

# A Numerical Study on Recuperative Finned-Tube Heat Exchangers

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## ABSTRACT

A recuperative heat exchanger is a crucial element in Joule-Thomson (JT) cryocoolers. The heat exchanger efficiency determines the cryocooler efficiency, and below a certain value of the heat exchanger efficiency the cryocooler is inoperative. Among a few configurations of heat exchangers, the helical finned-tube type is widely in use. In addition to the requirement for high heat transfer efficiency, there is also a demand from the heat exchanger for low pressure drops, especially at the low pressure stream. As JT cryocoolers are commonly integrated in miniature systems, the heat exchangers are required for miniature dimensions as well.

In the current research a numerical model for describing the heat exchanger is developed. The analysis includes calculation of the heat exchanger performance and provides an insight view into the fluid states and thermal characteristics of the heat exchanger under steady-state conditions. The analysis enables one to perform an extensive investigation of the heat exchanger and to optimize the dimensions for any target application.

## INTRODUCTION

A Joule-Thomson (JT) cryocooler consists of a heat exchanger, an expansion valve, and an evaporator. The current study focuses on analyzing the heat exchangers of JT cryocoolers, which have a significant affect on cryocooler performance. Various heat exchanger configurations are proposed in the literature tube in tube [1], matrix shape [2,3], and sintered heat exchangers [4]. However, the most popular configuration is the finned-tube type in which a rectangular cross-sectioned fin is wrapped on a helical tube, as presented in Figure 1. This configuration was proposed over 40 years ago [5,6] and ever since has been widely investigated. Chua et al. [7] performed a detailed analysis of a finned-tube heat exchanger with fixed dimensions against varying operating pressures and flow rates. Hong et al. [8,9] studied the pressure drop and heat transfer for finned-tube heat exchangers with varying dimensions and flow rates, while Gupta et al. [10-12] also investigated this type of heat exchanger.

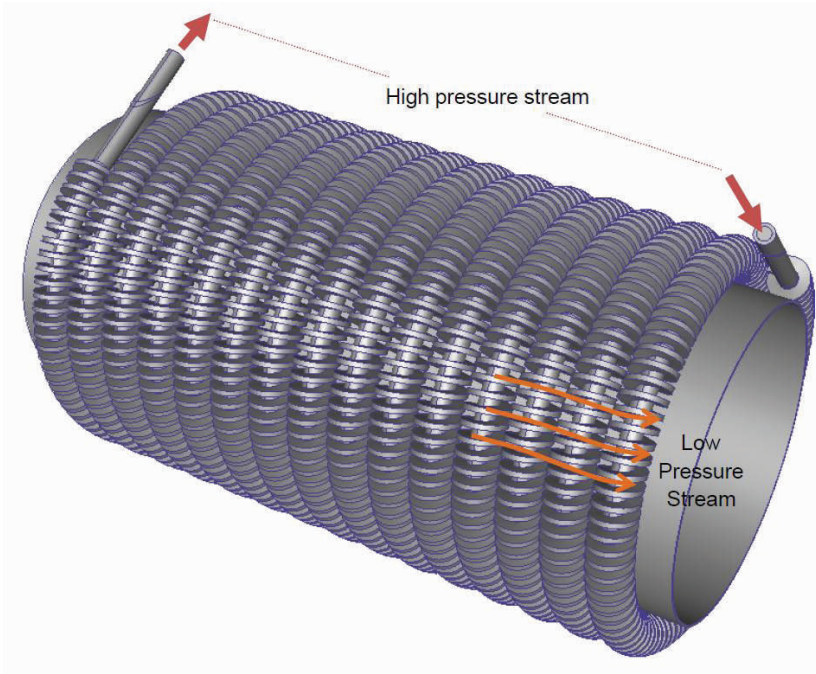
Finned-tube heat exchangers are also in use in Rafael. In this research a steady state analysis for our finned-tube heat exchangers is developed and implemented in Matlab code. The axial conduction and heat leaks to the environment are neglected. The analysis is capable of determining the pressure drops and the thermal performance of the heat exchanger with varying dimensions, gas pressures, and flow rates. A case study for a given configuration of the heat exchanger is explored to determine the fluid properties in the heat exchanger and the complete

heat exchanger performance. This configuration is further investigated at varying operating conditions. Finally, the finned-tube dimensions are optimized using a Design of Experiments (DOE) method for achieving the shortest total heat exchanger length and lowest pressure drops.

