A Numerical Model of Regenerator Based on Lagrange Description

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ABSTRACT

A one-dimensional theoretical model of a regenerator for a regenerative cryocooler has been developed. The model is based on the Lagrange description.

The model is helpful to explain the mechanisms involved in regenerative cryocoolers. In the model, gas and solid are divided into finite small parcels and their parameters are computed and recorded every calculation step. The calculation results conform to the first and the second laws of thermodynamics, which verifies the correctness of the model. T-s and P-v diagrams of some typical gas parcels are presented to show the details of the regenerative process.

INTRODUCTION

Regenerative cryocoolers, such as Stirling and pulse tube cryocoolers, are generally more compact than a recuperative cryocooler and are widely used in ground and space missions. In recent years great technical progress has been made in advancing the development of these coolers. For example, pulse tube coolers have been developed that can cool to 4.2 K [1], and a small Stirling cryocooler weighting only 300 g has been developed to supply a cooling power of 150 mW at 77 K with an input power of 3 W [2].

There are also parallel theoretical achievements on regenerative coolers. Bradley and Radebaugh have explained the refrigerant mechanism of a pulse tube cryocooler using an Enthalpy Flow Analysis [3]. Finkelstein developed the first program for NASA to analyze the thermodynamic process of the Stirling Cycle in the 1960s [4]. In the 1990s, Liang proposed the thermodynamic non-symmetry effect to illustrate the refrigerant process in pulse tube coolers, and the Lagrange description was introduced to explain the refrigeration principle of regenerative coolers [5]. In addition, the thermoacoustic effect has also been used to illustrate the mechanisms of regenerative cryocoolers.

Generally, models of the regenerator, which is one of the most important parts of a regenerative cryocooler, are based on an Euler description, and many numerical studies have been conducted successfully. In this way, Gas flow and heat transfer between gas and solid in a certain position can be easily demonstrated. But, the processes that a specific gas particle experiences are hard to monitor.

In a paper published in 1997, Liang proposed a theory to explain the heat-pump mechanism in a regenerator using a Lagrange description [6]. The model in that paper gives detailed information

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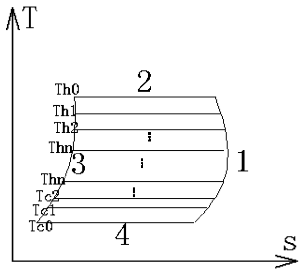

Figure 1. Suppositional thermodynamic process in the regenerator

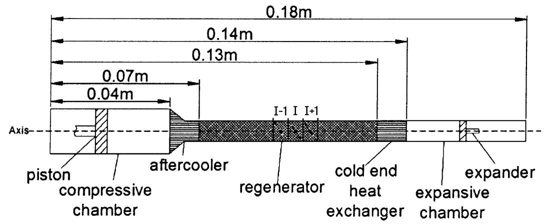

Figure 2. Geometrical model

on the thermodynamic processes involved with one gas parcel. However, the model is too simple and ideal to explain the regenerative process in a complete regenerator.

A Lagrange-description based model is also developed in this paper. In order to further understand the mechanisms involved in a regenerator with focus on the thermodynamics, the thermodynamic processes of the gas particles are studied from a systemic perspective. Energy and entropy conservation as well as thermal interaction between gas and solid are considered in this paper.

MODEL

Usually, a regenerative cooler is a closed system in which the gas is contained. So, it is possible to trace gas particles (small gas parcels) and investigate their individual physical processes.

Gas particles in a regenerative cryocooler experience distinct processes. And the processes depend on the heat transfer conditions around the particles. Cooling power is generated by the non-symmetry effect of gas particles in an expansive chamber [5]. Particles inside the regenerator work together and cooperate elegantly to pump heat from the cold end toward the aftercooler at the hot end [6]. The thermodynamic cycles (micro cycles) of the individual gas particles operating in different temperature ranges throughout the regenerator contribute to a macroscopic thermodynamic cycle — the equivalent overall thermodynamic cycle. As is shown in Figure 1, the macroscopic thermodynamic cycle 1-2-3-4-1 consists of many micro cycles.

