Passive Mechanical Device for Phase Shifting in a Pulse Tube Cryocooler

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ABSTRACT

A passive mechanical device for phase shifting was developed for a miniature Pulse Tube In-Line cryocooler developed earlier at the Rechler Cryogenic Laboratory, Technion-Israel Institute of Technology. A passive piston supported on flexure bearings was proposed for this mechanical device. In order to produce the desired phase shift in the regenerator, the piston mass, flexure bearings stiffness and damping coefficient of the mechanical system were obtained using a phasor analysis and a mechanical analogy, and by simulations using a commercial numerical solver for pulse tube cryocoolers (Sage®). In these calculations, other parameters of the existing cryocooler remained without changes. Simulation results were compared with those from the theoretical model. A prototype was manufactured and experiments were performed. The lowest temperature observed in the experiments was 158K. The discrepancy between theoretical results and experimental results was due to the limitations in the precision manufacturing and assembling of the passive mechanical phase shifter.

One of the difficulties of the instrumentation in the current experiment was measurement of the passive piston high frequency movement (at about 100 Hz) that had to be contact-less. It was proposed to use a laser beam and reflective surface mounted to the back side of the passive piston. The proposed measurement system was designed, manufactured and used in the experiments.

INTRODUCTION

A present worldwide trend focuses on the miniaturization of regenerative cryocoolers. It is evident that the pulse tube cryocooler is one of the best candidates for miniaturization. Downscaling the mechanical dimensions without a change in the operating parameters becomes problematic, as internal heat losses become dominant in the device. The method proposed to counteract these heat losses is to increase the fill pressure and the operating frequency of the miniature cryocoolers. A critical parameter for efficient operation is the proper phase shift between thr pressure and the mass flow rate at the different parts of the cryocooler.

One of the smallest pulse tube cryocoolers constructed to-date was designed at the Rechler Cryogenic Laboratory, Technion-Israel Institute of Technology by Sobol et al. [1]. An inertance tube and a reservoir were used for phase shifting in the cryocooler. This miniature pulse tube cryocooler achieved a no-load temperature of 99K, at a heat rejection temperature of 305K, and provided 400 mW cooling at

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110K at an operating frequency of 103 Hz. The main constraint of this cryocooler was phase shift limitation using the inertance tube. In miniature devices, the impedance created by the inertance tube has a large resistive component, which limits its ability to produce the required phase shift.

A possible solution for increasing the phase shift in the pulse tube cryocoolers was proposed by Matsubara et al. [2]. The concept was to use a passive mechanical device instead of an inertance tube and a reservoir. Theoretically, any desired phase shift could be produced by such a passive mechanical device and it could be miniaturized without deterioration of its performance, in contrast to the inertance tube.

In 2012 Lewis, Bradley and Radebaugh from NIST [3] developed and tested a miniature pulse tube with actively controlled piston installed at the hot end of the cryocooler instead of the inertance tube. The phase shift was controlled by software that locked the phase between the compressor and the piston at the hot end. It was shown that the efficiency of the pulse tube cryocooler with actively controlled piston was better than that of the same cryocooler with the inertance tube. The improvement of the efficiency was attained because the use of a controlled warm expander made possible flexible control in optimization of phase angles.

In the current research, the inertance tube and reservoir of the existing cryocooler [1] were replaced by a passive mechanical device. Design parameters of the existing cryocooler remain unchanged. The overall length of the assembly of aftercooler, regenerator, cold heat exchanger, buffer tube and hot heat exchanger is 31 mm. The regenerator is filled with SS#635 mesh screen and its length is 12 mm. The proposed design of the mechanical phase shifter is a passive piston supported on flexure bearings. The passive piston diameter is 5 mm. The main design parameters selected to produce the desired phase shift using the passive piston are: piston mass (50 gm), overall flexure bearings stiffness (16000 N/m) and damping coefficient whose source is oscillating gas (helium) in a precise clearance gap (20 µm) between piston and cylinder. These design parameters were obtained using phasor theory and compared to the simulation of the pulse tube cryocooler with the proposed mechanism for phase shifting by commercial software Sage®. Final optimization of the design parameters was performed by Sage®.