## NUMERICAL MODEL

A general view of the heat exchanger is shown in Figure 1. The heat exchanger consists of a helical finned-tube bounded on a mandrel and wrapped by an envelope, that is not shown in Figure 1. The high pressure stream flows through the inner tube, and the low pressure stream flows over the outer side of the finned-tube and is limited by the outer envelope. A sealing wire is used between the finned-tube and the outer envelope to force the low pressure stream to flow over the finned-tube.

There are several approaches for analyzing heat exchangers. An efficient method is by minimization of entropy generation, as for example has been used by Lerou et al. [13]. In the current research the analysis has been developed for determining the required heat-exchanger length for the desired heat-exchanger efficiency. In this analysis the heat exchanger has been divided into elements, where each element consists of a single bind of the helical finned-tube. The element has been chosen like this in order to equalize the mass flow rates of the high pressure and low pressure streams. Thus, the total length of the heat exchanger equals the width of a single bind,  $df$ , multiple by the number of required binds. A single element is presented in Figure 2 and schematically shown in Figure 3.



**Figure 1.** A general description of the finned-tube heat exchanger.

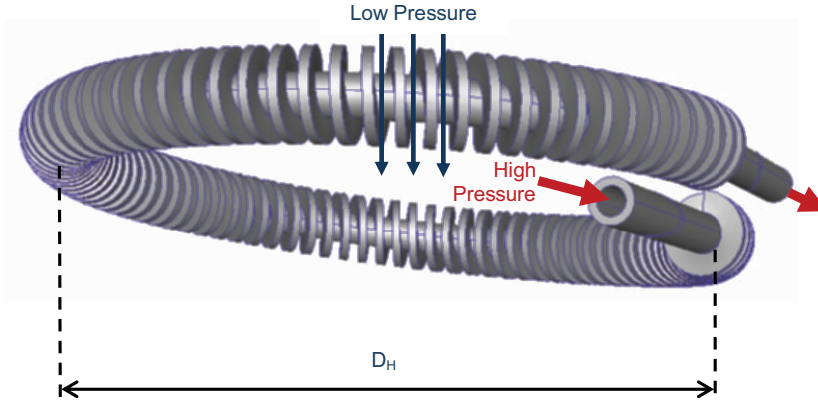


Figure 2. 3D view of a single element of the heat exchanger

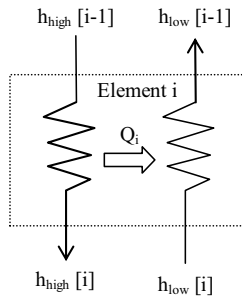


Figure 3. A schematic view of a single element with the enthalpies marking

### Physical dimensions

A section of the finned-tube with its dimensions is presented in Figure 4. The area for convective heat transfer for the high pressure stream is:

$$A_{high} = \pi \cdot d_{t,i} \times \pi \cdot D_H = \pi^2 d_{t,i} D_H \quad (1)$$

the area of the fins is given by:

$$A_{fin} = \pi^2 D_H \left[ \frac{d_f^2 - d_{t,o}^2}{2p_f} + d_f \left( 1 - \frac{t_f}{p_f} \right) \right] \quad (2)$$

the area for convective heat transfer for the low pressure stream is:

