Numerical Simulation of the Effect of Heat Conductivity in a 4 K Regenerator

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ABSTRACT

In a 4K cryocooler, the performance of the second stage is strongly dependent on the efficiency of the second stage regenerator. In order to improve the efficiency of the second stage regenerator, the effect of the heat conductivity of the regenerator material is analyzed by numerical simulation. The temperature profile in the second stage regenerator with an artificially increased heat conductivity of the regenerator material at the warm end is analyzed. Based on the simulation results, a novel and simple way, which is to replace the regenerator material at the warm end with a material having a higher heat conductivity, is proposed. By increasing the heat conductivity to 1×10^6 times that of lead, the P-V power is increased by 0.7 W at the first stage and 0.15 W at the second stage. Correspondingly, the cooling capacity is increased by 0.4 W at the first stage and 0.03 W at the second stage. The simulation results are reported in this paper.

INTRODUCTION

In a 4K cryocooler, the performance of the second stage is strongly dependent on the real gas effect and the efficiency of the second stage regenerator. The real gas effect can be estimated by the method suggested by de Waele and Xu et al.[1,2] Then, the regenerator loss at the second stage can be estimated by the enthalpy flow excluding the real gas effect.[3,4]

In order to improve the efficiency of a 4K pulse tube cryocooler, the extra enthalpy flow caused by the real gas effect was utilized by adding a heat exchanger to the second stage regenerator.[5]

However, it is difficult to extract cooling from the second stage of a 4K GM cryocooler because there is a large thermal conducting resistance caused by the clearance between the cylinder and the displacer, and the low thermal conducting displacer wall.

Xu, et al. reported that the performance of a 4K GM cryocooler can be improved by shifting the temperature profile at the warm end of the second stage regenerator to a higher level.[6] A novel and simple way to shift the temperature profile, which is to replace the regenerator material at the warm end of the second stage regenerator with a material having a lower heat capacity, was proposed and the effect was confirmed with a simulation method.[6]

In this paper, the temperature profile of the second stage regenerator with respect to the heat conductivity of the regenerator material and the mass variation in the void volume of the second stage regenerator are analyzed. Another novel and simple way for changing the temperature profile in the second stage regenerator is proposed. The effect of the novel way is analyzed by the simulation method we developed.[4]

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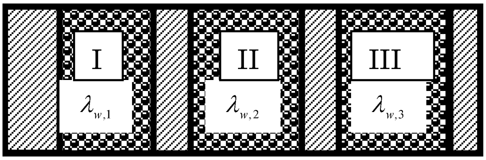

Figure 1. Schematic diagram of a second stage regenerator

SIMULATION

We reported a simulation method and confirmed the accuracy of the simulation results by comparing them with the results measured in a 1W 4K GM cryocooler.[4] The simulation method has been verified to be an effective way to improve the efficiency of a 4K GM cryocooler.[3-4, 6]

Figure 1 shows a schematic of the second stage regenerator, which is divided into three parts. Part I is at the warm end, Part II is at the middle, and Part III is at the cold end. The heat conductivities of the regenerator materials in the three parts are called $\lambda_{w,1}$, $\lambda_{w,2}$ and $\lambda_{w,3}$, respectively. During the simulation, the heat conductivity of the regenerator material is assumed to be equal to the normal value of lead in Part II, while the heat conductivity in Part I is assumed to be equal to or larger than that of lead. For the material in Part I, all other properties, such as density and heat capacity, are assumed to be the same as those of lead.

In the simulation model, the two-stage G-M cryocooler is divided into many elements and the state in each element can be calculated by solving the basic equations.[6] The energy conservation equations of gas flow and regenerator material are listed as follows:

The energy conservation equation of gas flow is,

$$\frac{\partial(m_i h_i)}{\partial t} + (\dot{m} h)_{f_{i+1}} - (\dot{m} h)_{f_i} + \alpha_i A_i (T_i - T_{wi}) - V_i \frac{dP_i}{dt} = 0 \qquad (1)$$

where α is the heat transfer coefficient and A is the heat transfer area.