For the comparison of the experimental and the theoretical model, it was necessary to measure the passive piston displacement at high frequencies. For this purpose, a contact-less displacement measurement system was designed, manufactured and implemented.

THEORETICAL ANALYSIS

Phasor Analysis for the Pulse Tube Cryocooler with Passive Piston

Phasor analysis is a common theoretical tool for investigation of different parameters in regenerative cryocoolers. Using such analysis eliminates time dependence from the equations which are transformed from the time-dependent space to the vector space and only phases and amplitudes of the flow oscillation become important. Radebaugh [4] introduced phasor analysis in a pulse tube refrigerator where the phase shifter was an orifice. The difference between [4] and the analysis introduced in this work is that the orifice and reservoir were replaced here by the passive piston (see Figure 1). The pressure phasor is permanently located along the real axis as a reference. Another difference is the following: in the regenerator the phase shift has to be close to zero for efficient performance of the pulse tube cryocooler. Thus, the phase angles 'upward' (counter-clockwise) from the regenerator are positive and the phase angles 'downward' (clockwise) from the regenerator are negative. Since the only difference between the pulse tube cryocooler in [4] and the one in this work is the passive piston, only the expressions of the mass flow in the passive piston would be introduced here. A more detailed analysis is given in [5].

The passive piston volume can be expressed as follows:

$$\vec{V}_p = V_{p0} + \frac{V_{p\_amp}}{2}\left[1+\sin\left(\omega t+\phi_p\right)\right] \tag{1}$$

where: V_p is the volume of the passive piston; V_{p0} is the 'dead' volume; V_{p_amp} is the swept, or stroke volume, and $V_{p\_amp}/2$ is the amplitude of the swept volume.

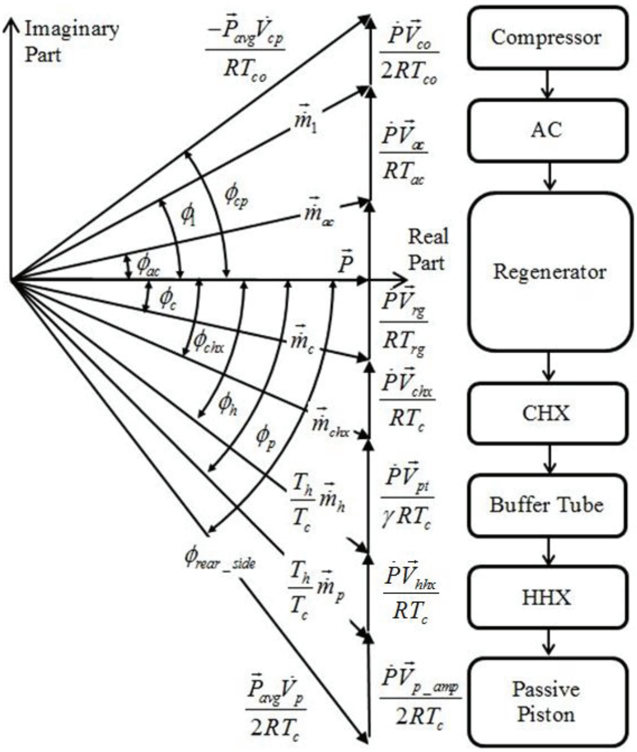

Figure 1. Schematic diagram for a Pulse Tube cryocooler equipped with a Passive Piston - in Vector space.

The passive piston mass conservation can be expressed as follows:

$$\vec{\dot{m}}_p = \frac{\vec{\dot{P}} V_{p\_amp}}{2RT_h} + \frac{P_a \vec{\dot{V}}}{RT_h} \tag{2}$$

where $\dot{m}_p$ is the mass flow, P_a is the mean pressure, T_h is the 'hot' temperature, $\dot{V}$ is the volumetric flow, and $\dot{P}$ is the pressure fluctuation in time.

Or,

$$\frac{T_h}{T_c}\vec{\dot{m}}_p = \frac{\vec{\dot{P}} V_{p\_amp}}{2RT_c} + \frac{P_a \vec{\dot{V}}}{RT_c} \tag{3}$$

where T_c is the 'cold' temperature.