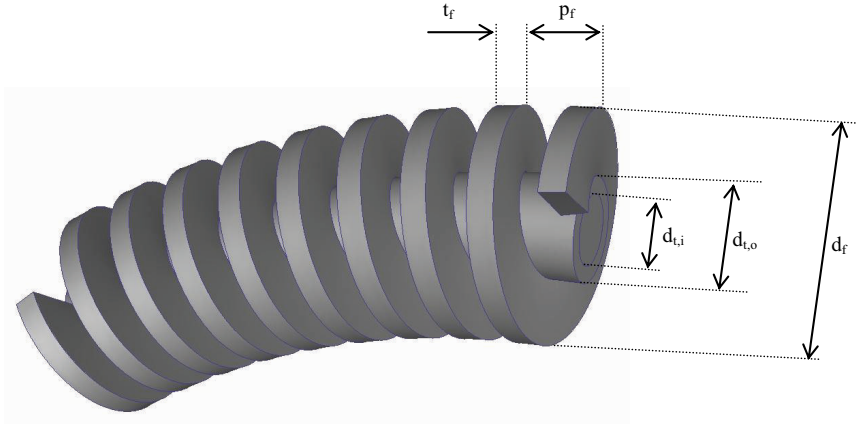
$$A_{low} = A_{fin} + \pi^2 D_H d_{t,o} \frac{t_f}{p_f} \quad (3)$$

the cross section area of the low pressure stream is:

$$A_{cross,low} = \frac{\pi}{4} \left[ (D_e - 2t_w)^2 - (D_e - 2d_f)^2 \right] - \pi \cdot D_H \left[ d_{t,o} \left( 1 - \frac{t_f}{p_f} \right) + d_f \frac{t_f}{p_f} \right] \quad (4)$$

the hydraulic diameter of the low pressure stream is:

$$D_{low} = \sqrt{\frac{4A_{cross,low}}{\pi}} \quad (5)$$



**Figure 4.** A section of the finned-tube

the fin length is determined by:

$$L_f = \frac{d_f - d_{t,o}}{2} \quad (6)$$

and the cross section area of the high pressure stream is:

$$A_{cross,high} = \frac{\pi \cdot d_{t,i}^2}{4} \quad (7)$$

### Thermal analysis

The governing equations of an element are:

$$h_{high}[i] = h_{high}[i-1] - \frac{Q_i}{\dot{m}} \quad (8)$$

$$h_{low}[i] = h_{low}[i-1] - \frac{Q_i}{\dot{m}} \quad (9)$$

$$Q_i = \varepsilon_i \cdot C_{\min} (T_{high}[i-1] - T_{low}[i]) \quad (10)$$

where  $i$  is the element index and  $C_{\min} = \dot{m} \cdot c_{p,\min}$  [14,15]. The heat exchanger effectiveness is calculated by:

$$\varepsilon_i = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]} \quad (11)$$

where  $C_r = C_{\min} / C_{\max}$

$$NTU = \frac{UA}{C_{MIN}} \quad (12)$$

$$UA = \frac{1}{\frac{1}{h_{high}A_{high}} + \frac{\ln\left(\frac{d_{t,o}}{d_{t,i}}\right)}{2\pi \cdot k_t \cdot L_t} + \frac{1}{\eta \cdot h_{low}A_{low}}} \quad (13)$$

$$\eta = 1 - \frac{A_{fin}}{A_{tube}} (1 - \eta_f) \quad (14)$$

where  $\eta$  is the fin efficiency and  $\eta_f$  is the adiabatic fin efficiency, which for rectangular fins is calculated by:

$$\eta_f = \frac{\tanh(mL)}{mL} \quad (15)$$

$$m = \sqrt{\frac{2h}{k \cdot t_f}} \quad (16)$$

The properties of the fluid are obtained from pre-determined tables derived from NIST-12<sup>TM</sup> data base. The properties of interest are: density, enthalpy, specific heat capacity, thermal conductivity, viscosity, and the Pr number.

Several correlations are proposed in the literature for determining the convective heat transfer coefficient for both streams [7,8,10,11]. In this research the correlation proposed by Hong et al. [8] for the high pressure stream has been adopted:

$$h_{high} = 0.023 \cdot \text{Re}^{0.8} \text{Pr}^{1/3} \left( 1 + 3.5 \frac{d_{t,i}}{D_H} \right) \left( \frac{k_{high}}{d_{t,i}} \right) \quad (17)$$

the correlation for calculating the convective heat transfer coefficient for the low stream is based on a previous work in which we have developed a finite elements analysis for JT cryocoolers [16]:

$$h_{low} = 0.26 \cdot \text{Re}^{0.75} \text{Pr}^{1/3} \left( \frac{k_{low}}{D_{low}} \right) \quad (18)$$

the Reynolds number is calculated as follows:

$$\text{Re} = \frac{\dot{m} \cdot D}{\mu \cdot A} \quad (19)$$

where for the high pressure stream  $D = d_{t,i}$  and  $A = A_{\text{cross,high}}$  and for the low pressure stream  $D = D_{\text{low}}$  and  $A = A_{\text{cross,low}}$ .

