Exergy Flow in Multistage Cryogenic Refrigerators with Application to Multistage Reverse Brayton Cryocoolers

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ABSTRACT

Exergy analysis of multistage cryogenic refrigerators is considered in this study using separation of pressure and thermal components of exergy through the thermodynamic cycle. This analysis provides a method for estimating the performance of multistage cryocoolers under a variable load condition in terms of an exergy-based figure of merit for each component of the system. A figure of merit for the expansion process is proposed that is convenient for quantifying the irreversibility of the expansion process at each stage and its effect on the performance of multistage refrigerators. The method is applied to multistage reverse Brayton cryogenic refrigerators to evaluate their thermodynamic performance under a variable load condition at each stage. The performance of the reverse Brayton cryocoolers employing auxiliary coolers at the intermediate stages is also analyzed.

INTRODUCTION

In some applications, it is important to be able to estimate the performance of multistage cryocoolers using a minimum number of free parameters. For example, the effect of load shifting from one stage to other stages or the required input power for specific cooling capacity at different temperature values can be estimated. The most important parameters controlling the performance of cryocoolers are the second-law efficiencies of the compressor, the heat exchangers and the expansion process. For instance, in many applications of heat exchangers in cryogenic refrigerators, no cooling would be provided if the effectiveness of the heat exchanger were less than 90 percent[1]. Exergy and second law analysis are powerful methods for quantifying the losses in cryogenic refrigerators and quantifying the tradeoff in the irreversibility of each component to optimize the performance of these systems[2]. Recently, using the separation of thermal and pressure components of exergy, we proposed a Figure of Merit (*FOM*) for heat exchangers with application to cryogenic refrigerators that represents the important effect of the heat exchangers as a component in overall efficiency of the refrigerator[3]. The analysis of the recuperative and regenerative heat exchangers with application to the design of cryocoolers has shown that a compromise between fluid friction and heat transfer irreversibilities results in a shallow optimum for the *FOM* of the heat exchangers[4,5]. Another major source of irreversibility in cryocoolers is due to the expansion process. This irreversibility occurs as a result of the process of expansion converting the pressure component of exergy to the thermal component in the refrigerator[6,7]. In this study we concentrate on quantifying the effect of expansion efficiency on the performance of multistage cryogenic refrigerators with application to multistage Reverse Brayton Cryogenic Refrigerators (RBCR). Exergy flow in a three stage RBCR will be presented and the irreversibility of the important components of the refrigerator is quantified.

Estimation of overall performance of a particular cryocooler can be based on the correlation of the performance of existing cryocoolers that has been reported[8]. However, the performance of a cryocooler as a result of a possible change in the efficiency of its major components cannot be estimated. In this study we would like to use exergy analysis to quantify the most important sources of irreversibilities to estimate the performance of multistage cryogenic refrigerators. In addition, we separate the pressure and thermal components of exergy flow in a three stage RBCR, including the effect of its major components, to determine its performance using exergy analysis.

EXERGY-BASED MODEL OF MULTISTAGE CRYOGENIC REFRIGERATORS

Exergy analysis of heat exchangers

The schematic of the n-th heat exchanger in a multistage cryogenic refrigerator with the pressure and thermal components of exergy at the inlet and exit of the heat exchanger is shown in Fig. 1. The thermal and pressure components of exergy at the inlet and exit of the heat exchanger can be obtained from the temperature and pressure of the streams at the inlet and exit of the heat exchanger. We assume these quantities are either given or can be obtained from the fluid flow and heat transfer analysis of the heat exchangers. The temperature of the inlet hot stream of the first stage is assumed to be the temperature of the environment and the inlet temperature of the cold stream of the last heat exchanger is assumed to be the temperature of the last cold reservoir. Two interactions with the reservoir are possible at the mid-stage between the two heat exchangers. The reservoir can cool the stream resulting in input exergy to the multistage cryocooler. This input exergy is provided by an auxiliary cooler. It should be pointed out that the thermal exergy associated with heat transfer is opposite to the direction of the heat transfer when the reservoir temperature is lower than the temperature of the environment. The midstage reservoir can also provide the required cooling load where the thermal exergy is taken from the multistage cryocooler reducing exergy input to the heat exchanger downstream. When a midstage load between two heat exchangers exists, an expansion process is assumed that reduces the temperature of the stream below the temperature of the midstage reservoir providing the desired cooling load. It is assumed in this study that the heat exchange between a stream and the reservoir is ideal and the reservoir has infinite thermal capacity so that the temperature of the exit stream is equal to the temperature of the reservoir.