The phasor analysis indicates the following: the optimum performance in the regenerator can be attained if the mass flow rate vector to the regenerator from the aftercooler would be equal to the mass flow rate vector from the cold heat exchanger. In this case these vectors would be symmetric about the real axis of the diagram. This causes the average mass flow to be in phase with the pressure in the regenerator. In the conventional Stirling cryocooler the same phasor arrangement would produce efficient performance of the device.

Therefore, from Figure 1, in order to obtain the optimal phase shift in the regenerator the following should hold simultaneously:

$$\begin{gathered} \phi_{ac} = -\phi_c \\ |\dot{m}_{ac}| = |\dot{m}_c| \end{gathered} \tag{4}$$

MECHANICAL ANALOGY

The solution obtained from the phasor analysis performed in the previous section does not include mechanical parameters that are necessary for the design of the passive piston. The proposed method for obtaining these design parameters is a mass-spring-damper analogy of the pulse tube cryocooler. The results from applying this method can be used together with results from the phasor analysis. The displacement of the gas acted upon by the compressor is represented by x_{cp} and that of the gas acted upon by the passive piston is represented by x_{pp} (which is assumed to be the same as the passive piston displacement). The gas in the pulse tube is represented as a spring with finite stiffness k_{pt} and negligible mass. The passive piston is represented as a mass, M_{pp}; spring stiffness, k_b, which can be a mechanical spring or a gas spring; and damping coefficient, c_{pp}. The compressor piston working area (the area interacting with the gas) is represented by A_{cp} and the passive piston working area is represented by A_{pp}. The pulse tube volume is represented as V_{pt} and the passive piston volume as V_b. A schematic mass-spring-damper model of the pulse tube cryocooler with a passive piston is shown at Figure 2.

The oscillation of gas in the pulse tube cryocooler can be described using this method by a second order ordinary differential equation (ODE):

$$M_{pp}\ddot{x}_{pp} + c_{pp}\dot{x}_{pp} + \left(k_b + k_{pt}\right)x_{pp} = k_{pt}\frac{A_{pp}}{A_{cp}}x_{cp} \tag{5}$$

where the ratio A_{pp}/A_{cp} represents an amplified ratio of spring stiffness in the pulse tube due to the different working areas of the compressor and the passive piston. The gas stiffness can be found from Hook's law and the expression for the polytropic compression.

Then, the equation (5) can be expressed as follows:

$$M_{pp}\ddot{x}_{pp} + c_{pp}\dot{x}_{pp} + x_{pp}k_{tot} = \frac{x_1 k_{pt} A_{pp}}{A_{cp}}\cos\left(\omega t\right) \tag{6}$$

where x_1 is the amplitude of the compressor piston displacement.

Initial conditions of the passive piston displacement and velocity are:

$$x_{pp}\left(0\right) = 0; \dot{x}_{pp}\left(0\right) = 0 \tag{7}$$

The solution for equation (6) is:

$$x_{pp}\left(t\right) = \tilde{A}\cos\left(\omega t\right) + \tilde{B}\sin\left(\omega t\right) \tag{8}$$

where
$$\tilde{A} = \frac{x_1 k_{pt} A_{pp}\left(k_{tot} - M_{pp}\omega^2\right)}{A_{cp}\left[\left(c_{pp}\omega\right)^2 + \left(k_{tot} - M_{pp}\omega^2\right)^2\right]}; \ \tilde{B} = \frac{x_1 k_{pt} A_{pp} c_{pp}\omega}{A_{cp}\left[\left(c_{pp}\omega\right)^2 + \left(k_{tot} - M_{pp}\omega^2\right)^2\right]}$$

The mass flow rate of the gas due to the piston oscillation can be expressed as follows:

$$\vec{\dot{m}}_g = \rho A_{pp}\dot{x}_{pp} \tag{9}$$

where: $\dot{m}_g$ is the mass flow rate of the gas, and ρ is the density of the gas.

