

Real Gas Effects on the COP of Regenerators Working at Low Temperatures

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ABSTRACT

Real gas properties impose profound effects on regenerators working at temperatures close to and below the critical point of the working fluid. The enthalpy flow associated with the pressure dependence, abbreviated as the pressure-induced enthalpy flow, generally affects the energy balance of regenerators. In this paper, the influence mechanism of the pressure-induced enthalpy flow on the COP has been analyzed. The main factor in keeping the energy balance of the regenerator is found to be the heat-associated enthalpy flow, which decreases the COP. Consequently, the temperature profile can be determined. The expression of the theoretical COP of the regenerator has been further derived. These discussions will contribute to the understanding of the working mechanism of the regenerator.

INTRODUCTION

Real gas properties impose profound effects on regenerators. The enthalpy flow associated with the pressure dependence, abbreviated as the pressure-induced enthalpy flow, is the significant real gas property, and generally induces profound heat losses at a temperature close to and below the critical point¹. Regenerator temperature profiles that are far from linear have been observed numerically and experimentally since the 1990s¹⁻⁵. This phenomenon has attracted great interest and significant progress has been achieved. The energy analysis and the expression of the COP has been discussed in previous study⁶⁻⁸. The important role of the heat-related enthalpy flow has identified based on the sources from open literature⁹. In this paper, the terms related to the loss mechanism will be clarified and the expression of the COP will also be derived.

SIGNIFICANCE OF THE PRESSURE-INDUCED ENTHALPY FLOW

The differential form of the specific pressure-induced enthalpy of real gases is expressed as

$$dh_p = v(1 - T\beta)dp \quad (1)$$

where the parameter β is the volume expansivity. In order to derive the magnitude of the pressure-induced enthalpy flow in the regenerator, some assumptions are made. A small pressure ratio is assumed, thus both the time-variation of volume expansivity and the time-variation of temperature

are regarded as constant during the integration of one cycle. The specific pressure-induced enthalpy is integrated over the pressure and the mean pressure is used as the reference pressure. The time-averaged pressure-induced enthalpy flow $\langle \dot{H}_p \rangle$ is calculated with the equation

$$\langle \dot{H}_p \rangle = (1 - T\beta) \frac{1}{T} \oint \dot{V}(p - p_0) dt \quad (2)$$

Compared with the expression of $\dot{P} \dot{V}$ power¹⁰, $\langle \dot{H}_p \rangle$ is expressed as

$$\langle \dot{H}_p \rangle = (1 - T\beta) \langle pV \rangle \quad (3)$$

It is clear that the pressure-induced enthalpy flow is proportional to the PV power with a coefficient of $(1 - T\beta)$. In cases of relatively large pressure ratios, the volume expansivity and the temperature vary significantly with time in one cycle. The average product of the volume expansivity and the temperature should be used in the calculation.

The pressure-induced enthalpy flow usually accounts for a large percent of the PV power at temperatures close to and below the critical point. The parameter of ^4He ¹¹ and ^3He ¹² has been calculated, when the pressure is 1 MPa, $(1 - T\beta)$ reaches the lowest point of -96% at 8.7 K for ^4He ¹¹, and reaches a value as high as 78% at 4.2 K, as shown in Figure 1. The real gas effect of ^3He is smaller than that of ^4He , but the parameter still reaches 69% at 4.2 K.

In order to estimate the variation of the pressure-induced enthalpy flow along the regenerator, the regenerator is assumed to be frictionless. Thus the PV power varies with the product of the compressibility factor and the temperature in the regenerator, as shown in the Eq. (4)

$$\frac{\langle pV \rangle_c}{\langle pV \rangle_h} = \frac{(ZT)_c}{(ZT)_h} \quad (4)$$

If the status at the hot end of the regenerator is used as the reference, the pressure-induced enthalpy flow at a certain temperature T inside the regenerator can be determined, and the difference of the pressure-induced enthalpy flow at two points of different temperature along the regenerator is calculated with the equation expressed as:

$$\Delta \langle \dot{H}_p \rangle = \Delta[(1 - T\beta)ZT](\langle pV \rangle_h / (ZT)_h) \quad (5)$$

The parameter $[(1 - T\beta)ZT]$ varies significantly with temperature, as shown in Figure 2. Three typical cases of PTCs^{2, 13-15} are compared⁹. The maximum variation of the pressure-induced enthalpy flow within the working temperature range is estimated to account for 2.6, 1.6, and 1.2 times the PV power at the cold end for these three PTCs. Such a large pressure-induced enthalpy flow will influence the energy balance significantly.

