Periodic Flow Hydrodynamic Resistance Parameters for Regenerator Filler Materials at Cryogenic Temperatures

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ABSTRACT

The regenerator is a critical component of all Stirling and Pulse Tube cryocoolers. It generally consists of a microporous metallic or rare-earth filler material within a container shell. The accurate modeling of the hydrodynamic and thermal behavior of different regenerator materials is crucial to the successful design and analysis of regenerative cryogenic systems. Due to the difficulty associated with experimental measurement at cryogenic temperatures, previous investigations that have used experimental measurements at steady and periodic flow conditions in conjunction with pore-level CFD analysis to determine the pertinent hydrodynamic parameters, namely the Darcy permeability and Forchheimer coefficients, have relied on experiments performed at ambient conditions. These results are assumed to be accurate for cryogenic temperatures since, for fully-developed flow, these parameters should depend only on the geometry of the porous medium (for Darcy permeability) and flow Reynolds number (for Forchheimer coefficient). There is a pressing need for the verification of the validity of the foregoing assumption. In this study, regenerators filled with spherical Er50Pr50 powder were assembled and tested under periodic helium flow at cryogenic temperatures. The mass flow and pressure drop data were correlated using a porous media CFD model to determine the Darcy Permeability and Forchheimer coefficients. These results are compared to the results of previous investigations at ambient temperature conditions, and the relevance of room-temperature models and correlations to cryogenic temperatures are critically assessed.

INTRODUCTION

Realistic prediction of the regenerator performance depends heavily upon the accurate modeling of the fluid-solid hydrodynamic and thermal interactions within the porous filler material of the regenerator. While it is theoretically possible to model the entire PTC system including the porous material on a microscopic level by directly solving the conservation equations, such an approach is computationally expensive and requires detailed knowledge of the microscopic geometry of the medium. The common approach is to convert the microscopic governing equations to the macroscopic level using volume averaging or a similar approach [1–5]. For the conservation of momentum under steady state conditions, for example, this yields the extended Darcy- Forchheimer model equation (1)

$$\left\langle \vec{u}_f \right\rangle = -\frac{\overline{\overline{K}}}{\mu_f} \cdot \left(\nabla \left\langle P_f \right\rangle^f - \rho_f \vec{g} \right) - \overline{\overline{F}} \cdot \left\langle \vec{u}_f \right\rangle \tag{1}$$

where $\overline{\overline{K}}$ represents the Darcy permeability tensor and $\overline{\overline{F}}$ represents the Forchheimer inertial coefficient tensor, which are frequently referred to as hydrodynamic resistance parameters. These closure parameters are needed for CFD-assisted optimization of the regenerator design.

There are currently two main methods for determining the permeability and inertial coefficients for a particular medium which are pore-level CFD simulation and experimentation. In the first approach microscopic or pore-level CFD models of the regenerator are developed to determine the pressure drop characteristics by comparing the results to the macroscopic volume-averaged equations to determine the closure parameters. This technique has been employed successfully by multiple researchers including [6]. CFD simulations have also been used to model the heat transfer and thermal dispersion in porous media [7, 8]. This approach is limited because the real geometric configuration of a porous medium is often quite complicated and variable. Instead, experimental regenerator testing has been utilized extensively by researchers.

Using experiments and a one-dimensional porous media flow model Harvey [9] determined the hydrodynamic resistance parameters for metal foam, #400SS mesh, #325SS mesh, and 60μm perforated disks. Cha [10, 11] determined the steady and oscillatory flow closure parameters in the radial and axial directions for a variety of porous media using a CFDassisted methodology to iteratively determine the closure parameters. Cha also showed that oscillatory and steady flow closure parameters can be different. This method was later pursued by Clearman [12] and Landrum [13]. Other aspects of pulse tube cryocoolers (PTCs) were investigated by Conrad [14], Pathak [15], and Mulcahey [16].

