5] would limit the temperature range to temperatures above 63.15 K (the triple-point temperature), resulting in a lower maximum torque. Furthermore, the system would operate in the range of unstable solutions, so provisions are required to force the system to run in stable operation. Finally, using liquid nitrogen, involves the risk of cavitation in the cryofan.

In the remainder of this paper non-ideal situations will be considered where temperature differences due to imperfect heat exchange and flow resistances in the coil cryostats and the heat exchanger are taken into account.

PRESSURES

For pressure drops Δp in channels with length L and hydraulic diameter D_h we use the relation

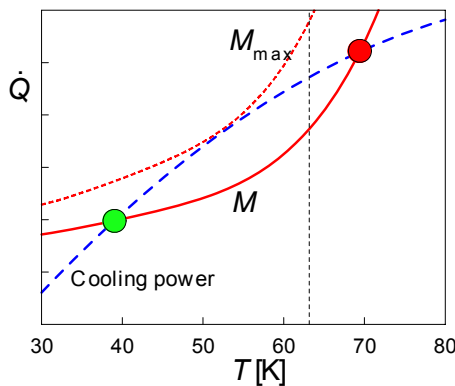$$\Delta p = f_r \frac{1}{2} \rho v^2 \frac{L}{D_h}. \tag{1}$$

Figure 3. Dashed curve: cooling power of the cryocooler; Solid curve: AC loss of the motor for a given torque M. The left and the right dots indicate the high- and low-temperature solutions respectively. The left dot is the stable solution. The dotted curve gives the AC-loss curve at maximum torque. The vertical line gives the triple point of nitrogen.

Here f_r is the Darcy friction factor for turbulent flow given by

$$f_r = \frac{0.4}{R_e^{0.25}}. \tag{2}$$

For laminar flow in an infinitely wide slit $f_r = 96/R_e$. The gas velocity is $v = \dot{V}_t/A$ with A the total flow area of the flow channels involved and the total volume flow in the circuit given by

$$\dot{V}_t = \dot{n}_t \frac{RT}{p} \tag{3}$$

where $\dot{n}_t$ is the molar flow rate, R is the molar ideal gas constant, and p and T the average pressure and temperature respectively. The Reynolds number is given by

$$R_e = \frac{\rho v D_h}{\eta} \tag{4}$$

with η is the viscosity, approximated by the relation

$$\eta = 0.519 T^{0.64} \ \mu\text{Pas} \tag{5}$$

and ρ the density given by

$$\rho = \frac{M_m p}{RT}. \tag{6}$$

Here M_m is the molar mass of the working fluid. For slits with width W_s and spacing t_s

$$D_h = \frac{2 W_s t_s}{W_s + t_s}. \tag{7}$$

The pressure difference over the fan is in good approximation

$$p_3 - p_2 = \Delta p_{max} \left[1 - \left(\frac{\dot{V}_t}{V_{max}} \right)^{p_w} \right]. \tag{8}$$

We use the relations $\Delta p_{max} = \frac{1}{2} c_f \rho \ (D_f - D_0)$ and $V_{max} = V_f (D_f - D_0)^2$. In these expressions D_f is the fan diameter and the values of the parameters $c_f = 10^5$ m/s², $V_f = 4$ m/s, $D_0 = 19$ mm, and $p_w = 2.4$ are obtained from the specifications of the range of fans produced by the company Cryozone[6]. The value of c_f is for the rated rotation speed ω of the fan. Note that Δp_{max} depends quadratically on ω while V_{max} is proportional to ω and the fan height.

GAS TEMPERATURES

In the circuit we consider four different gas temperatures (see Figs.2 and 4). The AC losses cause an increase in the temperature of the gas according to

$$\dot{Q}_{AC} = \dot{n}_t C_p \left(T_{g2} - T_{g1} \right). \tag{9}$$

The power P_f the power applied at the fan causes a temperature rise at the fan given by

$$P_f = \dot{n}_t C_p \left(T_{g3} - T_{g2} \right). \tag{10}$$

The temperature effect of the cryostat heat leak is given by

$$\dot{Q}_{cr} = \dot{n}_t C_p \left(T_{g4} - T_{g3} \right). \tag{11}$$

Furthermore

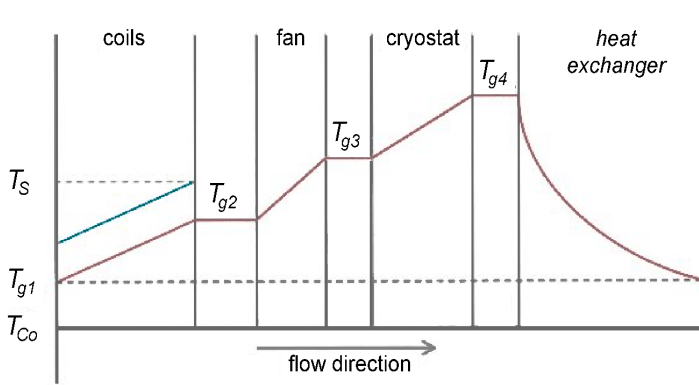$$\dot{Q}_{Co} = \dot{n}_t C_p \left(T_{g4} - T_{g1} \right). \tag{12}$$