Therefore,

$$\vec{\dot{m}}_g\left(t\right) = \vec{\dot{m}}_p\left(t\right) = \rho A_{pp}\omega\left[-\tilde{A}\sin\left(\omega t\right) + \tilde{B}\cos\left(\omega t\right)\right] = \rho A_{pp}\omega\sqrt{\tilde{A}^2 + \tilde{B}^2}\sin\left(\omega t + \tilde{\phi}_{in}\right) \tag{10}$$

Or,

$$\vec{\dot{m}}_p\left(t\right) = \rho A_{pp}\omega\sqrt{\tilde{A}^2 + \tilde{B}^2}\cos\left(\omega t + \tilde{\phi}_{in} - \frac{\pi}{2}\right) \tag{11}$$

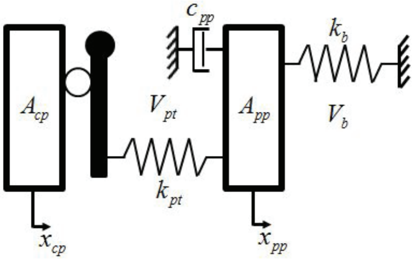

Figure 2. Schematic mass-spring-damper model of pulse tube cryocooler with a passive piston.

Where,

$$\tilde{\phi}_{in} = \arctan\left(-\frac{\tilde{B}}{\tilde{A}}\right)$$

The pressure of the piston on the gas, $\vec{P}_p$, can be found from the following:

$$\vec{P}_p = \sqrt{\tilde{C}^2 + \tilde{D}^2}\cos\left(\omega t + \phi_{pp} - \frac{\pi}{2}\right) \tag{12}$$

Where,

$$\phi_{pp} = \arctan\left(-\frac{\tilde{C}}{\tilde{D}}\right);\ \tilde{C} = \frac{k_{pt}x_1}{A_{cp}} - \tilde{A}\frac{k_{pt}}{A_{pp}};\ \tilde{D} = \tilde{B}\frac{k_{pt}}{A_{pp}}$$

In Figure 1 the reference for the real axis is pressure. In other words, the pressure vector has to be coincident with the x direction, namely the phase of the pressure has to be zero. Thus, it is necessary to shift the phase in equation (12) to be consistent with the discussion in the previous Section.

Therefore,

$$\dot{\vec{P}}_p = -\omega\sqrt{\tilde{C}^2 + \tilde{D}^2}\sin(\omega t) \tag{13}$$

$$\vec{\dot{m}}_p(t) = \rho A_{pp}\omega\sqrt{\tilde{A}^2 + \tilde{B}^2}\cos\left(\omega t + \phi_{\dot{m}_p}\right) \tag{14}$$

where, $\phi_{\dot{m}_p} = \tilde{\phi}_{in} - \phi_{pp}$ is the phase shift of the mass flow rate. So,

$$\phi_{\dot{m}_p} = \arctan\left(-\frac{\tilde{B}}{\tilde{A}}\right) - \arctan\left(-\frac{\tilde{C}}{\tilde{D}}\right) \tag{15}$$

This result can now be substituted into the results from the previous section. In conclusion, the following should hold simultaneously for the optimal performance of the pulse tube cryocooler with passive piston:

$$\arctan\left(\frac{\tilde{G}}{\tilde{H}}\right) = -\arctan\left(\frac{\tilde{M}}{\tilde{N}}\right) \Rightarrow \frac{\tilde{G}}{\tilde{H}} = -\frac{\tilde{M}}{\tilde{N}} \tag{16}$$

and

$$\sqrt{\tilde{M}^2 + \tilde{N}^2} = \sqrt{\tilde{G}^2 + \tilde{H}^2} \Rightarrow \tilde{M}^2 + \tilde{N}^2 = \tilde{G}^2 + \tilde{H}^2 \tag{17}$$

Where,

$$\tilde{G} = \frac{P_{avg}}{RT_{co}}\frac{V_{co}}{2}\omega\sin\left(\phi_{pp}\right);\ \tilde{H} = \frac{P_{avg}}{RT_{co}}\frac{V_{co}}{2}\omega\cos\left(\phi_{pp}\right) + \left[\frac{V_{co}}{2} + V_{ac}\right]\frac{\omega\sqrt{\tilde{C}^2 + \tilde{D}^2}}{RT_{co}}$$

$$\tilde{M} = \frac{T_h}{T_c}\rho A_{pp}\omega\sqrt{\tilde{A}^2 + \tilde{B}^2}\cos\left(\phi_{\dot{m}_p}\right)$$

$$\tilde{N} = \frac{\omega\sqrt{\tilde{C}^2 + \tilde{D}^2}}{RT_c}\left[V_{hhx} + \frac{V_{pt}}{\gamma} + V_{chx}\right] - \frac{T_h}{T_c}\rho A_{pp}\omega\sqrt{\tilde{A}^2 + \tilde{B}^2}\sin\left(\phi_{\dot{m}_p}\right)$$