The energy conservation equation of the regenerator material is,

$$V_{wi} c_{wi} \frac{\partial T_{wi}}{\partial t} = -\lambda_{wi} A_{wi} \frac{\partial^2 T_{wi}}{\partial x^2} dx + \alpha_i A_i (T_i - T_{wi}) \qquad (2)$$

where V_w is the volume of material, c_w is the volumetric specific heat capacity of the material, A_w is the cross-section area of the material, and λ_w is the heat conductivity of the material.

Since the regenerator is filled with spheres, the equivalent heat conductivity of the regenerator packing bed is much smaller than that of a solid material. The equivalent heat conductivity of the regenerator material can be calculated by the following correlation,

$$\lambda_{w,e} = 1.1\lambda_g \left(\frac{\lambda_w}{\lambda_g}\right)^{0.25} \qquad (3)$$

where λ_g is the heat conductivity of helium gas.

SIMULATION RESULTS AND DISCUSSIONS

Simulations have been performed using the parameters of a RDK-408D2 1W 4K two-stage GM cryocooler manufactured by Sumitomo Heavy Industries, Ltd. (SHI). For typical calculations, the cold-head is operated at 1.0 Hz, the high and low pressures are 2.25 MPa and 0.76 MPa, respectively.

The inner diameters of the first and second stage cylinder are 82 mm and 35 mm, respectively. The diameter of the second stage regenerator is 30 mm. The normalized length of Part I, II and III are about one-third for each. The second stage regenerator is filled with lead and $HoCu_2$ spheres. The porosity of the second stage regenerator is assumed to be 0.3 for all calculations. The average sphere diameter is 0.44 mm for lead and 0.33 mm for $HoCu_2$.

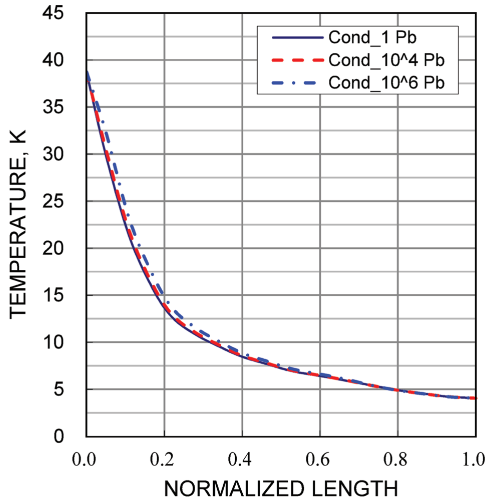

Figure 2. Temperature profile in the second stage regenerator.

Temperature Profile in the Second Stage Regenerators

Figure 2 shows the simulation results of the temperature profile in the second stage regenerator. The heat conductivity of regenerator material in Part I is assumed to be $1x10^2$, $1x10^4$ and $1x10^6$ times that of lead. As shown in Fig. 2, when the heat conductivity in Part I is assumed to be larger than that of lead, the temperatures in the second stage regenerator increase. Accordingly, the average temperature in the second stage regenerator increases.

Pressure Variation in Expansion Volumes

Figures 3a and 3b show the pressure variation in the first and the second stage expansion volumes, respectively. As shown in the figures, when the heat conductivity in Part I is assumed to be $1x10^6$ times that of lead, the pressures in the expansion volumes goes down slightly more slowly after approaching the high pressure and then rises up slightly more slowly after approaching the low pressure. See Reference 6 for more details about the mechanism. As shown in Fig. 2, the tempera-