Figure 4. Red curve: gas temperature profile in the circuit; blue line: superconductor temperature. Note that the temperature variations are small compared to the absolute temperature.

The heat exchanger consists of N_s parallel slits of the dimensions W_s by t_s and length L_s. We assume that the body temperature is equal to the cryocooler temperature. The difference between the gas temperature T_g and the body decreases exponentially as can be shown as follows: the heat flow from the gas to the body is given by

$$\mathrm{d}\dot{Q}_w = \kappa N_{us}\frac{\mathrm{d}A}{D_{hs}}(T_g - T_{Co}) \tag{13}$$

with N_{us} is the Nusselt number in the slits of the heat exchanger. In case of turbulent flow

$$N_u = 0.020 R_e^{0.79}. \tag{14}$$

In case of laminar flow in a slit $N_u = 140/17$. The thermal conductivity of helium is approximated by

$$\kappa = 3.9 T^{0.64} \text{ mW/K.} \tag{15}$$

For narrow slits

$$\mathrm{d}A = 2 N_s W_s \mathrm{d}h \tag{16}$$

so with

$$D_{hs} = \frac{2 W_s t_s}{W_s + t_s} \tag{17}$$

we get

$$\frac{\mathrm{d}\dot{Q}_w}{\mathrm{d}h} = -\kappa N_{us} N_s \frac{W_s + t_s}{t_s}\left(T_g - T_w\right). \tag{18}$$

Since

$$\frac{\mathrm{d}\dot{Q}_w}{\mathrm{d}h} = \dot{n}_t C_p \frac{\mathrm{d}T_g}{\mathrm{d}h} \tag{19}$$

we have

$$\frac{\mathrm{d}T_g}{\mathrm{d}h} = \frac{\kappa N_{us} N_s}{\dot{n}_t C_p}\frac{W_s + t_s}{t_s}(T_g - T_{Co}). \tag{20}$$

Introducing the characteristic length

$$L_{ex} = \frac{C_p \dot{n}_t}{\kappa N_{us} N_s} \frac{t_s}{W_s + t_s} \qquad (21)$$

Gives

$$L_{ex} \frac{dT_g}{dh} = T_g - T_{Co}. \qquad (22)$$

With the appropriate boundary conditions we get, for the temperature difference between inlet and exit temperature of the heat exchanger

$$T_{g4} - T_{g1} = \left(1 - \exp\left(-\frac{L_s}{L_{ex}}\right)\right)(T_{g4} - T_{Co}). \qquad (23)$$

SUPERCONDUCTOR TEMPERATURE

The left side of Fig.4 gives the temperatures of the gas and the superconductor. In the coil cryostat there are four channels in which the working fluid flows (two channels on each side of the two halves of the coil). See Fig. 5. Due to the heat produced in the coil, the inner parts will be hotter than the wall, but these internal temperature differences are small and will be neglected. The heat flow from the superconductor to the gas is supposed to be homogeneous. This is correct if the coil temperature is reasonable constant. The temperature difference (ΔT_{Sg}) between the gas and the superconductor is given by

$$\Delta T_{Sg} = \frac{D_{hc}}{N_{uc}\kappa N_c 4 W_c L_c} \dot{Q}_{AC}. \qquad (24)$$

Here N_{uc} is the Nusselt number in the coil cryostats, L_c the length of a channel, and N_c the number of coils. The temperature of the hottest spot of the superconductor is at the point where the gas leaves the coil cryostat
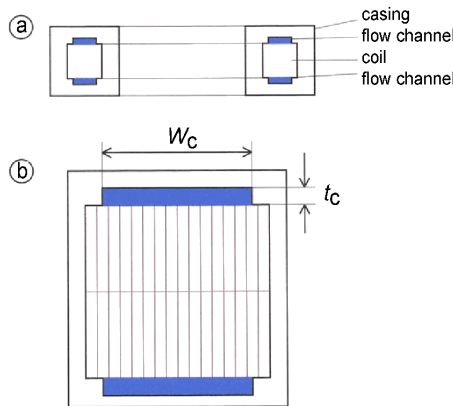
$$T_S = \Delta T_{Sg} + T_{g2}. \qquad (25)$$

Figure 5. Top: cross section of a superconducting coil in its container. The dark-blue regions are the flow channels for the working fluid; the light-red regions are the superconducting tapes; the grey regions are the casing. Bottom: half of a coil in its container. Compare with the right half of Fig.1.