Equations (16) , (17) were solved for the passive piston mass, spring stiffness and damping coefficient that are included in the definition of $\tilde{G}, \tilde{H}, \tilde{M}, \tilde{N}$ by Matlab® software. In the following section these results would be compared with the numerical solution from Sage® software that is dedicated to solving and optimizing different configurations of pulse tube cryocooler.

PULSE TUBE CRYOCOOLER DESIGN IN SAGE®

Overview

For the Stirling-based cycle cryocoolers the most commercial software for analysis is Sage®, developed by Gedeon Associates [6]. Sage® has a visual interface in which different standard components such as pistons, and heat exchangers are assembled into a complete machine. The optimization possibility of the design is a great advantage of Sage®, which makes it a very useful tool for cryocooler design. The software employs theoretical models and empirical correlations in calculations of different parameters. The assembly has a modular form where each component of the system is organized logically inyo a hierarchical tree structure. In Figure 3 the root level editing window of Sage® with the final model of the Passive Piston Pulse Tube cryocooler is shown. It is based on the model of the earlier design developed by Sobol et al. [1].

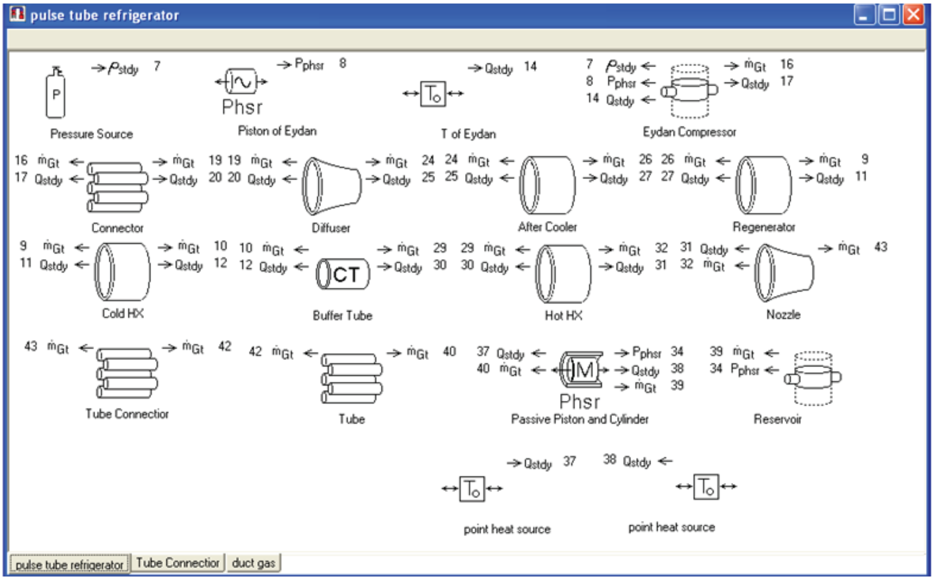

Figure 3. Root-level editing window of Sage® with the final model of the Passive Piston Pulse Tube cryocooler.

Optimization

Available parameters for optimization are limited because the present design of the Passive Piston Pulse Tube cryocooler is based on the already designed and manufactured Inertance Tube Pulse Tube cryocooler [1]. Most parameters such as dimensions, frequency, compressor piston displacement, mesh properties, frequency and temperatures of the previous design had to remain unchanged. The parameters available for optimization are passive piston physical parameters which determine piston dimensions such as piston mass, spring stiffness and damping coefficient. The objective function in the optimization was the maximum net cooling power at the cold end of the pulse tube model. The diameter of the piston in the phase shifter was selected to be 5 mm. The entire pulse tube cryocooler was analyzed to attain the objective function. Several combinations of optimized parameters were found. Due to the manufacturing restrictions, the values of parameters found in the optimization process were rounded off to acceptable values.