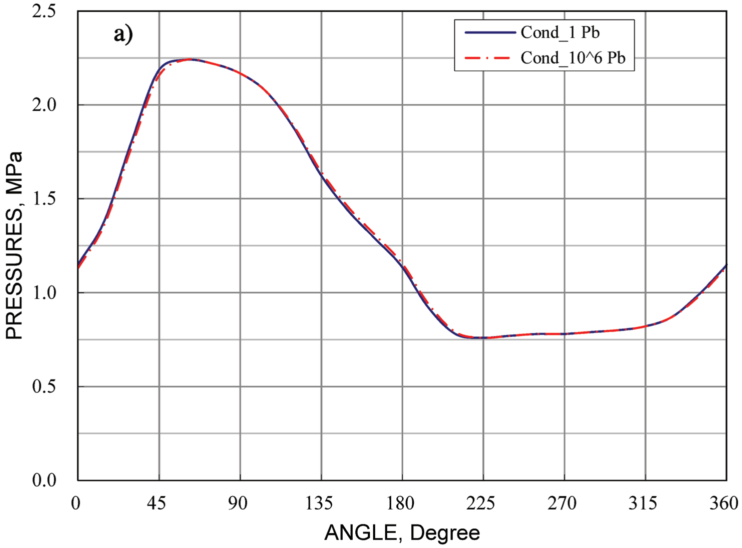

Figure 3a. Pressure variation in the first-stage expansion volume.

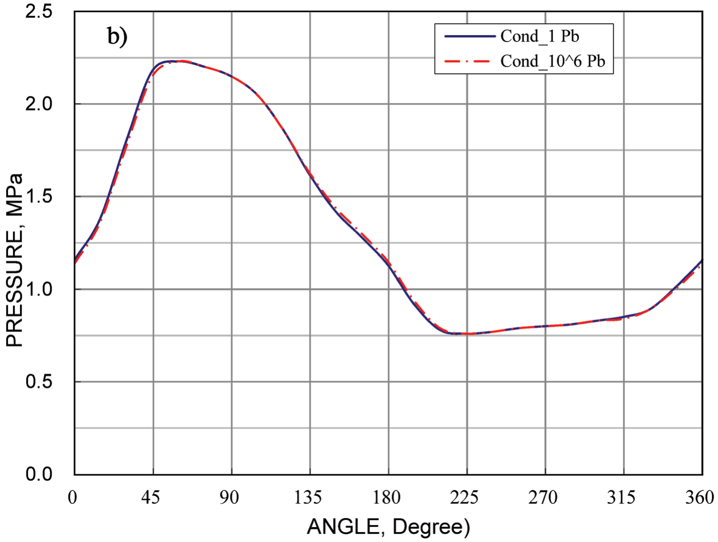

Figure 3b. Pressure variation in the second-stage expansion volume.

tures slightly increase in the region from 8 K to 30 K, where the density change caused by pressure change is large. Accordingly, the average temperature in the second stage regenerator increases and a smaller amount of gas is needed to fill the void volume of the second stage regenerator. Therefore, the pressure in the cold head goes down slightly more slowly. Similarly, since the average temperature in the second stage regenerator is increased and a smaller amount of gas is needed to fill the void volume of the second stage regenerator, the pressure in the cold head rises up slightly more slowly.

The pressure change caused by an increased heat conductivity is relatively small compared to that caused by a reduced heat capacity reported in Ref. 6. The reason can be analyzed from Equation (3). The equivalent heat conductivity of the regenerator packing bed is strongly dependent on the heat conductivity of the helium gas. In contrast, the impact on the equivalent heat conductivity caused by the heat conductivity of the regenerator material is relatively small.

P-V Power and Cooling Capacity

Figure 4 shows the P-V diagram of the first and the second stage expansion volumes. As shown in Fig.4, when the heat conductivity in Part I is assumed to be $1x10^6$ times that of lead, the areas of the first and second-stage P-V diagrams increase slightly. The reason is that the pressures in the expansion volumes go down slightly more slowly after approaching the high pressure and rise up slightly more slowly after approaching the low pressure. Therefore, the phase shift between the pressure and the expansion volume is improved, and the P-V power is slightly increased.