POWERS

The cryostat heat leak $\dot{Q}_{Cr}$, which includes the heat leak through the three current leads from the electrical power supply to the coils, is not a strong function of the operation temperature and can be calculated from the over-all motor design. The fan power, needed to circulate the gas, is equal to

$$P_f = (p_3 - p_2)\dot{V}_t. \tag{26}$$

The cooler temperature T_{Co} as function of the cooling power is approximated by a series expansion

$$T_{Co} = \sum_{i=0}^{3} q_i \dot{Q}_{Co}^i. \tag{27}$$

The total AC loss as function of the superconductor temperature T_S and the torque M is also approximated by a series expansion

$$\dot{Q}_{AC} = \sum_{i,j} a_{ij} M^i T_S^j \tag{28}$$

So we assume that the AC loss is determined by the highest temperature of the superconductor. This is a pessimistic assumption. In the steady state

$$\dot{Q}_{Co} = \dot{Q}_{AC} + P_f + \dot{Q}_{Cr}. \tag{29}$$

CURRENT LEADS

The current leads run through vacuum, so they are not precooled by vapor as in a bath cryostat. In this Section the minimum heat load by the current leads on the low-temperature regions will be calculated[7]. In a section of the wire with length dl the amount of Ohmic heat production is

$$d\dot{Q} = \rho_e \frac{dl}{A_w} I^2 \tag{30}$$

where I is the rms value of the current, A_w the cross section of the wire, and ρ_e the specific electrical resistivity of the wire material. Multiplying this expression with

$$\dot{Q} = -\kappa_w A_w \frac{dT}{dl}, \tag{31}$$

with κ_w the thermal conductivity of the wire material, gives

$$\dot{Q} d\dot{Q} = -\rho_e \kappa_w I^2 dT. \tag{32}$$

Note that A_w has dropped out of the expression, so the following derivation holds for wires with non-uniform cross section. Integrating from the low temperature T_L to the high temperature T_H gives a relation between the heat flow $\dot{Q}_L$ at the cold end and the heat flow $\dot{Q}_H$ at the hot end.

$$\dot{Q}_L^2 = \dot{Q}_H^2 + 2I^2 \int_{T_L}^{T_H} \rho_e \kappa_w dT. \tag{33}$$

The minimum $\dot{Q}_L$ is found if the heat flow at the hot end is zero ($\dot{Q}_H = 0$). In that case

$$\dot{Q}_{Lmin}^2 = 2I^2 \int_{T_L}^{T_H} \rho_e \kappa_w dT. \tag{34}$$

With $\rho_e \kappa_w = L_L T,$ with L_L the Lorenz number which is practically temperature independent, and since $T_H^2 \gg T_L^2$ we get

$$\dot{Q}_{Lmin} \approx IT_H \sqrt{L_L}. \tag{35}$$

This relation shows that the minimum heat load at the cold end via the current leads is proportional to the current and for the rest it is practically independent of everything else. A typical value of LL is 20 nWΩ/K2, so TH=290 K gives $T_H\sqrt{L_L} \approx 0.041$ W/A. With three current leads this gives 0.123 W/A. The Lorenz number of all pure metals is practically the same. The Lorenz number for alloys is higher than of pure metals so, making the current leads from alloys, which have a low thermal conductivity but a high resistivity, will not result in a smaller heat load.

Eliminating $\dot{Q}$ in the expression for $d\dot{Q}$ gives

$$\frac{dT}{dl}\left(-\kappa_w A_w \frac{dT}{dl}\right) = \frac{I^2 L_L}{\kappa_w A_w} T. \tag{36}$$

With this relation the temperature profile in the wire can be calculated with the boundary condition $T(l{=}0) = T_H$ and $T(l{=}L_w) = T_L$ (with L_w the length of the wire). In the optimum situation $(dT/dl)_{l=0}{=}0$. Assuming $\kappa_w A_w$ constant and $T_L \ll T_H$ results in a simple relation for the cross section of the wire A_w which gives the minimum heat load at the design current I

$$A_w = \frac{2}{\pi} \frac{I\sqrt{L_L}}{\kappa_w} L_w. \tag{37}$$

RESULTS

The relations, given above, form a closed set that can be solved numerically or with algebraic programs such as Maple. The input parameters are: q_i, determined by the cryocooler, a_{ij}, determined by the properties of the superconducting tape and the motor design, the torque M, the heat exchanger parameters (N_s, W_s, t_s), the coil-cryostat parameters (N_c, W_c, t_c), the cryofan diameter and frequency (D_f, ω), the parameters of the working fluid (η, κ, M_m, C_p), and, finally, the cryostat heat leak $\dot{Q}_{Cr}$. In the rest of this paper some typical calculated results will be presented.