The Figure of Merit (*FOM*) of the n-th heat exchanger is defined by,

$$FOM_n = \frac{E_{cn}^{p} + E_{cn}^{th}}{E_{hn}^{p} + E_{hn}^{th}}, \tag{1}$$

Figure 1. Schematic of the heat exchanger with parameters used in the model.

where E is the exergy, the superscripts $_p$ and $_{th}$ denote the pressure and thermal components respectively, and the subscripts $_{hn}$ and $_{cn}$ are the hot and cold sides of the n-th heat exchanger, respectively. It should be pointed out that the magnitude of the thermal component of exergy increases at the cold side of heat exchangers in cryogenic applications and its direction is into the heat exchanger. Both heat transfer through a temperature difference and fluid friction result in the reduction of the exergy at the cold side of the heat exchanger, see Fig. 1.

Given the inlet temperatures of the streams, the effectiveness of the heat exchanger, and assuming a balanced heat exchanger, the exit temperatures of the heat exchangers can be determined. For example, assuming that the working fluid behaves as an ideal gas, the *FOM* for the n-th heat exchanger can be written as[4],

$$FOM_n = \frac{(T_{hn,out} - T_{cn,in}) - T_o \ln(T_{hn,out}/T_{cn,in}) + T_o(1-1/\gamma)\ln(\mathrm{Pr}_n)}{(T_{hn,in} - T_{cn,out}) - T_o \ln(T_{hn,in}/T_{cn,out}) + T_o(1-1/\gamma)\ln[\dfrac{\mathrm{Pr}_n/(1-\Delta P_{hn}/P_{hn,in})}{(1-\Delta P_{cn}/P_{cn,in})}]}, \tag{2}$$

where To is the temperature of the environment, γ is the specific heat ratio, Pr_n is the pressure ratio, and ΔP_{hn} and ΔP_{cn} are the pressure drop for the hot and cold streams, respectively. Using the definition of heat exchanger ineffectiveness $\lambda=1-\varepsilon$ with $\lambda \ll \varepsilon$ and $\Delta P_{hn} \approx \Delta P_{cn} = \Delta P_n/2$, Equation (2) can be simplified,[9]

$$FOM_n = \frac{[(T_{hn,in} - T_{cn,in}) - T_o(T_{hn,in}/T_{cn,in} - 1)]\lambda_n + T_o(1-1/\gamma)\ln[\mathrm{Pr}_n - \Delta P_n/P_n]}{[(T_{hn,in} - T_{cn,out}) - T_o(1 - T_{cn,in}/T_{hn,in})]\lambda_n + T_o(1-1/\gamma)\ln(\mathrm{Pr}_n)}. \tag{3}$$

Extensive numerical calculations of the regenerative and recuperative heat exchangers in well-designed cryogenic refrigerators have shown that the *FOM* is of the order of 0.7[5]. For a recent high performance recuperator, it has been reported that the *FOM* is of the order 0.85[10].

Exergy analysis of expansion process

Another important source of irreversibility in refrigerators is the exergy destruction in the expansion process. This process converts pressure exergy into thermal exergy for the purpose of transferring heat from the thermal reservoir. In this study we consider a model for the analysis of the expansion process between the n-th and (n+1)-th heat exchangers as shown in Fig. 2. The control volume shown in Fig. 2 consists of the n-th expansion process and the n-th cold box heat exchanger interacting with the n-th thermal reservoir. If heat is transferred from the reservoir to the control volume the temperature at the end of the expansion process must be smaller than the temperature of the reservoir. Otherwise heat must be transferred from the working fluid to the thermal reservoir. In this case an auxiliary cooler is required for the energy balance of the reservoir. We assume no irreversibility due to fluid flow in the heat transfer process in the cold box heat exchanger. The free parameters in the exergy analysis of the control volume of Fig. 2 are the difference in the pressure components of exergy driving the process and the exergy efficiency of the expansion process. The efficiency of expansion and throttling processes have been subject of a few studies[6,7]. The expansion process can produce work that can be used to reduce the input work of the compressor and should be considered as a product of the process. In application to pulse tube refrigerators this work is lost and must be included in the irreversibility of the expansion process.