Simulation Results

The results from the Sage® optimization were compared to the results from the theoretical analysis. Clearance gap width was calculated only by Sage®. The damping coefficient and clearance gap in both solutions were constant and independent of piston mass at the examined range of piston mass values.

However, the spring stiffness behavior was different. In Figure 4 a linear fit was calculated for both solutions. The goodness of fit was about 1.0; therefore, it can be concluded that the spring stiffness had a linear behavior at the tested range of piston mass values. A linear fit function was also calculated and its graph was shown for the analytical solution from theoretical analysis, as well as for the Sage® solution. It is necessary to discuss the difference between the graph of analytical solution and of Sage® solution. Both solutions have a linear behavior. Also, it is quite simple to realize that analytical and numerical solutions are parallel. However, the shifting from the x axis of these two functions is quite different. The reason for such behavior is that the analytical model neglects the compressibility and the helium gas stiffness; therefore, the calculated spring stiffness in the case of the analytical solution also contains in it the real gas stiffness. In contrast, in the Sage® software the real gas compressibility and stiffness is taken into account, while the model is solved or optimized. Thus, the value of spring stiffness from the analytical solution is greater than the one

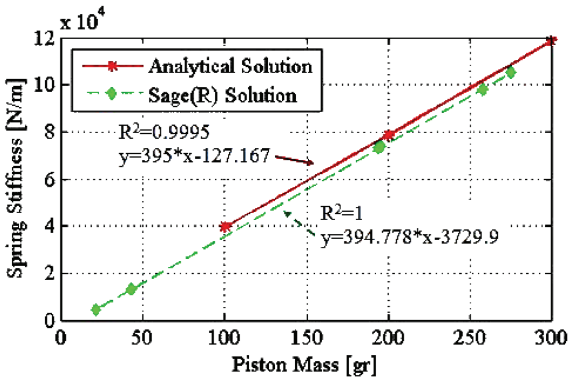

Figure 4. Spring stiffness vs. piston mass.

Table 1. Final parameters for Passive Piston model.

Piston Mass	Spring Stiffness	Damping Coefficient	Clearance Gap Width
50 gm	16000 N/m	1 x 10^{-3} N*s/m	20 μm

from the Sage® solution at the same mass value for each mass. Piston mass of 50 gm was chosen as design point for the passive piston design. In summary, final parameters for piston design are summarized Table 1.

EXPERIMENTAL SETUP

The parameters to be measured are the temperature, pressure and the passive piston displacement. Temperatures were measured at the aftercooler, cold heat exchanger and hot heat exchanger (see Figure 5). At the cold heat exchanger the temperature had to be measured with high precision; therefore, a diode gauge was employed, with accuracy of ±0.1K. Aftercooler and hot heat exchanger temperature accuracy are less critical; therefore, K-Type thermocouples were used. Pressure oscillations were measured at the interface between the compressor and aftercooler, buffer tube and hot heat exchanger, and at the inlet to the passive piston compression volume (see Figure 5).

Piston Displacement Measurement System

The main difficulty in the experiment was the measurement of the passive piston displacement without the physical contact of any mechanical part with the oscillating piston. The values of the displacement have to be measured at the same times as the pressure values at different cryocooler points. Using values of the passive piston displacement, the velocity of the passive piston can be calculated using differentiation by a finite differences method. The velocity of the passive piston can provide an estimate for the gas volumetric flow rate at the passive piston. This parameter is necessary for evaluation of the phase shift between the volumetric flow of the gas and pressure of the gas in the passive piston. The importance of this parameter was explained previously.

For piston displacement measurement an optical system was used. The operating principle of the system is as follows: a laser beam passes through the glass at the rear side of the cylinder cover and reflects from the reflective surface on the rear side of the piston; then the reflected beam passes back through the glass and impinges on the position sensitive detector (PSD). The laser beam main specifications are: power: 0.9mW, wavelength: 655 nm. The active area size of the PSD main specification is 1x12 mm. The PSD is based on a p-n junction that produces current and voltage which depend on the location of the laser beam impingement. A schematic concept of the piston displacement measurement is shown in Figure 6 and its mounting to the passive piston assembly is shown in Figure 5. After the post-processing and amplification, the output voltage signal can be collected by a data acquisition program.