The simulation results of the cooling capacity and the P-V power when the heat conductivity of the regenerator material in Part I of the second stage regenerator is increased are shown in Table 1. The enthalpy flux from the first and the second stage regenerator is also shown in Table 1. As shown in Table 1, when the heat conductivity is increased, the P-V power and the cooling capacity at both stages slightly increase. By increasing the heat conductivity to $1x10^6$ times that of lead, the P-V power is increased by 0.7 W at the first stage and 0.15 W at the second stage. The enthalpy flux from the regenerator is increased by 0.1 W at the first stage, but increased 0.12 W at the second stage. Accordingly, the cooling capacity is increased by 0.4 W at the first stage and 0.03 W at the second stage. Apparently, the cooling capacity of the first stage is strongly dependent on the P-V power, while the cooling capacity of the second stage is mainly dependent on the regenerator efficiency. Although the effect is relatively small, the tendency is the same as that with a reduced heat capacity at the warm end of the second stage regenerator.

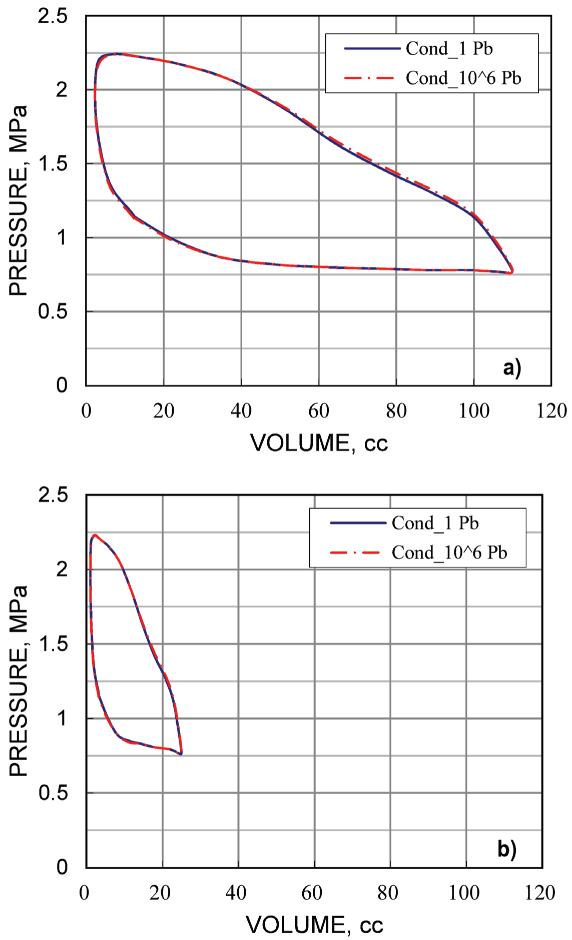

Figure 4. P-V diagram of the expansion volumes. (a) first stage; (b) second stage.

Table1. Simulation Results with Increased Heat Conductivity in Part I of the Second Stage Regenerator.

Heat Conductivity Ratio to Lead, $\lambda_{w,1}/\lambda_{w,Pb}$	**First Stage Cooling Capacity*,** Q_{c1} **(W)**	**Second Stage Cooling Capacity*,** Q_{c2} **(W)**	**First Stage P-V Power,** W_{e1} **(W)**	**Second Stage P-V Power,** W_{e2} **(W)**	**Enthalpy Flux from First Stage Regenerator,** $\dot{H}_{r1}$ **(W)**	**Enthalpy Flux from Second Stage Regenerator,** $\dot{H}_{r2}$ **(W)**
1	62.9	1.28	91.0	19.52	39.5	17.93
$1\text{x}10^2$	63.0	1.29	91.1	19.53	39.6	17.94
$1\text{x}10^4$	63.1	1.30	91.2	19.57	39.6	17.97
$1\text{x}10^6$	63.3	1.31	91.7	19.67	39.6	18.05

*Cooling capacity before excluding shuttle and radiation loss.

CONCLUSIONS

A novel and simple way to shift the temperature profile is proposed, that is to replace the regenerator material at the warm end of the second stage regenerator with a material having a higher heat conductivity.

According to the simulation results, by increasing the heat conductivity to $1\text{x}10^6$ times that of lead, the P-V power is increased by 0.7 W at the first stage and 0.15 W at the second stage. The cooling capacity is increased by 0.4 W at the first stage and 0.03 W at the second stage.

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