

# Cascading Three Pulse Tube Coolers with Work Recovery

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## ABSTRACT

A cascade pulse tube cooler (PTC) consists of sub-PTCs that are connected by transmission tubes in between; it can recover the PV work at the warm end of the pulse tube to drive the subsequent stages. The more cascading stages it has, the closer the efficiency will approach Carnot efficiency. In this paper, a cascade PTC consisting of three sub-PTCs is presented, and experimental results show that while working at 233 K, the cooling powers at the three stages are 164.9 W, 70.7 W and 18.0 W, respectively. This results in a total cooling power of 253.6 W, 39.9% over a PTC without cascading.

## INTRODUCTION

A pulse tube cryocooler (PTC) cannot work with Carnot efficiency due basically to the expansion work that has to be dissipated thermally at the warm end of the pulse tube. This dissipation is especially prominent at high cooling capacity or at high temperatures, as it reduces the COP and limits the application of PTCs above 120 K. This intrinsic limitation limits its capability of reaching Carnot efficiency even in the ideal case. That is, the ideal efficiency of the PTC is only  $T_c/T_h$ , which is lower than the Carnot efficiency  $T_c/(T_h-T_c)$ .<sup>1</sup> Therefore, how to recover this amount of dissipated work becomes a critical issue in a highly efficient PTC.

Previous studies have been carried out to recover this amount of PV work: one way is by introducing moving parts,<sup>2-5</sup> and another is by conducting loop configurations.<sup>6-9</sup> Also, G. Swift proposed a quarter-wavelength pulse tube cooler in which a second pulse tube cooler was added after the quarter-wavelength pulse tube to recover the expansion work, this idea provides a new way to recover the PV work without introducing either moving parts or streaming.<sup>10,11</sup>

We started our program studying cascade PTC with work recovery in 2012. In 2013, our single stage PTC reached 181.3 W cooling power at 233 K.<sup>12</sup> In 2014, our two-stage cascade PTC obtained 241.6 W cooling power (175.0 W from the first stage and 66.6 W from the second stage).<sup>13,14</sup>

In the work reported here, we build a three-stage cascade PTC.

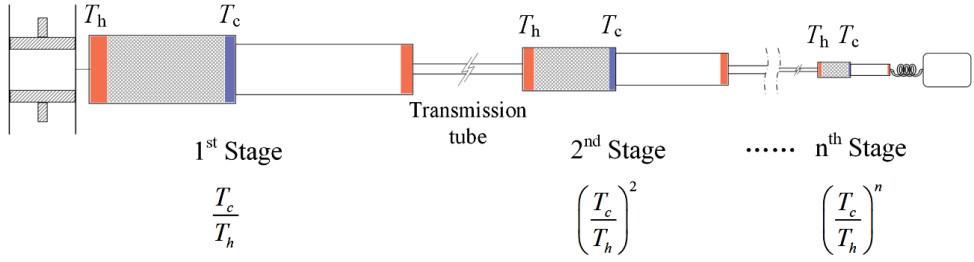


Figure 1. Schematic diagram of a multi-stage cascade PTC

## THEORETICAL ANALYSIS

Figure 1 shows a schematic diagram of a multi-stage cascade PTC. It consists of  $n$ -stages of sub-PTCs with transmission tubes in between (here  $n$  is a positive integer number not less than 2). Each sub-PTC works at the same hot end and cold end temperatures  $T_h$  and  $T_c$ , respectively. The linear compressor provides the original driving power, the acoustic power at the outlet of the former stage is transferred to the latter one by the transmission tube. Assuming that there is no loss in each stage of sub-PTC, then the cooling efficiency of each stage is  $T_c/T_h$ . We can obtain the total cooling efficiency of the whole system:<sup>13,14</sup>

$$COP_n = \frac{Q_{c1} + Q_{c2} + \dots + Q_{cn}}{E_1} = \frac{T_c}{T_h} + \left(\frac{T_c}{T_h}\right)^2 + \left(\frac{T_c}{T_h}\right)^3 + \dots + \left(\frac{T_c}{T_h}\right)^n = \frac{T_c}{T_h - T_c} \left[ 1 - \left(\frac{T_c}{T_h}\right)^n \right] \quad (1)$$

when  $n$  goes to infinity, the COP will become:

$$COP_{n \rightarrow \infty} = \frac{T_c}{T_h - T_c} \quad (2)$$

That is, the efficiency of an infinite stage cascade PTC will be the Carnot efficiency. This theoretical analysis with ideal hypothesis is not only beautiful but also meaningful, because there may be many different ways similar to this to get to Carnot efficiency for a refrigerator. Equation (2) in this paper shows us one of such refrigeration methods which approaches Carnot efficiency step by step, and it is theoretically realizable learning that each term in Equation (2) corresponds to each stage of sub-PTC in Fig. 1.