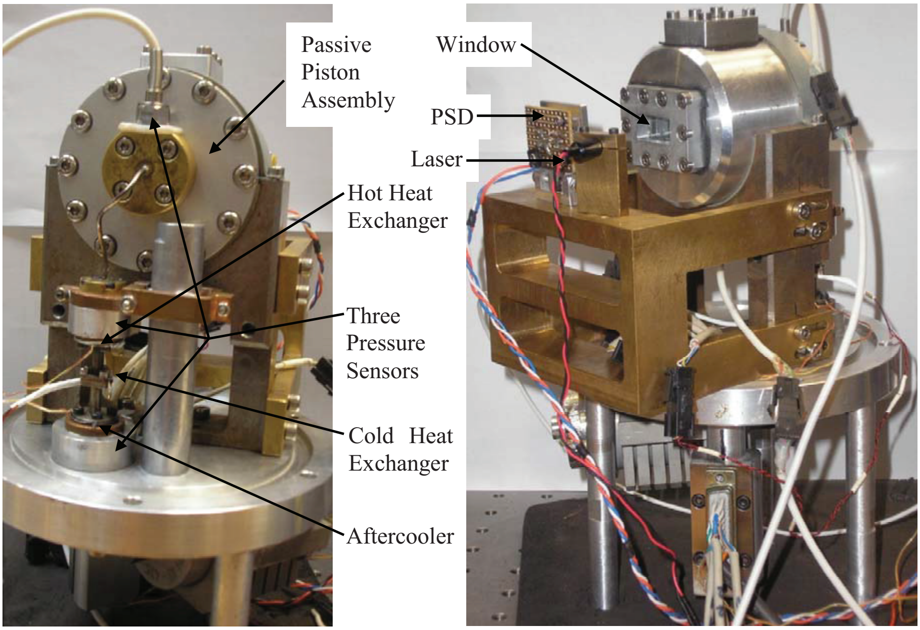

Figure 5. Cryocooler exclusive of vacuum chamber with passive piston displacement measurement system.

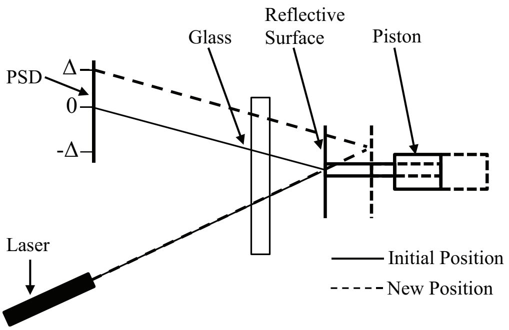

Figure 6. Schematic of piston displacement measurement.

In order to eliminate the convection heat transfer at the cold end it is convenient to perform the experiments with the cryocooler in a vacuum chamber. However, due to the design restrictions, the passive piston displacement could only be measured without the vacuum chamber.

RESULTS

The experiments were performed at the environmental temperature of 296K. Fill pressure in the refrigerator was set to 40 bar. Cryocooler performance was checked at several frequencies of the compressor near the design point of 100 Hz. The target pressure ratio was 1.3 (as set in the design). The temperature recorded was the lowest temperature obtained at the cold heat exchanger. In order to produce vacuum in the vacuum chamber a Varian® vacuum system was used.

Passive Piston Displacement Measurement Results

As mentioned before, due to design restrictions, the passive piston displacement measurement was performed without the vacuum chamber. Figure 7 shows the variation of pressure, piston dis-

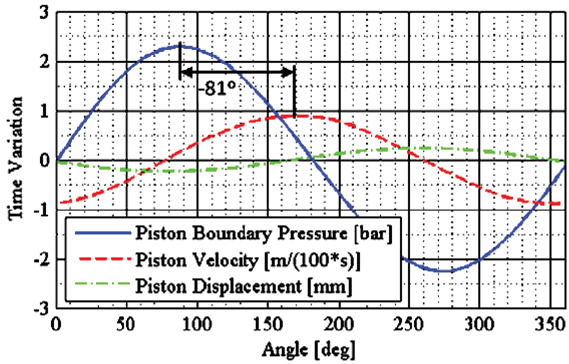

Figure 7. Pressure at the inlet to the passive piston, passive piston displacement and velocity at frequency of 120 [Hz].

placement, and piston velocity at the inlet to the passive piston at frequency of 120 Hz. The phase shift at the boundary of the passive piston between piston velocity and pressure oscillation, which is the same as the phase between mass flow of the gas and pressure oscillation, was about -81°.