The analysis begins at the warm end of the heat exchanger where  $T_{\text{high}}[i-1]$  equals the ambient temperature,  $p_{\text{high}}[i-1] = p_{\text{high}}[i] = p_{\text{high}}$ ,  $p_{\text{low}}[i-1] = p_{\text{low}}[i] = p_{\text{low}}$ , and  $T_{\text{low}}[i-1] = T_{\text{high}}[i-1] - \Delta T$ , where  $\Delta T$  is set according to the desired heat exchanger efficiency (in the current case it equals 2°). The three variables,  $T_{\text{high}}[i]$ ,  $T_{\text{low}}[i]$ , and  $Q_i$ , are determined by solving equations 8 ÷ 10.

When the temperature distribution in the heat exchanger is obtained, the pressure drops at both streams are determined. There are several methods for calculating the pressure drops in a helix finned-tube heat exchanger. In the current study a merge of the equations suggested by Hong et al. [8] and Chua et al. [7] is implemented. The pressure drops at the high and low pressure streams are calculated, respectively, by:

$$\frac{dp_{high}}{dx} = - \left( \frac{f_{high}}{2 \cdot d_{t,i}} \rho U^2 \right) - \frac{d\rho U^2}{dx} \quad (20)$$

$$\frac{dp_{low}}{dx} = \left( \frac{2 \cdot f_{low}}{D_{low}} \rho U^2 \right) + \frac{d\rho U^2}{dx} \quad (21)$$

the friction factors are calculated based on the suggestions of Timmerhaus & Flynn [17]. In order to take into account the fins influence the correlation is modified by adding  $X_A = 1 - t_f/p_f$ :

$$f_{high} = \frac{0.184}{\text{Re}^{0.2}} \left( 1 + 3.5 \frac{d_{t,i}}{D_H} \right) \quad (22)$$

$$f_{low} = \frac{2}{X_A^2 \text{Re}^{0.15}} [0.088 - 0.16 \cdot X_L] \quad (23)$$

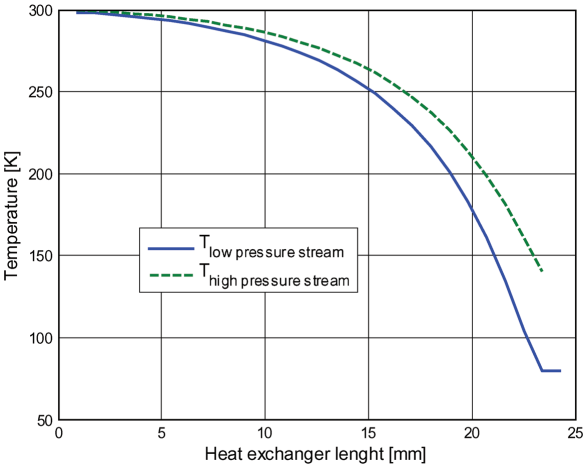
where  $X_L = d_{t,o}/d_f$ .