The exergy balance for the control volume given in Fig. 2 where work is produced in the expansion process can be written as,

$$E_n^{th} + E_n^p = E_{n+1}^{th} + E_{n+1}^p + E_{Qn} + W_n + I_n, \tag{4}$$

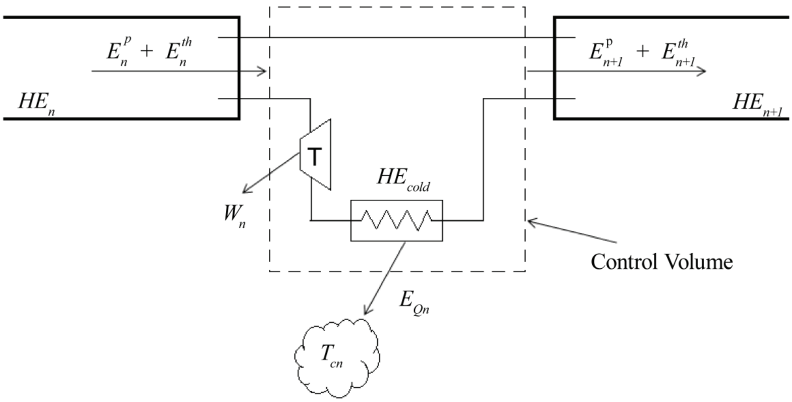

Figure 2. Schematic of the control volume for the n-th expansion process between n-th and (n+1)-th heat exchangers used in the model.

where E_{Qn} is the magnitude of the thermal exergy transferred to the n-th cold reservoir. We define the second-law efficiency of the expansion process using the following relation,

$$\eta_n = 1 - I_n/(E_n^p - E_{n+1}^p), \tag{5}$$

where I_n is the irreversibility of the expansion process. Assuming no irreversibility due to the fluid flow in the n-th cold box, the thermal exergy balance can be written as

$$E_{n,exp}^{th} - E_n^{th} = E_{Qn} + E_{n+1}^{th} - E_n^{th}, \tag{6}$$

where $E_{n,exp}^{th}$ is the thermal exergy after the expansion process. It should be pointed out that if $E_{n,exp}^{th} < E_{n+1}^{th}$ no cooling can be produced at the load temperature and an auxiliary cooler must be used to maintain the thermal reservoir at the temperature of T_{n+1}. We introduce a parameter Fn for the expansion process to estimate work produced by the process,

$$F_n = W_n/(W_n + E_{n,exp}^{th} - E_n^{th}). \tag{7}$$

From Equations (6) and (7) the parameter F_n can be written as,

$$F_n = W_n/(W_n + E_{Qn} + E_{n+1}^{th} - E_n^{th}). \tag{8}$$

Using Equations (4), (5) and (8), we can find an expression for the magnitude of exergy transfer to the cold reservoir,

$$E_{Qn} = \left(E_n^{th} - E_{n+1}^{th}\right) + \eta_n(1 - F_n)(E_n^p - E_{n+1}^p). \tag{9}$$

For oscillating regenerative processes the quantities are time dependent and should be integrated over the period of oscillation. An estimate of the parameter F_n can be obtained for an ideal gas using separation of the work produced and the irreversibility during the expansion process. Assuming an ideal gas equation of state for the working fluid, the change in the pressure component of exergy per unit mass in the expansion process can be related to the pressure ratio during the expansion.

$$E_n^p - E_{n+1}^p = RT_o ln(Pr_n), \tag{10}$$

where R is the gas constant and P_{rn} is the pressure ratio for the n-th expansion process. Using the definition of adiabatic efficiency for the expansion process, η_{an}, and the exergy efficiency of the expansion process given by Equation (5), the following relation for parameter F_n can be obtained.

$$F_n = T_n\eta_{an}(1 - Pr_n{}^{\left(\frac{1}{k}-1\right)})/\{T_o ln[1 - \eta_n(1 - Pr_n{}^{\left(\frac{1}{k}-1\right)})]\} \ , \tag{11}$$

where k is the specific heat ratio of the working fluid. The relation between η_n and η_{an} can be written as,

$$\eta_{an} = [1 - Pr_n{}^{\eta_n\left(\frac{1}{k}-1\right)}]/\left[1 - Pr_n{}^{\left(\frac{1}{k}-1\right)}\right]. \tag{12}$$

From Equations (11) and (12) the following relation for the parameter F_n in terms of η_n can be obtained,

$$F_n = (T_n/T_o)[1 - Pr_n{}^{\eta_n\left(\frac{1}{k}-1\right)}]/[\eta_n\left(1 - \frac{1}{k}\right) ln\ (Pr_n)]. \tag{13}$$

Equation (12) shows that $\eta_{an} > \eta_n$. In many cryogenic refrigerator applications, where the pressure ratio is not large, they are numerically close. Using the expansion of the functions in Equation (13) in series, the parameter F_n can be approximated in most applications by $F_n \approx T_n/T_o$.