Semi-Ideal Regenerator Model

In this paper, a model for the regenerator is developed based on the Lagrange description. As shown in Figure 2, the model geometry includes a compression chamber, expansion chamber, aftercooler, cold end heat exchanger, and regenerator. The geometric parameters and heat transfer coefficients are listed in Table 1.

As is shown in Figure 2, the gas and solid between the piston and the expander are divided into small parcels along the axis of the model. The parcels are used to imitate particles. P-v and T-s diagrams of gas parcels in the above parts are demonstrated, which is useful to reveal the details of the regenerative cryocooler's principle. The regenerative process and heat-pump effect in the regenerator and the generation of cooling power in the cold end heat exchanger are shown clearly in this investigation.

To simplify the real processes, the following assumptions are made:

1) The model is one-dimensional and the physical parameters of it are lumped along the axis of symmetry, as shown in Figure 2.

Table 1. Dimensions of geometric model

	Compressive chamber	Aftercooler	Regenerator	Cold end heat exchanger	Compressive chamber
Area (mm^2)	176	176–34	34	34	34
Length (mm)	40	10	60	10	40

2. Suppose there exists a piston in a compressive chamber and an expander in an expansive chamber and their velocities are sinusoidal.
3. The temperature of the aftercooler is 300 K and the temperature of the cold end heat exchanger is 80 K.
4. Solid parcels are static and gas parcels oscillate axially.
5. Solid regions are viewed as porous medium. The heat transfer behavior of the gas and solid depend on a given heat transfer coefficient.
6. Losses caused by flow resistance of the gas and heat conduction along the length are negligible.
7. Working gas is considered as an ideal gas.
8. Gas and solid parameters are uniform across the cross section. The thermal status of the parcels is determined by that in their centers. Solid parcels are viewed as heat reservoirs.
9. The oscillating direction of the gas is perpendicular to the cross section. Gas parcels exchange heat with the solid parcel only under the condition that the position of the gas parcel center is between the two interfaces of solid parcel.
10. There is no inner heat source in the regenerator.

Mathematical Formulation

As is mentioned above, the velocities of piston and expander are sinusoidal.

For piston, $u_p = U_p \sin(\omega t - \frac{\pi}{3})$

For expander, $u_e = U_e \sin(\omega t - \frac{\pi}{2})$

Since this model is based on Lagrange description, each gas parcel's mass is constant every moment. The mass equation of a certain gas parcel is

$$\frac{dm_g}{dt} = \frac{d(\rho_g V_g)}{dt} = 0 \tag{1}$$

Here, m_g is mass of gas parcel, ρ_g density of gas parcel, V_g volume of gas parcel

According to the assumption that flow resistance is negligible, the momentum conservation equation of any gas parcel can be expressed as:

$$\frac{du_g}{dt} = -\frac{1}{\rho_g}\frac{\partial p}{\partial X} \tag{2}$$

Where u_g is velocity of gas parcel, p is pressure.

On the basis of the first law of thermodynamics and assumption 9, the energy equations of gas and solid parcel can be derived:

$$\frac{d(m_g c_{v_g} T_g)}{dt} = -p\frac{dV_g}{dt} + q_g \tag{3}$$

$$\frac{d(m_s c_{v_s} T_s)}{dt} = q_s \tag{4}$$

Where c_{v_g} is specific heat of gas parcel, q_g is heat that transfers to gas parcel, T_g is temperature of gas parcel. Similarly, c_{v_s} is specific heat of solid parcel, q_s is heat which is transferred to solid parcel, T_s. is temperature of solid parcel.