RESULTS AND DISCUSSION

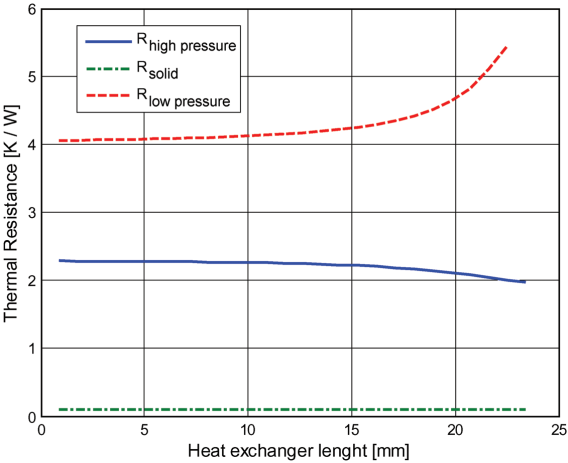
A Case Study

The first case study is for the dimensions of one of the finned-tube heat exchangers that are in use in Rafael. The flow rate is 0.2 g/s, the high pressure is 40 MPa, and the envelope diameter is 10 mm. Figure 5 presents the temperature distribution in the heat exchanger for the high and low pressure streams. The required heat-exchanger length obtained is 23.4 mm and the temperature of the high pressure stream that leaves the heat exchanger is 140.3 K. These results fit our knowledge based on many years of experience.

While exploring the heat exchanger, the model provides insight into many parameters of the fluids: the velocity profiles of both streams in the heat exchanger, the Reynolds numbers, the Prandtl numbers, and more. The overall heat transfer coefficient, UA, of the heat exchanger consists of three thermal resistances as previously described in Equation 13: the convective resistance of the high pressure stream, the conductive resistance of the solids, and the convective heat transfer of the low pressure stream. Figure 6 describes these three resistances from which one should conclude that the convective resistance of the low pressure stream is the highest and the one that must be improved.



**Figure 5.** Temperature distribution for high and low pressure streams in the heat exchanger for the case study.



**Figure 6.** Thermal resistances in the heat-exchanger for the case study

Case Study Heat Exchanger at Varying Operating Conditions

Using the dimensions of the existing heat exchanger, the following parameters have been varied parametrically:

- Mass flow rate, 0.02 to 1.0 g/s
- High pressure, 10 to 80 MPa

Two resulting parameters are the main concern: the required heat exchanger total length, and the pressure drop in the low pressure stream (which determines the cooling temperature of a JT cryocooler). The results are presented in Figures 7 and 8 as a function of the mass flow rate and high pressure, respectively.

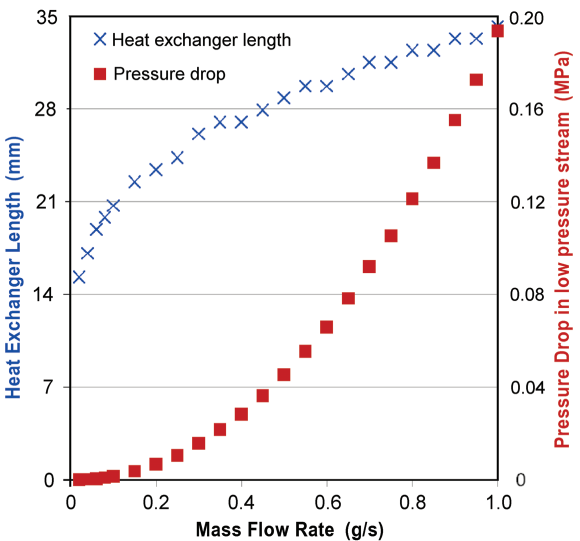


Figure 7. Pressure drop in the low pressure stream and heat exchanger length dependence on the mass flow rate, for the existing heat exchanger

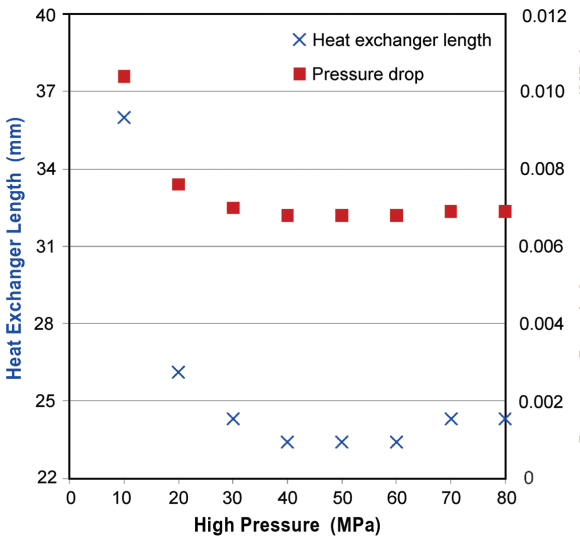


Figure 8. Pressure drop in the low pressure stream and heat exchanger length dependence on the high pressure, for the existing heat-exchanger