A three stage cascade PTC was designed based on REGEN<sup>15</sup> and Sage<sup>16</sup>. In this work, we used an existing linear compressor of model CFIC 2s132 (the rated power is 500 W at 60 Hz, 2.5 MPa charging pressure) to obtain a relatively high cooling power, and a cooling temperature of 233 K (-40°C) was chosen for the first step.

The key point in designing a cascade PTC is to ensure good phase relations inside each stage. Figure 2 shows the phasor diagram of the entire cascade cooler, in which three vectors labeled  $p_1$ ,  $p_2$  and  $p_3$  represent the pressure waves in the 1st stage, 2nd stage, and 3rd stage, respectively. The vectors with open-headed arrows represent the mass flow in the 1st stage, the vectors with blunt-headed arrows represent that in the 2nd stage, while the vectors with sharp pointed arrows represent that in the 3rd stage. It is shown that the reversal of phase relation between pressure wave and mass flow by the transmission tubes plays an important role, so that appropriate phase relations inside all three stages are satisfied at the same time. That is, at the warm end of the regenerator, the mass flow leads the pressure wave; while at the cold end of the regenerator, the pressure wave leads the mass flow. The pressure wave of the 1st stage is turned by 167.6 degrees while entering into the 2nd stage; at the same time, the mass flow is turned by 112.0 degrees. Similarly, the pressure wave of the 2nd stage is turned by 162.0 degrees while entering into the 3rd stage; at the same time the mass flow is turned by 97.7 degrees.

Table 1 lists the simulation results of the cascade PTC based on a Sage model. It includes three operation modes: single-stage operation, two-stage cascade operation, and three-stage cascade

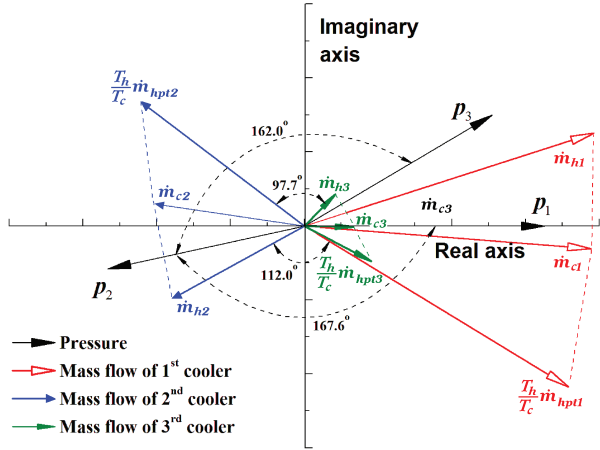


Figure 2. Phasor diagram of the three-stage cascade PTC

Table 1. Calculation results of three-stage cascade PTC based on Sage code

	Single stage <sup>12</sup>	Two-stage cascade <sup>13,14</sup>			Three-stage cascade			
		1 <sup>st</sup>	2 <sup>nd</sup>	In total	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	In total
Electric power input / W		500						
Acoustic power input / W	389.9	379.1	125.7	379.1	348.9	143.3	46.0	348.9
Cooling power at 233 K / W	196.3	184.5	65.1	249.6	172.3	75.4	24.1	271.8
COP	<b>0.3926</b>	<b>0.4992</b>			<b>0.5436</b>			
Percent of Carnot / %	11.29	14.35			15.63			

operation, all with the same electric power input of 500 W. The three-stage cascade PTC is expected to achieve a total cooling power of 271.8 W at 233 K; the corresponding cooling efficiency will be improved 38.5% compared with the single stage PTC and 8.9% compared with the two-stage cascade PTC.