Numerical method is introduced to solve the above equations. At time j, the parametric equations of gas parcel i can be expressed as

$$\rho_{gi}^{j-1} V_{gi}^{j-1} = \rho_{gi}^{j} V_{gi}^{j} \tag{5}$$

$$\frac{u_g^j - u_g^{j-1}}{\Delta t} = -\frac{1}{\rho_f}\left(\frac{p_i^j - p_{i-1}^j}{X_i^j - X_{i-1}^j}\right) \tag{6}$$

$$\frac{m_{gi} c_{v_g} (T_{gi}^j - T_{gi}^{j-i})}{\Delta t} = -p_i^j \frac{V_{gi}^j - V_{gi}^{j-1}}{\Delta t} + q_g \tag{7}$$

$$\frac{m_s c_{v_s}\left(T_{si}^j - T_{si}^{j-1}\right)}{\Delta t} = q_s \tag{8}$$

$$q_g = h(T_{sk}^j - T_{gi}^j) \tag{9}$$

$$q_s = \sum_{l=m}^{n} h(T_{gl}^j - T_{si}^j) \tag{10}$$

Where h is heat transfer coefficient, X_i^j is the position of mass centre of gas parcel i at time j. Other parameters which are referred to in the above formulations are defined in the following expressions

Volume of gas parcel i at time j can be written as:

$$V_{gi}^{j} = V_{gi}^{j-1} + (u_{i+1}^{j} A_{X_{i+1}^{j}} - u_{i}^{j} A_{X_{i}^{j}})\Delta t \tag{11}$$

Here, A is sectional area.

Central position of gas parcel can be gotten by

$$Xc_i^j = \frac{\int_{X_i^j}^{X_{i+1}^j} X dV}{V_i^j} \tag{12}$$

Density and displacement of the interface of gas parcel are

$$\rho_f(Xc_i^j - Xc_{i-1}^j) = \rho_{gi}^j(Xc_i^j - X_i^j) + \rho_{gi-1}^j(X_i^j - Xc_{i-1}^j) \tag{13}$$

$$X_i^j = X_i^{j-1} + u_i^j \Delta t \tag{14}$$

RESULTS AND DISCUSSION

Although the model is constructed using the Lagrange description, the computation results are discussed from two aspects: the Euler description (for parts of the model), and the Lagrange description (using gas parcels).

Discussion Based on Euler Description

The aftercooler, regenerator, and cold end heat exchanger are divided into seven control volumes, shown in Figure 3, for the purpose of investigating the energy and entropy conservation in this model. During calculation, parameters related to enthalpy and entropy of the control volumes, as shown in Figure 3, are periodically and instantaneously recorded, separately.

Table 2 lists the calculation results of cold end energy flow of the control volumes of a cycle. The calculation results are consistent with energy conservation. Since the temperatures of the aftercooler and cold end heat exchanger are kept constant, the internal energy change of the solid in the control volumes "a" and "f" are zero. For other control volumes, the internal energy change of the solid in them is equal to the heat absorbed by the gas, as solids in these regions only exchange heat with the gas.

Because the heat transfer process in the regenerator is not ideal, the net time average enthalpy flow in the regenerator is not zero. It flows from the regenerator to the cold end heat exchanger. What's more, the cooling power of the cold end heat exchanger is less than the PV work of the expander, and the heat released into the aftercooler is less than the input PV power from the piston, for there exists net work flow from the piston to the expander (not loss). This work flow is caused by the non-symmetry of the temperature fluctuation that results from having finite heat capacity of the solid fillers in the regenerator. Figure 4 shows the temperature fluctuation of the gas in a short length of the regenerator near the cold end. During a cycle, the fluctuation is about 2K.

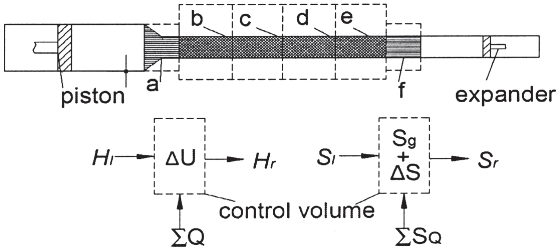

Figure 3. Sketch of control volumes.