The superconductor and cooler temperatures as functions of the torque are given in Fig.6 for a system with two AL600 coolers. It shows that the temperatures rises fast near the maximum
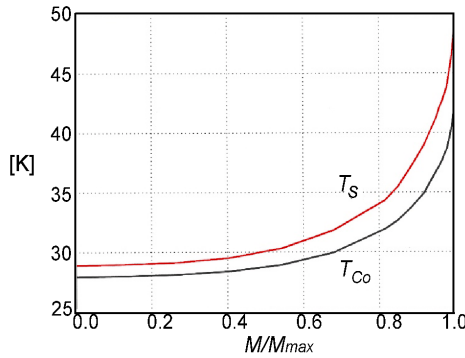
Figure 6. Calculated T_S and T_{Co} as functions of M divided by the maximum torque.

torque. At a cooler temperature of about 42 K there is no stable solution and the systems runs out of control in a process that has similarities with a quench.

Fig.7 is a plot of T_S as function of the slit width t_s in the heat exchanger for a heat-exchanger length $L_H = 10$ cm. Note the vertical scale. The dependence of T_S on t_s is not very strong. This is good for designers, so one has some freedom in the choice of the slit width. Fig.8 is a plot of T_S as function of the slit length L_H of the heat exchanger. Below L_H values of 10 cm T_S becomes rather large. Above 10 cm T_S is low enough and not a strong function of L_H. Around $L_H = 40$ cm (not shown in the figure) T_S is a minimum.

Comparing the performance of the system for the three gases in consideration (He, Ne, H2) shows that the maximum torque for helium is 2 % lower than for hydrogen and for neon it is about 12% lower. As helium has no safety problems helium is the preferred working fluid.

DISCUSSION

In general the performance of the system is not a strong function of the system parameters, so it should not be too difficult to design a system that operates as desired. In addition to the engineering requirements come aspects such as weight and volume, reliability, ease of handing, and, last but not least, cost (cryocooler, superconducting tapes, heat exchanger, manufacturing). By comparing the maximum torques, reached with different cryocoolers, one can quantify the cost of a higher system performance. These will be subject of future investigations.
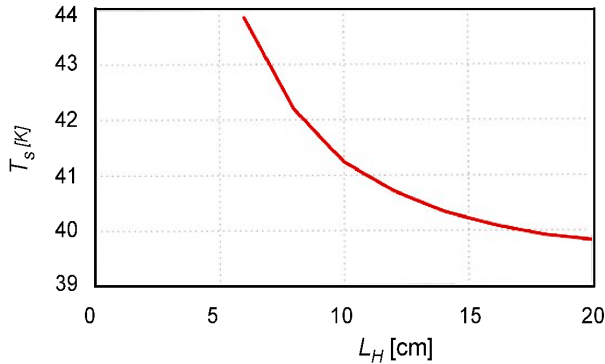

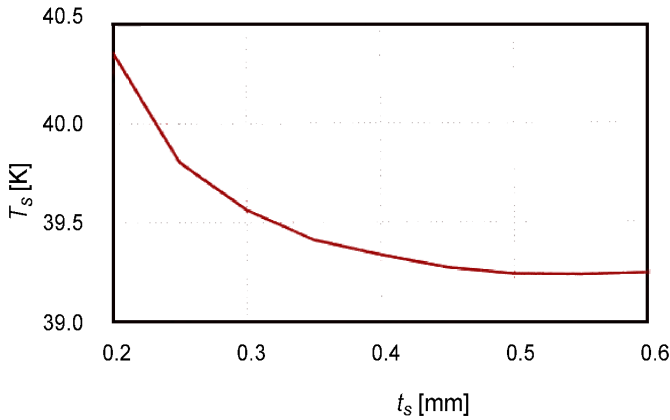**Figure 7.** Calculated T_S as function of the slit width in the heat exchanger.

Figure 8. Calculated T_S as function of the length L_H of the slits of the heat exchanger.

ACKNOWLEDGMENT

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