EXPERIMENTAL SYSTEM

To demonstrate the feasibility of the basic idea, experiments were carried out. A three-stage cascade PTC setup was designed, as shown in Figs. 3 and 4. The system consists of a linear compressor, a 1st stage PTC, a transmission tube I, a 2nd stage PTC, a transmission tube II, a 3rd stage PTC. Each PTC consists of an aftercooler, a regenerator, a cold heat exchanger, a pulse tube, and a secondary hot heat exchanger. The 1st stage PTC and the 2nd stage PTC are connected by a 7 m long transmission tube with inner diameter of 14.2 mm; while the 2nd stage PTC and the 3rd stage PTC are connected by a 6.3 m long transmission tube with inner diameter of 10.0 mm.

The CFIC 2s132 model linear compressor is driven by an AC power supply to regulate the frequency and power output. The four hot end heat exchangers located at the warm ends of the regenerators and pulse tubes are cooled by water at room temperature. Non-vacuum expanded perlite is used for thermal insulation.

Table 2 lists the main parameters of the cascade PTC. The fill matrix in the three regenerators is a 200 mesh stainless steel with a porosity of 0.6704. The measurement system includes measurements of temperature, pressure and cooling power. Two rhodium-iron resistance thermometers are mounted on the cold end of the 1st stage PTC, two platinum resistance thermometers are mounted on the cold end of the 2nd stage PTC, while three rhodium-iron resistance thermometers are mounted on the cold end of the 3rd stage PTC, all calibrated with accuracy of ±0.1 K. Four pressure sensors are employed to measure the pressures at the compressor back space, the inlet of the 1st stage, the inlet of the 2nd stage, and the inlet of the 3rd stage, as shown as P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub> and P<sub>4</sub> in Fig. 3. For P<sub>1</sub>, P<sub>3</sub> and P<sub>4</sub>, KISTLER 211B3 piezoelectric pressure transducers are used to measure the amplitude and phase of the oscillating pressure; for P<sub>2</sub>, an Entran EPX piezoresistive pressure transducer is used to