In Figure 3, H_1 is the enthalpy flow passing the left face, H_2 is the enthalpy flow passing the right face, ΔU is the internal energy change, $\sum Q$ is the heat transferred to the control volume, S_l is the entropy flow passing the left face, S_r is the entyopy flow passing the right face, $\sum S_Q$ is the entropy flow introduced to the control volume by heat transfer, S_g is the entropy generation in the control volume, and ΔS is the internal entropy change in the control volume.

Equation (15) shows that the calculation conforms to energy conservation.

$$\frac{W_p + Q_c - W_E - Q_H + \sum \Delta U_{Gas} + \sum \Delta u_{Solid}}{\min\ (W_p, Q_c, W_E, Q_H)} = 0.22\% \tag{15}$$

Entropy parameters, including entropy flow, entropy generation, and entropy change of the seven control volumes are also monitored for the purpose of checking entropy conservation. Table 3 shows the details.

Entropy generation is introduced by heat-transfer between solid and gas. The entropy generation increases along the length of regenerator. For a certain temperature difference and heat transfer, it introduces more entropy generation in lower temperature than that in high temperature. It is easy to check out that the data shown in Table 3 satisfy entropy conservation.

Figure 5 shows time average enthalpy and entropy flow vs. position. In regenerator, the reason why the enthalpy flow is not zero and entropy flow is not constant is because of the imperfection of regenerator**.**

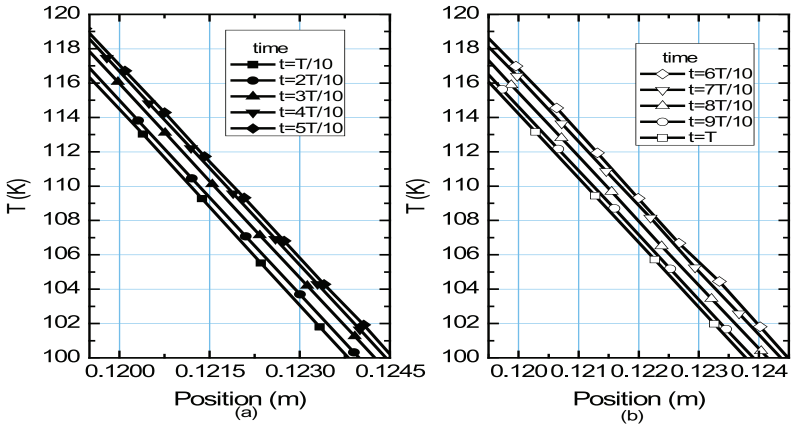

Figure 4. Gas temperature fluctuation in certain region of regenerator for a cycle. (a) first half-cycle, (b) second half-cycle

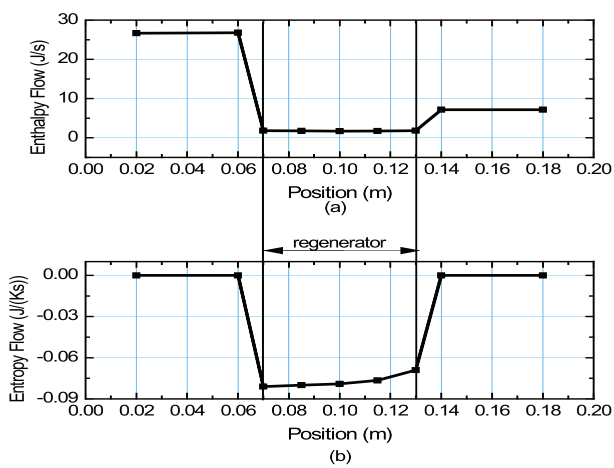

Figure 5. Time average enthalpy flow and entropy flow along the length of regenerator. (a) enthalpy flow (b) entropy flow