Increase of the flow rate requires longer heat exchangers and also increases the pressure drop in the low pressure stream. However, the trends of the two relations are different, as shown in Figure 7. The high pressure has minimal effect on the heat exchanger length and the pressure drop, except for extreme low values. It seems that the pressure drop follows the heat exchanger length and does not depend directly on the high pressure. According to Figure 8, the high pressure has to be maintained above 30 MPa.

### Changing the Finned-Tube Heat Exchanger Dimensions

The option of using the same inner tube of the heat exchanger with different fins was also explored. The following parameters were examined:

- $d_f$ , 0.5 to 1.2 mm
- $p_f$ , 0.18 to 0.3 mm
- $t_f$ , 0.05 to 0.15 mm

And, the following parameters were kept constant:

- Mass flow rate, 0.2 g/s
- $D_o$ , 10 mm
- $p_{high}$ , 40 MPa

The investigation was performed using Mode-Frontier software, using a MOGA-II optimizer and the SOBOL method for DOE. The results were computed as a function of  $d_f$ . The results showed that the required heat exchanger length decreased dramatically with a reduction in the fin height. This is associated with the increased velocity that increases the convective heat transfer coefficient due to the reduction in the hydraulic diameter of the low pressure stream. However, this is at the expense of increased pressure drop in the low pressure stream, which becomes significant at very low values of  $d_f$ . According to the analysis results,  $d_f$  has to be equal to about 0.7 mm. The other parameters,  $p_f$  and  $t_f$ , were found to have only a weak influence on the heat exchanger performance. After a careful examination, we arrived at the conclusion that in order to reduce the pressure drop in the low pressure stream,  $p_f$  has to be maximized, and  $t_f$  has to be minimized. And, in order to reduce the heat exchanger length,  $p_f$  has to be minimized, and  $t_f$  has to be maximized.

Changing the inner tube of the finned-tube heat exchanger is executed by changing the inner diameter of the tube and calculating the tube thickness according to:

$$\sigma = \frac{p \cdot d_{i,d}}{2 \cdot t_i} \quad (24)$$

A second case was analyzed in which the inner tube diameter was varied, and the outer diameter was changed accordingly to keep the total finned-tube diameter constant. Examining the pressure drop in the low pressure stream, the inner diameter of the tube shouldn't exceed about 0.38 mm, while, according to the pressure drop at the high pressure stream, the inner diameter of the tube needs to be larger than about 0.23 mm. From the heat-exchanger length point-of-view, the inner diameter of the tube needs to be as large as possible. Therefore, in this case, an inner diameter of 0.38 mm is desired.

In summary, the analysis enables an extensive exploration of the heat exchanger: for example, changing the inner diameter of the tube while keeping the fins height constant, changing the envelope diameter, changing the helix diameter, and much more.

### CONCLUSIONS

A numerical model has been developed for analyzing helical finned-tube heat exchangers. These types of heat-exchangers are commonly in use for Joule-Thomson cryocoolers. The model determines the required heat-exchanger length and the pressure drops in both streams.



The model illuminates many characteristics of the fluids in the heat exchanger: temperature, pressure, velocity, density, Pr number, Re number, and more. It also provides thermal characteristics, such as heat capacity, convective heat transfer coefficient, NTU, effectiveness, and more.

A detailed investigation of a case study that refers to the existing finned-tube heat exchanger in use in Rafael was presented. The finned-tube diameter seems to be the most crucial dimension and, in the current case study, has to be equal to about 0.7 mm.

A Design of Experiments method was applied to explore new options for finned-tube heat exchangers. Our existing tube was examined with various options for fins: width, height, and pitch. The results show that the most influential parameter is the fin height. For the currently tested conditions, the finned-tube diameter has to be about 0.7 mm. Finally, various options for changing the dimensions of the tube were explored.

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