Rare-earth alloy regenerator fillers suitable for applications in multistage PTCs where the regenerator of the final stage may operate at temperatures in the 10 – 20K range or lower include ErPr (erbium-praseodymium) alloy, which has a high heat capacity at and below 20K. Pathak [17, 18] performed experiments with a packed-bed of 69μm average diameter Er0.5Pr0.5 (alloys with 50% Er and 50% Pr) spheres in a prototypical regenerator for steady and oscillatory flows of helium at room temperature for a range of frequencies, charge pressures, and gas velocities. The Darcy permeability, which in general depends only on the geometry, remained relatively constant. The Forchheimer coefficient varied with the mass flow rate and was correlated in terms of the pore-based Reynolds number.

A key shortcoming of these and many other published studies are that their tests were conducted at room temperature. In theory, hydrodynamic resistance parameters should not be affected by the operating temperature if the geometric configuration of the porous medium remains the same, experimental verification of the forgoing assumption is needed. The objective of this work is thus to determine the Darcy permeability and Forchheimer inertial coefficients for Er0.5Pr0.5 powder, at cryogenic temperatures for a range of pressure and velocity amplitudes and compare the results to those obtained at ambient temperature conditions.

EXPERIMENTAL SETUP

The test section is shown schematically in Figure 1. The test section consists of a stainless regenerator sandwiched between a heat exchanger and a 7.24cc stainless steel surge volume with a Q-drive model 2S132W pressure wave generator (PWG) upstream connected via a 0.91m stainless steel transfer tube. The cold heat exchanger (CHX), is thermally attached to with the 1st stage of a Sumitomo model RDK-408D2 GM cryocooler with a copper bus bar. The first stage cold head of the Sumitomo GM cryocooler can deliver 50W of cooling at 43K, while the second stage provides 1W cooling power at 4.2K. The interior of the CHX is packed with #100 mesh copper screens (64.7% porosity). The regenerator is packed with Er0.5Pr0.5 micro spheres with 55μm mean diameter and 37% porosity. The ErPr powder is retained by #400 SS mesh screens at the inlet and outlet which are reinforced by two #60 SS screens. PCB piezotronics brand pressure sensors (models 102A05 and 102A10) are installed in specially designed ports on the upstream and downstream sides of the CHX and SV. The experimental setup and the GM cryocooler are mounted inside of a vacuum sealed dewar with modular feedthroughs (Figure 2). This dewar is capable of maintaining an insulating vacuum of 10^{-6} torr.

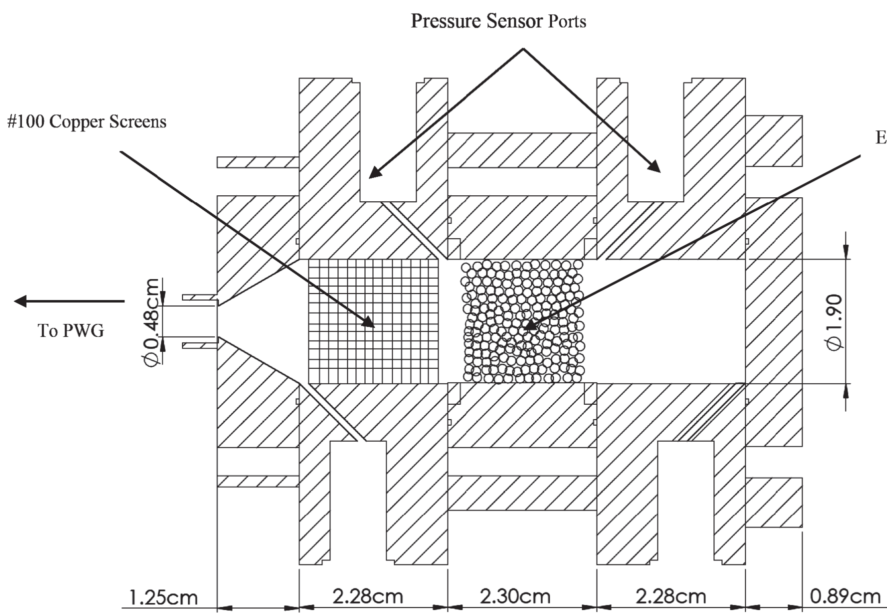

Figure 1. Experimental setup schematic

Instantaneous pressure measurements were made upstream and downstream of the regenerator test section in 2 second durations at a sample rate of 25.6kHz for a total of 51200 samples. The oscillatory pressure measurements were analyzed using Matlab's Fast Fourier Transform (FFT) capability for the fundamental frequency and the next four harmonics of the fundamental. Temperature was measured using a Cryo-con S900 silicon diode thermally mounted with cryogenic rated epoxy to the CHX. The mass flow between the regenerator test section and the surge volume (SV) is calculated by considering the instantaneous pressure and temperature within the SV and assuming that the surge volume is rigid and adiabatic. A simple analysis then leads to