Table 2 Time average energy parameters for control volumes.

	a	b	c	d	e	f	note
H_l(W)	26.769	1.818	1.764	1.694	1.748	1.794	W_P=26.367 W_E=7.170 Q_H=24.510 Q_C=5.333
H_r(W)	1.818	1.764	1.694	1.748	1.794	7.150	
ΔU_{gas}(W)	-0.00363	-0.09736	-0.0914	-0.126	-0.0830	-0.0231	
$\sum Q_{gas}$ (W)	-24.910	-0.146	-0.161	-0.0728	-0.0361	5.333	
$\sum Q_{solid}$ (W)	24.910	0.146	0.161	0.072	0.0362	-5.333	
ΔU_{solid}(W)	0	0.146	0.161	0.0728	0.0362	0	

Table 3 Time average entropy parameters for control volumes.

	a	b	c	d	e	f	note
S_l(W/K)	-2.903E-06	-0.0810	-0.0802	-0.0790	-0.0763	-0.0692	$\sum S_g$ =0.0143
S_r(W/K)	-0.0810	-0.0802	-0.0790	-0.0763	-0.0692	1.38E-06	
ΔSgas(W/K)	7.788E-05	-0.00048	-0.00075	-0.00089	-0.00169	-0.00058	
S_{Qgas}(W/K)	-0.0810	0.000285	0.000488	0.00180	0.00541	0.0686	
S_{Qsolid}(W/K)	0.0830	0.000532	0.000734	0.000417	0.0004	-0.067	
ΔSsolid	0	0.000532	0.000734	0.000417	0.0004	0	
S_g(W/K)	0.00213	0.000817	0.00122	0.00232	0.00581	0.00197	

In Table 2, H_1 is the enthalpy flow passing the left face, H_2 is the enthalpy flow passing the right face, ΔU_{gas} is the internal energy change of the gas, $\sum Q_{gas}$ is the heat transferred from solid to gas, $\sum Q_{solid}$ is the heat transferred from gas to solid, ΔU_{solid} is the internal energy change of the solid.

In Table 3, S_l is the entropy flow passing the left face, S_r is the entropy flow passing the right face, ΔS_{gas} is the internal entropy change of the gas, S_{Qgas} is the entropy flow to the gas caused by heat transfer. S_{Qsolid} is the entropy flow to the solid caused by heat transfer, ΔS_{solid} is the internal entropy change of the solid, S_g is the entropy generation in the control volume.

The results shown in Table 2 and Table 3 are combined to examine the model.

The Carnot efficiency is

$$\eta_c = \frac{80K}{220K} = 0.36364$$

Efficiency, η_t, of this model is

$$\eta_t = \frac{Q_c}{Q_H - Q_c} = 0.27241$$

Net work W (input power subtract the loss) is

$$W = Q_H - Q_C - T_0 \Delta S_g = 15.287 J/s$$

Then net efficiency η is

$$\eta = \frac{Q_C}{W} = 0.34886$$

It can be seen that the net efficiency η is smaller than the Carnot efficiency, which may be caused by the losses that turn to heat, which also require work to pump them to the aftercooler. This result conforms to the Second Law of thermodynamics.

Discussion Based on Lagrange Description

No gas parcels go through the each part of the model and follow the ideal cycle described in classical thermodynamics. For example, gas parcels in expansive chamber undergo adiabatic processes while parcels in regenerator experience isothermal processes.

To get an insight into the working processes in the regenerator, thermodynamic processes of gas parcels between compressor piston and expander are investigated in this paper. The work loss in this model is caused only by heat-transfer temperature difference. The cycles that gas parcels follow are reversible.