ENERGY BALANCE IN THE REGENERATOR

By applying the first law of thermodynamics¹⁰ to the regenerator section between heat exchangers at steady state, i.e. no heat exchange with external systems, the relationship of the heat conduction,

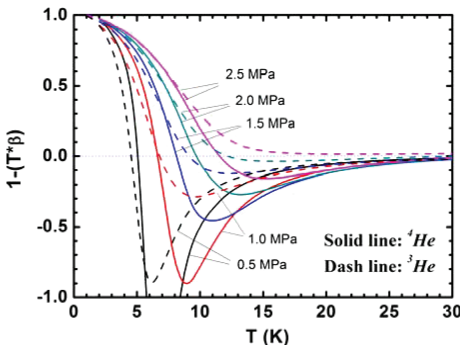


Figure 1. Dependence of parameter $(1 - T\beta)$ on temperature for ^4He and ^3He

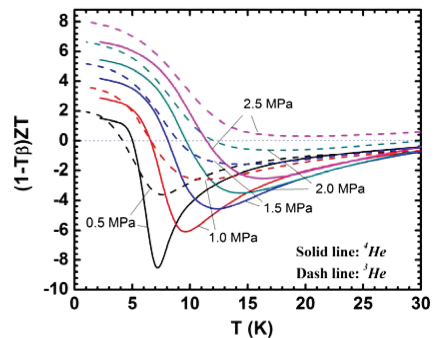


Figure 2. Dependence of parameter $(1 - T\beta)ZT$ on temperature for ^4He and ^3He

including that of gas, matrix and wall, the heat-associated enthalpy flow and the pressure-induced enthalpy flow is expressed as:

$$d(\langle \dot{H}_p \rangle + \langle \dot{H}_T \rangle + \dot{Q}_{\text{cond}})/dx = 0 \quad (6)$$

If it is integrated over the length, the formula is rewritten as:

$$\langle \dot{H}_p \rangle + \langle \dot{H}_T \rangle + \dot{Q}_{\text{cond}} = \dot{Q}_{\text{tot}} \quad (7)$$

The total heat flow \dot{Q}_{tot} must be kept constant over the length. As $\langle \dot{H}_p \rangle$ varies, \dot{Q}_{cond} and/or $\langle \dot{H}_T \rangle$ must change accordingly in order to keep the energy balance.

The pressure-induced enthalpy flow usually dominates the total heat flow of the last-stage regenerator working between below 50 K and around 4.2 K. \dot{Q}_{tot} should be larger than the maximum $\langle \dot{H}_p \rangle$ since neither \dot{Q}_{cond} nor $\langle \dot{H}_T \rangle$ can be negative. The maximum $\langle \dot{H}_p \rangle$ usually occurs at the cold end in this temperature range, as shown in Figure 1. According to Eq. (7), the maximum of the sum of \dot{Q}_{cond} and $\langle \dot{H}_T \rangle$ must be larger than the maximum variation of $\langle \dot{H}_p \rangle$ within the working temperature range, expressed as:

$$(\langle \dot{H}_T \rangle + \dot{Q}_{\text{cond}})_{\text{max}} \geq \langle \dot{H}_p \rangle_{\text{max}} - \langle \dot{H}_p \rangle_{\text{min}} \quad (8)$$

Which factor of $\langle \dot{H}_T \rangle$ and \dot{Q}_{cond} dominates the responsibility of the energy balance? According to the calculation⁹ of three cases of PTCs, the maximum of the heat conduction is one to two-orders smaller than the maximum variation of the pressure-induced enthalpy flow in these cases. This conclusion can be extended to general cases. The only term that can keep the energy balance in practical circumstances is the heat-associated enthalpy flow. The heat-associated enthalpy flow can be in of the same magnitude of the pressure-induced enthalpy flow, and it is able to conserve the energy balance. Evidence has been found⁹ from the numerical calculation for PTC in reference² and another PTC in reference¹⁶. In most cases, the heat conduction can be omitted since it constitutes a very small fraction. This point is used in the derivation in the following content.

FUNCTION OF THE REGENERATOR TEMPERATURE PROFILE

The temperature profile of regenerators deviates from linear to meet the requirement of energy balance. The relationship between the heat-associated enthalpy flow and the temperature gradient is proportional, which has been derived in a previous study⁹. The factor between the two terms is noted as k_T . This factor is derived to be:

$$k_T = -\frac{0.5946 \left((1/\tau)(1-\phi)\rho_m c_{p,m} + h_{ht}\phi/d_h \right) c_{p,g} d_h^{1.3} Pr^{2/3} \cos\delta (PV)^{0.65}}{\pi^{1.3} \left((1/\tau)(1-\phi)\rho_m c_{p,m} \right) a_2 \mu^{0.3} \phi^{0.3} d_{reg}^{0.6} (Zvcos(\phi-\theta))^{0.65}} \quad (9)$$