$$\dot{m} = \frac{V}{\gamma RT}\frac{dP}{dt} \tag{2}$$

The time derivative of the SV pressure can be determined numerically from the collected data to calculate the instantaneous mass flow rate into the SV. A FFT is then performed in order to develop an equation for mass flow rate as a function of time.

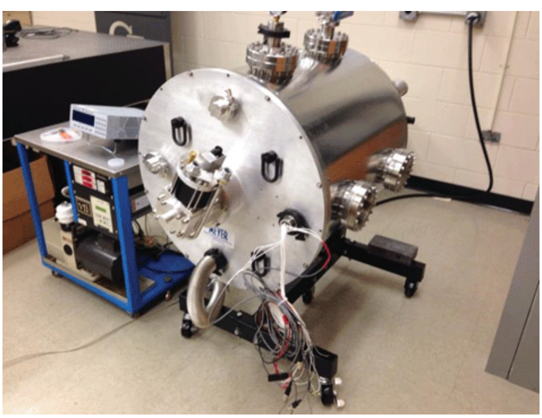

Figure 2. Cryogenic vacuum chamber with modular feedthroughs

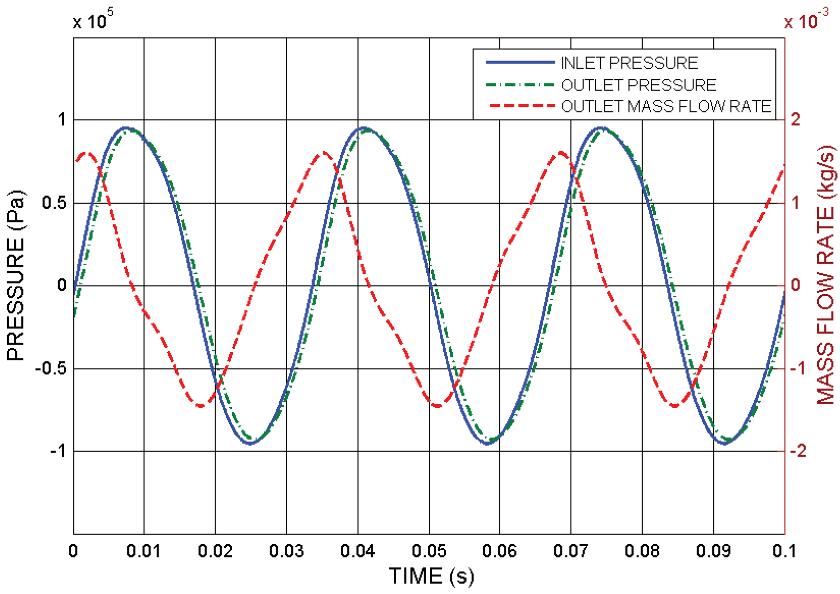

Figure 3. Inlet and outlet pressure oscillations with outlet mass flow rate at 75K and 30VAC PWG input

Some of the results are characterized using the pore based Reynolds number, which uses the square root of the permeability as the length scale.

$$\mathrm{Re}_K = \frac{|\dot{m}|\,K^{0.5}}{\mathrm{m}A} \qquad (3)$$

Figure 3 shows typical upstream and downstream pressure oscillations along with the calculated mass flow rates for the same PWG input power at 75K. Figure 4 and Figure 5 show the amplitude of the instantaneous pressure drop across the regenerator versus the pore based Reynolds number for all samples. The center line represents a linear fit to the experimental data, and the bounding lines represent the upper and lower limits of the data based on the random uncertainty.