Thermodynamic processes of some typical gas parcels are chosen to represent the thermodynamic process of regenerator. The parcels are numbered on the length of the model. For example, NO.35 is near the aftercooler, NO.115 is near cold end heat exchanger and NO.75 is near the middle point of regenerator.

Figure 8(a) and Figure 8(b) show the temperatures of some gas parcels versus position. Because of limited heat capacities of solid in regenerator, the temperature of parcels is nonsymmetrical when they oscillate in the regenerator.

For gas parcels oscillating in regenerator, their heat-transfer behaviors depend on the lengthwise temperature gradient of the filler. Generally, if compressed, the parcels give out heat to the filler nearby, and if expanded, they absorb heat from the filler or solid nearby. Figure 8 shows that during a period the effect for a certain gas parcel is absorbing heat from the fillers at low temperature and rejecting heat to the filler at high temperature.

For gas parcels motion in compressive chamber or expansive chamber, they undergo adiabatic processes and transport work without any loss. Nevertheless, once gas parcels in the two chambers go to adjacent heat exchanger, they may be suddenly heated or cooled. This is so called temperature jump. The temperature jump results in PV work loss.

Figure 7(b) shows the process of gas parcels oscillating between expansive chamber and cold end heat exchanger. Because of the thermodynamic nonsymmetry, gas parcels flow into cold end heat exchanger with enthalpies lower than those of their going into pulse tube. As a result, temperature of parcels which goes from expansive chamber to cold end heat exchanger is lower than temperature of cold end heat exchanger. In this way, cooling power is generated at the cold end heat exchanger. In this view, temperature jump in the cold end heat exchanger is inevitable.

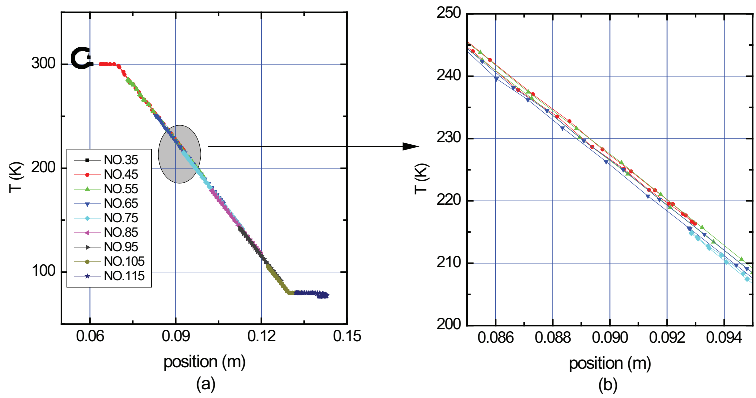

Figure 6. Temperature vs position of some typical parcels: a) overall view, (b) detail view

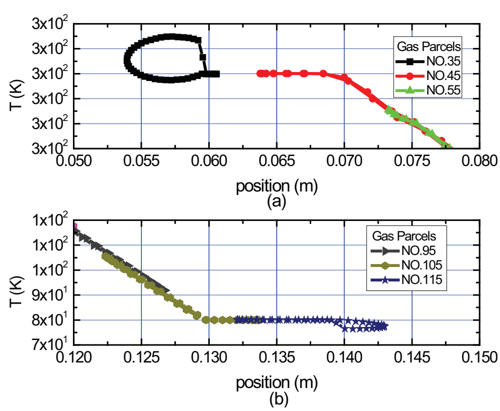

Figure 7. Temperature fluctuation vs position of gas parcels oscillating near the two ends of the regenerator: (a) Parcels near aftercooler, (b) Parcels near cold end heat exchanger

Figure 7(a) shows the temperatures of gas parcels that shuttle through compressive chamber and aftercooler. Unlike cold end heat exchanger, temperature jump is not beneficial. Since the two heat transfer differences are inevitable, the thermodynamic efficiency of this kind of cooler is lower than the Carnot efficiency

Figure 8~9 show the computational results of T-s and P-v diagrams of some typical gas parcels. The physical behaviors of gas parcels depend on not only the phase shifts between their displacements and pressures, but also the condition of heat transfer nearby.