RESULTS AND DISCUSSION

Estimation of performance of single stage cryogenic refrigerators

We first consider the application of the method in estimation of performance of single stage cryogenic refrigerator using only a few free parameters. The exergy input to the cryocooler is in the form of electric power and it is partially destroyed in the compressor and aftercooler while it is converted to the pressure component of exergy with a very small thermal exergy. The effect of the compressor and aftercooler is taken into account, defining an exergetic efficiency for the combined compressor and aftercooler. The input exergy to the heat exchanger is partially destroyed by fluid friction and heat transfer and exits the cold side of the heat exchanger. The efficiency of the heat exchanger is given by the *FOM* defined in this study and is estimated by Equation (3). Higher order methods can be used to quantify the effect of the heat exchanger. The other two parameters needed for estimation of the *FOM* of the heat exchanger are its effectiveness and the pressure ratio across the heat exchanger. The effect of the pressure drop across the heat exchanger can be estimated by the efficiency of the pressure component of exergy given by η_p with $1 \geq \eta_p \geq FOM$[9]. The other major process that affects the performance of cryocoolers is the expansion process converting the available pressure exergy at the exit of the heat exchanger to thermal exergy so that heat can be extracted from the cold reservoir. Dropping the subscript n from Eqn. (9) for single stage cryogenic refrigerators, the thermal exergy delivered to the thermal reservoir at the temperature Tc can be written as,

$$E_Q = W_{COMP}\,\eta_{COMP}\,\{FOM - \eta_p[1 - \eta(1 - F)]\}. \tag{14}$$

FOM in the above equation is given by Equation (3) for the heat exchanger in application to single stage refrigerators where the subscript n should be dropped. The magnitude of *FOM* decreases as the cold end temperature T_c is reduced. Therefore, a very low value of heat exchanger ineffectiveness is required for positive cooling load from the cold reservoir. The parameter F is given by Equation (13) where the subscript n is dropped for application to the single stage refrigerators. The effect of expansion efficiency η on F given by Eqn. (13) is interesting and can be approximated with reasonable accuracy by T_c/T_o in most applications as indicated previously. The no-load temperature of a cryocooler is the consequence of the extended bracket in Eqn. (14) reaching a value of zero as the *FOM* is reduced. This no load temperature is approximately independent of the compressor power as shown in Eqn. (14). In addition, because of the definition of the magnitude of cooling exergy given by, $E_{Q=}Q\left(\frac{T_o}{T_c}-1\right)$, and the variation of the *FOM* and F with T_c as given by Equations (3) and (13), it can be shown that the cooling capacity is approximately a linear function of cold end temperature T_c usually observed in developing the load curves for cryocoolers. It should be pointed out that for the case of expansion processes when no work is produced, the factor $\eta(1-F)$ is denoted by η_{exp} used previously[9]. In this case the factor F does not have to be introduced.

Exergy flow in a multistage RBCR

As an example of the application of exergy analysis developed in this study we consider the exergy flow and second law analysis of a three stage RBCR. The parameters used in the analysis are given in Table 1 for the relevant major components. The working fluid is taken to be helium and it is assumed to behave as an ideal gas for the range of temperature considered in this analysis. Optimization and second-law analysis of a two-stage RBCR for different configurations has been recently reported[11]. The average pressure used in the example is three atmospheres with the pressure ratio of two across the compressor. Assuming that the heat exchangers are balanced with given values of effectiveness and the pressure drop ratio at the hot and cold streams given in Table 1, the temperature and pressure at the inlet and exit of all heat exchangers can be determined. It should be pointed out that the inlet temperature at the hot side of each heat exchanger is assumed to be equal to the corresponding temperature of the thermal reservoir given in Table 1. It is assumed that the heat exchanger effectiveness of each cold box heat exchanger is high with no pressure drop for the fluid passing through it. Given the pressure and temperature at the inlet and the exit of each heat exchanger, the value of the *FOM* for the heat exchanger can be calculated using Eqn. (3).

The refrigerator is driven by a compressor followed by a highly effective aftercooler. Three heat exchanger and expander combinations as shown in Fig. 2 follow the aftercooler. The expanders are connected to the three thermal reservoirs with fixed temperatures given in Table 1. The adiabatic efficiency of each expander is also given in the table.