CFD ANALYSIS

The methodology has been discussed before [10, 18] and will be discussed briefly here. The CFD code ANSYS Fluent [19] is used to model the regenerator test section as a porous medium

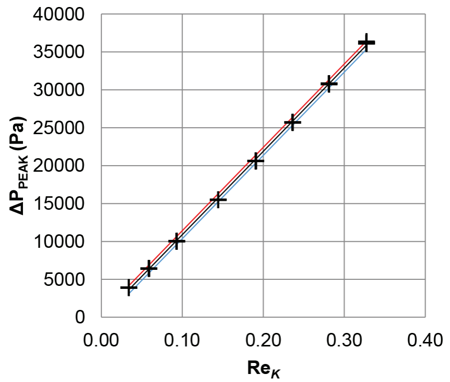

Figure 4. Regenerator pressure drop amplitude vs. Re_K at 300K

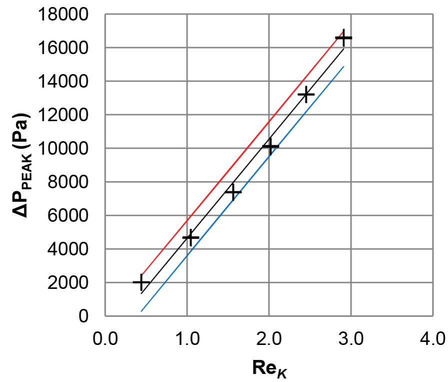

Figure 5. Regenerator pressure drop amplitude vs. Re_K at 75K

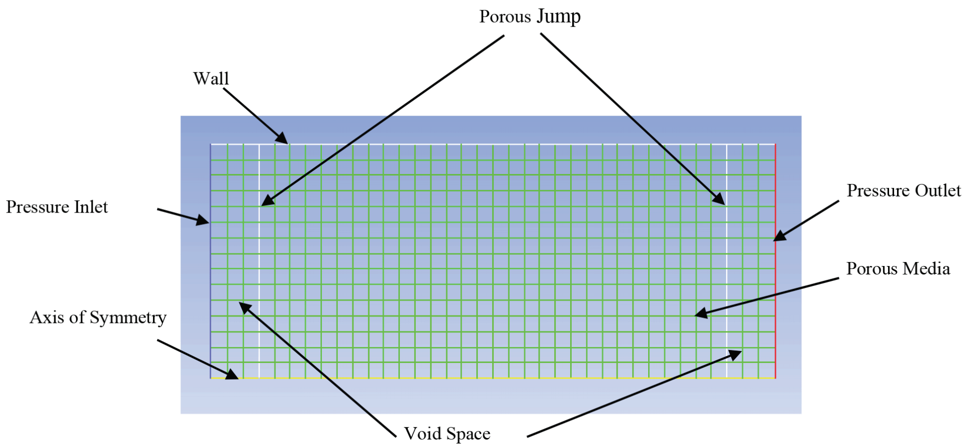

Figure 6. 2D axisymmetric geometry in Fluent

with helium as the working fluid. The governing equations for mass, momentum, and energy conservation in Fluent are as follows:

$$\frac{\partial \varepsilon\rho}{\partial t} + \nabla\cdot(\varepsilon\rho\,\vec{u}) = 0 \tag{4}$$

$$\frac{\partial(\varepsilon\rho\vec{u})}{\partial t} + \nabla\cdot(\varepsilon\rho\vec{u}\vec{u}) + \varepsilon\nabla P - \nabla\cdot(\varepsilon\bar{\bar{\tau}}) = -\left(\frac{\mu}{\bar{\bar{\beta}}}\cdot\vec{u} + \frac{\bar{\bar{C}}\rho}{2}\cdot\left|\vec{u}\right|\vec{u}\right) \tag{5}$$

$$\nabla\cdot[(\varepsilon k_f + (1-\varepsilon)k_s)\nabla T + (\bar{\bar{\tau}}\cdot\varepsilon\vec{u})] = \frac{\partial}{\partial t}(\varepsilon\rho_f E_f + (1-\varepsilon)\rho_s E_s) + \nabla\cdot(\varepsilon\vec{u}(\rho_f E_f + P)) \tag{6}$$