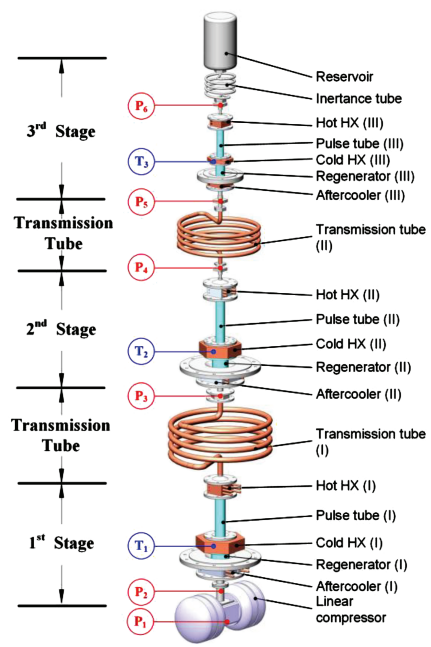


Figure 3. Schematic drawing of three-stage cascade PTC

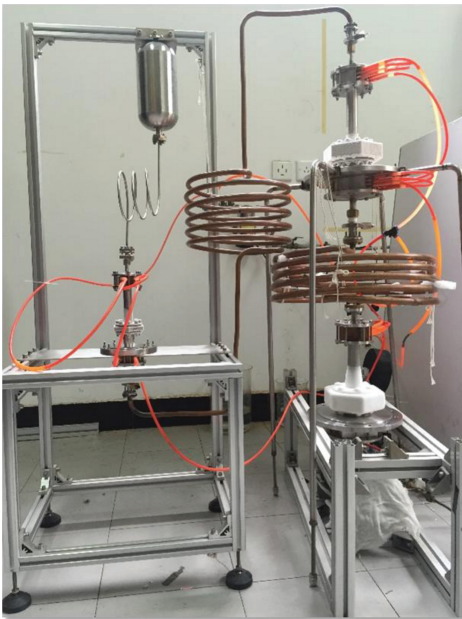


Figure 4. Experimental setup of three-stage cascade PTC

Table 2. Main parameters of the three-stage cascade PTC

	Part Name	Dimension
1 <sup>st</sup> Stage	Regenerator	53.7 mm i.d., 37.7 mm long
	Pulse tube	30.5 mm i.d., 134.3 mm long
	Inertance tube (in single-stage operation)	8 mm i.d., 2.67 m long
	Reservoir (in single-stage operation)	450 cm <sup>3</sup>
	Transmission tube I	14.2 mm i.d., 7 m long
2 <sup>nd</sup> Stage	Regenerator	47.6 mm i.d., 48 mm long
	Pulse tube	27 mm i.d., 150 mm long
	Inertance tube (in two-stage cascade operation)	6 mm i.d., 1.4 m long
	Reservoir (in two-stage cascade operation)	1000 cm <sup>3</sup>
3 <sup>rd</sup> Stage	Transmission tube II	10.0 mm i.d., 6.3 m long
	Regenerator	27.0 mm i.d., 53 mm long
	Pulse tube	19.6 mm i.d., 100 mm long
	Inertance tube	4 mm i.d., 1.3 m long
	Reservoir	1000 cm <sup>3</sup>

measure the dynamic as well as the mean pressure. Real-time data is collected using LabVIEW in a PC. To measure the cooling power, four 50 Ω resistors capable of providing 200 W heating power are installed on the cold end of the 1st stage, three 50 Ω resistors capable of providing 150 W heating power are installed on the cold end of the 2nd stage, and another three 500 Ω resistors capable of providing 29.4 W heating power are installed on the cold end of the 3rd stage.

RESULTS AND DISCUSSION

During the preliminary test, after about 7.5 hours, the temperatures of the three PTCs come to steadystate; their final temperatures were 144.8 K, 146.0 K and 188.5 K, respectively. If compared with single-stage operation (final temperature of 99.7 K) or two-stage cascade operation (120.8 K and 127.2 K, respectively), these are higher than before. This is mainly due to the sacrifice of the 1st and the 2nd cooler, e.g. their phasor diagrams are compressed to some degree. We compare the

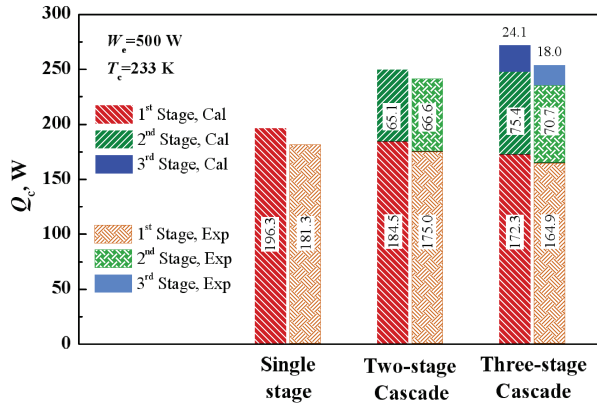


Figure 5. Comparison of cooling power

cooling capacities among three operations modes. Those are: singlestage operation, two-stage cascade operation, and three-stage cascade operation. Here the electric power input to the linear compressor is fixed at 500 W, and the cold end temperature is fixed at 233 K. As shown in Fig. 5, the single-stage PTC can supply a cooling power of 181.3 W, while for the two-stage cascade PTC, the cooling power of the 1st and 2nd stages are 175.0 W and 66.6 W, thus 241.6 W cooling capacity in total. In three-stage operation, the cooling powers of the three stages are 164.9 W, 70.7 W and 18.0 W, making a total cooling power of 253.6 W. The cooling efficiency is improved by 39.9% compared with single stage PTC and 5.0% compared with the two-stage cascade PTC. This positively demonstrates the feasibility of the cascade concept. In addition, the cooling capacity agrees quite well with the calculation results as shown in Figure 5; this verifies our design. In this work, the cooling temperature is higher than the cryogenic temperature ( $\leq 120$  K) reflecting the existing conditions in our lab. But for most cryogenic applications such as HTS and LNG, lower temperatures and even higher cooling capacities are required; in such cases, the cascade PTC can be more attractive.

It should be noticed that although extra cooling power can be obtained, the 1st stage PTC is always playing a dominant role. It is found that the 1st stage cooler deteriorates as the cascading stage number increases. Its no-load temperature goes up, and its cooling capacity goes down as shown from Fig. 5 — from both calculation and experiment. This means the 1st stage cooler sacrifices itself to some extent. Figure 6 shows the calculated phasor diagrams of the 1st stage PTC under three operating modes. We can see that the phasor diagram of the 1st stage is compressed more and more in both two and three-stage cascade operation. For the phase span between the mass flow at the warm end of the regenerator and the warm end of the pulse tube, this value is about  $79^\circ$  in single-stage operation, which is reduced to about  $61^\circ$  in two-stage cascade operation, and even further reduced to about  $49^\circ$  in three-stage cascade operation. As a result, the cooling performance of the 1st stage declines. From this point of view, the number of cascading stages maybe not the more the better in practice. It is beneficial before the loss of the former stages become even larger than the benefit of the additional cascading stages.