Cryocooler Performance Measurement

As mentioned earlier, in order to decrease heat transfer by convection at the cold end, the experiment was performed in a vacuum chamber. The temperature at the cold heat exchanger was measured at several frequencies around the design point. The results are shown in Figure 8. The lowest temperature of 157.8K was reached at a frequency of 120 Hz. The pressure ratio at the aftercooler inlet was measured at the same frequencies (see Figure 8). The pressure ratio at high frequencies decreased from the design point of 1.3 to 1.2.

CONCLUSIONS

The present research has proved the viability of the concept of using a passive mechanical mechanism as a phase shifter of the flow in a pulse tube cryocooler. There was fine agreement between analytical and numerical solution by the Sage® software. However, the temperatures at the cold end reached in the experiments were higher than those reached with the cryocooler described in [1]. Also, the pressure ratio was lower than the design value. This was due to manufacturing and assembly inaccuracies that added parasitic friction between piston and cylinder. However, the phase shift between piston velocity and pressure oscillation that was measured at the inlet to the passive piston demonstrated the feasibility of the concept.

Passive piston displacement contactless measurement system was proposed and used in the

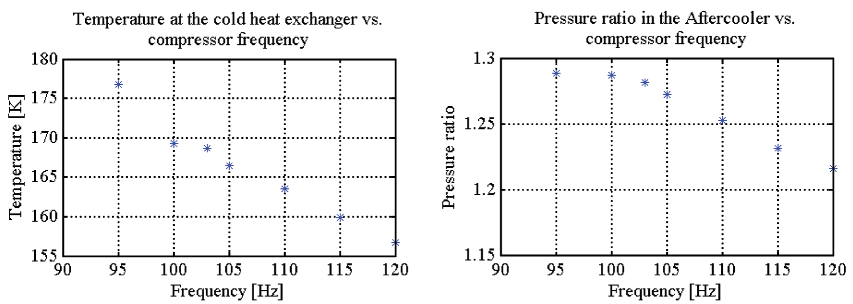

Figure 8. Results of the experiment in vacuum chamber.

experiments. The advantage of the proposed system was in the contactless concept and ability to measure small displacement values at high frequencies, due to the high accuracy of the PSD. The results of measurements helped to understand gas behavior in the pulse tube cryocooler operating at high frequencies.

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REFERENCES

1. Sobol, S., Katz, Y. and Grossman, G., ”A Study of a Miniature In-Line Pulse Tube Cryocooler,” *Cryocoolers 16*, ICC Press, Boulder, CO (2011), pp. 87-95
2. Matsubara, Y. and Miyake, A., ”Alternative Methods of the Orifice Pulse Tube Refrigerator,” *Proceedings of the 5th International Cryocooler Conference*, (1989), pp. 127-135
3. Lewis, M., Bradley, P. and Radebaugh, R., “Experiments with Linear Compressors for Phase Shifting in Pulse Tube Cryocoolers,” *Adv. in Cryogenic Engineering*, Vol. 57, Amer. Institute of Physics, Melville, NY (2012), pp. 1600-1607
4. Radebaugh, R., “A Review of Pulse Tube Refrigeration,” *Adv. in Cryogenic Engineering*, Vol. 35B, Springer US (1990), pp. 1191-1205
5. Radchenko, D., “Investigation of a Passive Mechanical Mechanism for Phase Shifting of the Flow in a Pulse Tube Cryocooler,” M.Sc. Thesis, Faculty of Mechanical Engineering, Technion, 2015
6. Gedeon, D., “Sage® - Stirling Cycle Model Class Reference Guide,” Gedeon Associates, Athens, OH, USA, April 2006