Considering the energy balance, and substituting the expression for $\langle \dot{H}_p \rangle$ and $\langle \dot{H}_T \rangle$, the governing equation for the temperature profile is expressed as:

$$(\langle pV \rangle_h / (ZT)_h) \frac{d[(1-T\beta)ZT]}{dx} - k_T \frac{d^2 T}{dx^2} = 0 \quad (10)$$

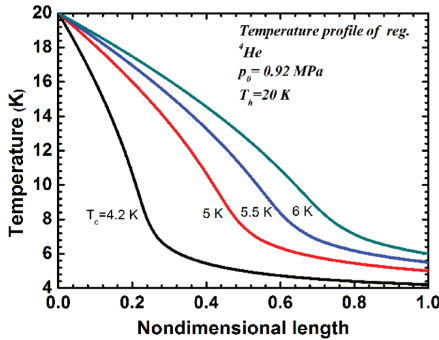


Figure 3. Calculated temperature distribution along the regenerator

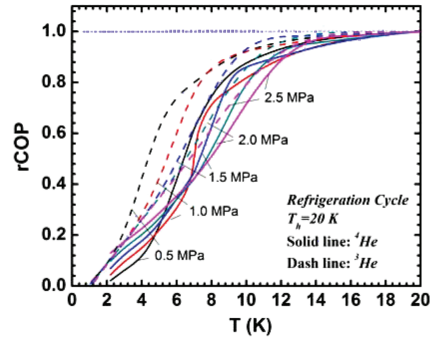


Figure 4. Dependence of the relative COP of the refrigerator cycle on the cold end temperature

An example of the temperature distribution is shown in Figure 3 based on the calculation of a PTC¹⁴ with a hot end temperature of 20 K. The nonlinearity dominates as the cold end temperature approaches 4.2 K. The temperature profile is in accordance with previous experiment observation⁴.

DERIVATION OF THE THEORETICAL COP

Since the total enthalpy flow is kept constant in the regenerator section between the heat exchangers, the heat-associated enthalpy flow at any location can be derived relative to the point when the pressure-induced enthalpy flow reaches a maximum of the regenerator section, as the equation shows:

$$\langle \dot{H}_p \rangle + \langle \dot{H}_T \rangle = (\langle \dot{H}_p \rangle + \langle \dot{H}_T \rangle)_{\max(\dot{H}_p)} \quad (11)$$

The most theoretical scenario of the regenerator, i.e. with least losses and not against the thermodynamic laws, can be ideal heat transfer when needed, the heat-associated enthalpy flow at that point equals to zero. Thus, the local heat-associated enthalpy flow can be rewritten as:

$$\langle \dot{H}_T \rangle = \max(\langle \dot{H}_p \rangle) - \langle \dot{H}_p \rangle \quad (12)$$

Substitute the equation to the expression for the refrigeration power available at the cold end⁹. The refrigeration power is expressed as

$$\dot{Q}_c = \langle pV \rangle_c - \max(\langle \dot{H}_p \rangle) \quad (13)$$

The COP of the regenerator is expressed as:

$$\text{COP} = \frac{\langle pV \rangle_c - \max(\langle \dot{H}_p \rangle)}{\langle pV \rangle_h - \langle pV \rangle_c} \quad (14)$$

Substituting Eq. (3), (4) for the pressure-induced enthalpy flow and for the PV power, the COP can be rewritten as

$$\text{COP} = \frac{(ZT)_c - \max[(1-T\beta)ZT]}{(ZT)_h - (ZT)_c} \quad (15)$$

The relative COP, the COP divided by the Carnot COP, for ⁴He and ³He at the pressure range of 0.5-2.5 MPa is plotted in Figure 4. The hot end temperature is set to be 20 K. The COP is always smaller than the Carnot COP, because there is a heat loss named heat-associated enthalpy flow along the regenerator except the point with the maximum pressure-induced enthalpy flow.

Note that the theoretical COP for ⁴He between a temperature below about 5 K and 20 K increases with the pressure. This is right in a vast pressure range of 0.1-10 MPa. The theoretical COP at 2.5 MPa is 50% higher than that at 1 MPa. Certainly there is a barrier of matrix heat capacity in practical cases. Still it means that the measured COP could be increased if the matrix can be improved.

This calculation is in accordance with the analysis in reference⁶. Although the expansion work of the pulse tube cryocooler is regarded to be dissipated in the calculation, the COP for the regenerator should be the same.

CONCLUSION

In this paper, the significance of the pressure-induced enthalpy flow and the main factor in keeping the energy balance of the regenerator have been discussed. Consequently, the temperature profile can be derived based on the energy equation. We have further derived the expression of the theoretical COP of the regenerator, taking into account of the real gas effects. These discussion may contribute to the understanding of the working mechanism of the regenerator.

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