The pressure waves at different positions shown in Fig. 3 are measured at 233 K under cascade operation, as shown in Fig. 7. The pressure ratios in the 1st, 2nd and 3rd stages are 1.140, 1.164 and 1.157, respectively. It is shown that the phase angle between P1 and P2 is  $138.0^\circ$ , for the mass flow in the compression space and P1 should be  $90^\circ$  out of phase. This indicates  $48.0^\circ$  between the mass flow and the pressure wave at the entrance to the 1st stage. What is more important, it is also shown in Fig. 6 that the measured phase difference between three sub-PTCs, those are  $170.8^\circ$  between 1st and 2nd stage, while  $165.3^\circ$  between 2nd and 3rd stage, which coincide well with the calculated  $167.6^\circ$  and  $162.0^\circ$  (see Fig. 3). This verifies the phase reversion function of the transmission tubes, which is a key design in the whole system.

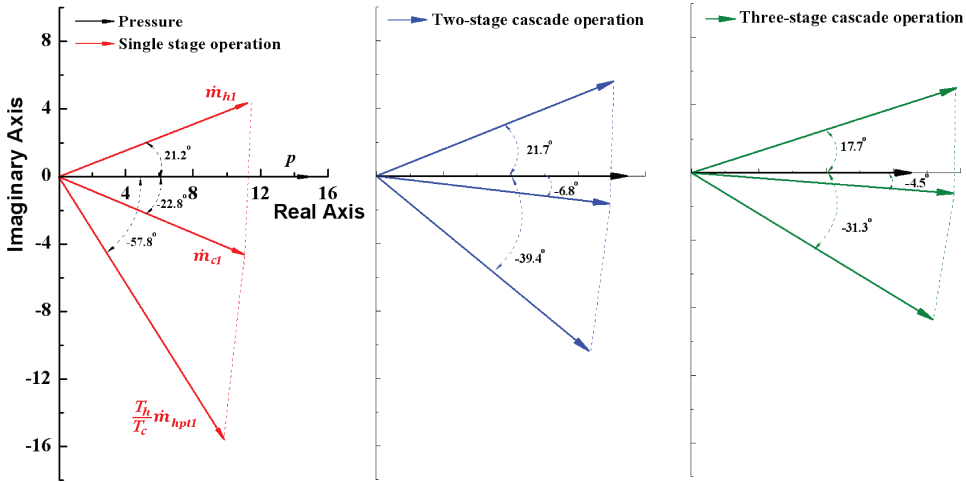


Figure 6. Calculated phasor diagrams of the 1st stage PTC under three different operating modes

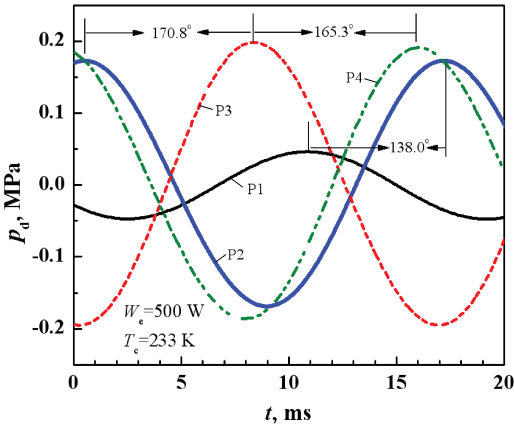


Figure 7. Measured pressure waves

CONCLUSIONS

A three-stage cascade PTC has been designed and tested. These preliminary tests show that the cooling powers of the three stages are 164.9 W, 70.7 W and 18.0 W, respectively, making a total cooling power of 253.6 W at 233K. The cooling efficiency is improved by 39.9% compared with a single stage PTC and 5.0% compared with a two-stage cascade PTC. This demonstrates the concept of cascade PTCs with work recovery, and lays a good foundation for much lower temperatures with potential use in HTS and LNG applications.

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