Table 1. Data used for exergy analysis of a three-stage RBCR

Components	Temperature K	Effectiveness	Adiabatic efficiency	Pressure drop ratio or pressure ratio
Compressor, COMP			0.7	2
Aftercooler, AF	300	1		0.02
Heat exchanger 1, HE1		0.99		0.02
Expansion 1, EXP1	120		0.6	0.3
Heat exchanger 2, HE2		0.99		0.02
Expansion 2, EXP2	60		0.6	0.3
Heat exchanger 3, HE3		0.99		0.02
Expansion 3, EXP3	30		0.6	calculated

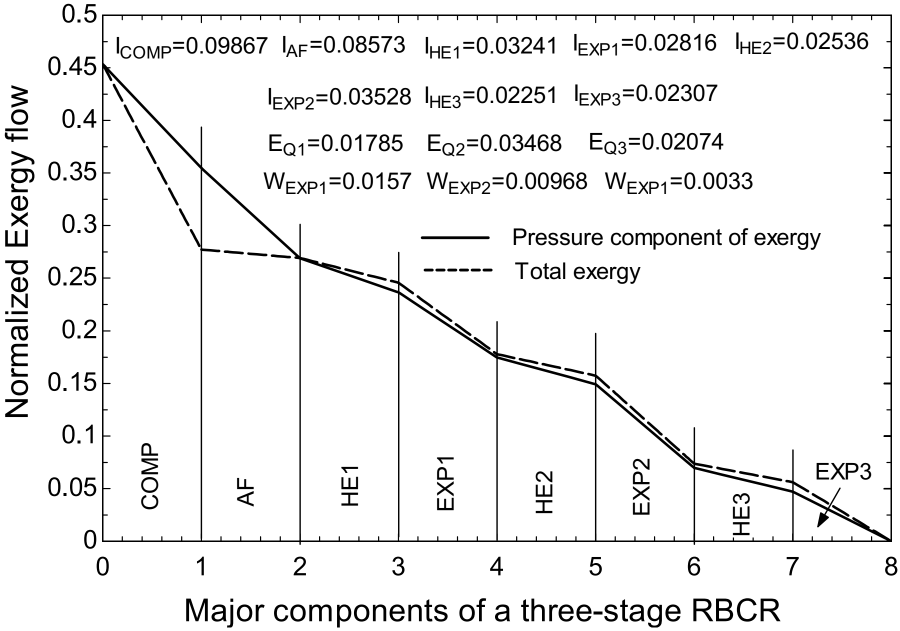

Figure 3. Normalized total and pressure components of exergy flow, normalized irreversibility of the components and the normalized exergy of the products for a three-stage RBCR

Figure 3 gives the result of the exergy flow analysis of the three-stage RBCR. The figure shows the total exergy flow and the pressure exergy flow at the inlet and exit of each component and is connected linearly to show the exergy flow through the refrigerator. The exergy flow is normalized by dividing the exergy flow rate by the thermal capacity rate $\dot{m}T_oC_p$. The pressure exergy developed by the compressor drives the system while the thermal exergy produced in the compressor is totally destroyed in the aftercooler. The irreversibility of the compressor and aftercooler are the two major losses of the refrigerator, accounting for about 40 percent of the losses for the example used in this study. The thermal exergy flow is not shown in the figure and can be obtained by subtracting the pressure exergy from the total exergy. The total exergy and the pressure exergy flow cross each other at the exit of the aftercooler, resulting in a negative thermal exergy component downstream of the aftercooler. This indicates that a net thermal exergy enters each heat exchanger and is destroyed as a result of the heat transfer through a finite temperature difference in each heat exchanger. The slope of the pressure component of exergy in each heat exchanger is indicative of the fluid friction and the pressure drop in the heat exchanger and the corresponding irreversibility. The pressure drop component of exergy in each expander results in the thermal exergy of the product transferred to the thermal reservoir. The cooling capacity at each stage can be controlled by the pressure ratio of each expander and its second-law efficiency. The pressure ratio for the expansion process is calculated from the exit pressure at the hot side of the heat exchanger and the pressure drop ratio given in Table 1. The pressure drop ratio is defined as a fraction of the total pressure change across the compressor. The magnitude of the irreversibility of each component and the exergy transferred to the reservoirs are also given in Fig. 3. Assuming that the work of expansion process is recoverable, the exergetic efficiency of the refrigerator can be obtained as the ratio of the total exergy transferred to the thermal reservoirs and the net compressor power. This results in a value for the efficiency of 0.17 for the three-stage RBCR used in this study. The calculation is performed using EES software[12].

CONCLUSIONS

A relation between the adiabatic efficiency of the expansion process and the exergetic efficiency for the expansion was developed. A method based on the separation of the thermal and pressure components of exergy is developed to relate the work produced and the thermal exergy change in the expansion process. The method can be used to estimate the performance of multistage cryogenic refrigerators under a variable load condition or when an auxiliary cooler must be used. For application to single stage refrigerators, a simple equation is developed capable of estimating its performance in terms of the most important free parameters. The method is applied to a three-stage RBCR to quantify the pressure and thermal components of the exergy flow in the components of the refrigerator. The irreversibility of each component and its relation to the exergy flow at its boundary is determined.

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