Cooling power generated in the cold end heat exchanger is integrated actions of all gas parcels. The parcel expands near the heat exchanger and absorbs heat, Q_1, in the cold end exchanger. Then it goes to regenerator with heat rejecting, Q_2, to nearby filler caused by compressing. The net effect is that heat in the cold end heat exchanger is pumped to filler at high temperature. On the basis of the second thermal Law, Q_1 is less than Q_2. After being stored in the filler for a short time, the heat Q_2 will be carried by other gas parcels to filler whose temperature is higher. This is so called regenerative process and heat-pump effect. Heat will be absorbed by parcel and stored in the filler alternately until it is transferred to aftercooler. The envelope of all curves in Figure 8 and 9 stand for the thermodynamic cycle of the regenerator. In a word, the regenerative process which happens in regenerator is the effect of all gas parcels' thermodynamic cycles.

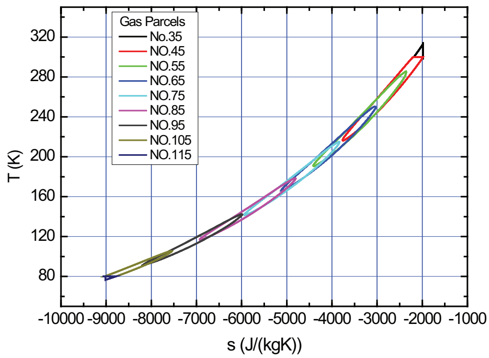

Figure 8. T-s diagram of some typical gas parcels

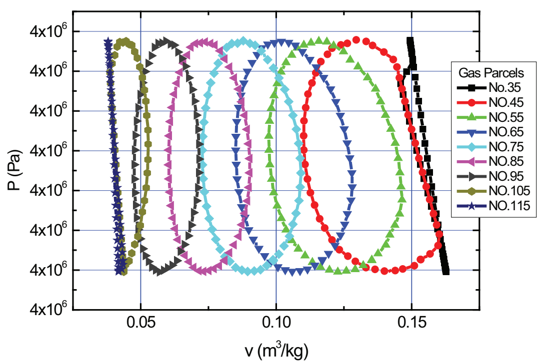

Figure 9. P-v diagrams of some typical gas parcels

CONCLUSIONS

A model has been developed using the Lagrange description. The computational results of this model conform to the first and second laws of thermodynamics very well. Regenerator imperfections including the limited heat capacity of the gas and solid and the heat-transfer temperature difference between the two substances results in the following phenomena:

1. The limited heat capacity of the regenerator filler material causes gas temperature fluctuation and net enthalpy flow at certain positions in the regenerator. In this process, there is no work loss.
2. Heat-transfer temperature difference between gas and filler also causes net enthalpy flow in regenerator. Here, work loss is introduced.
3. Due to the heat-transfer temperature difference, the COP of this model is less than that of Carnot.
4. For certain heat transfer temperature differences, entropy generation at low temperature is bigger than that at high temperature.

The macroscopic cycle of a regenerator can be considered as many microscopic cycles of individual gas parcels. Investigation of the cycles of the individual gas parcels is therefore equivalent to investigation of the regenerative principle of the regenerator as a whole. Since parameters of certain gas parcels are monitored every calculation step, their thermodynamic cycle can be demonstrated. T-s and P-v diagrams for some typical gas parcels are presented in this paper; this is helpful to explain the principles of the regenerative process from a thermodynamic perspective. The regenerative process or heat-pump effect is demonstrated clearly in this paper. During a cycle, gas temperature jumps in the aftercooler or cold-end heat exchanger are inevitable. The temperature jump in the cold-end heat exchanger generates cooling power with work loss, while the temperature jump in the aftercooler has no benefit.

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