where $\bar{\bar{\beta}}$ (m^2) represents the viscous resistance tensor and $\bar{\bar{C}}$ (m^{-1}) the inertial resistance tensor of porous media. These equations are of course three-dimensional. The experiments involve axisymmetric flow, however, and 2-D flow simulations are performed using polar cylindrical coordinates. One can compare the momentum equation, Equation (5), with the 1-D momentum equation, Equation (1). If the porous medium is isotropic or the axial direction is a principle direction of the porous medium, then:

$$K = \varepsilon^2\beta \tag{7}$$

$$F = \frac{CK^{0.5}}{2\varepsilon^3} \tag{8}$$

For the packed bed of ErPr powder, isotropy is assumed. For flow through screens previous studies have shown that velocities in the radial direction are typically very small in comparison with the axial velocities and that CFD simulation results are insensitive to errors and inaccuracies in radial permeability and Forchheimer coefficients [10–13]. As a result, when axial resistance parameters are studied, for CFD simulations isotropic porous media are assumed.

The 2-D, axisymmetric geometry used in CFD simulations is shown in Figure 6. A mesh size of 0.635mm was used for all modeling based on mesh convergence and grid-independence tests. A user defined function (UDF) was created to apply the oscillatory pressure boundary conditions to the inlet and outlet. The #400 SS retaining mesh of the regenerator is modeled as a porous jump using the values of $\beta = 2.5295\times10^{-11}\,\mathrm{m}^2$ and $C = 120000\,\mathrm{m}^{-1}$ from Cha [10].

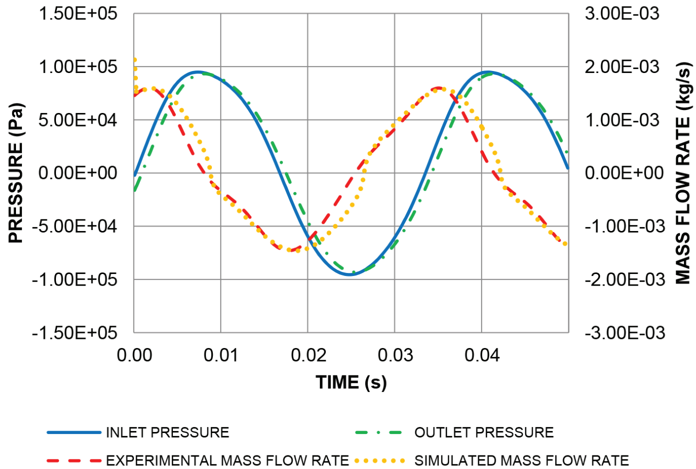

Figure 7. Simulated mass flow rate at 300K and 30VAC PWG input (Re_K=0.26)

Due to the inherent limitations of our measurement equipment, we were unable to measure the pressure drop across the regenerator within reasonable uncertainty limits at sufficiently low mass flow rates that would justify neglecting the inertial resistance. We therefore have used the permeability determined by Pathak [18], $K = 1.93\times10^{-11}\,\mathrm{m}^2$, which corresponds to a viscous resistance of $b = 1.41\times10^{-10}\,\mathrm{m}^2$. Subsequent parametric calculations confirmed that this assumption was justified. Once the proper boundary and operating conditions are supplied to CFD simulations, the inertial resistance is varied systematically until the simulated mass flow rate amplitude at the outlet matches the experimental value. Figure 7 and Figure 8 show some typical results. The simulations show excellent agreement with respect to both the magnitude and phase of the oscillatory mass flow rate.

RESULTS AND DISCUSSIONS

The results for all simulated data points are summarized in Figure 9 where the Forchheimer inertial resistance coefficient, *F*, is displayed as a function of the pore-based Reynolds number defined in Equation (3). The data in this study is also compared to the data of Pathak [18], which were obtained with Er0.5Pr0.5 powder at room temperature. It can be noted that the data obtained in the present study agree with Pathak's measurements very well in terms of trends. In terms of

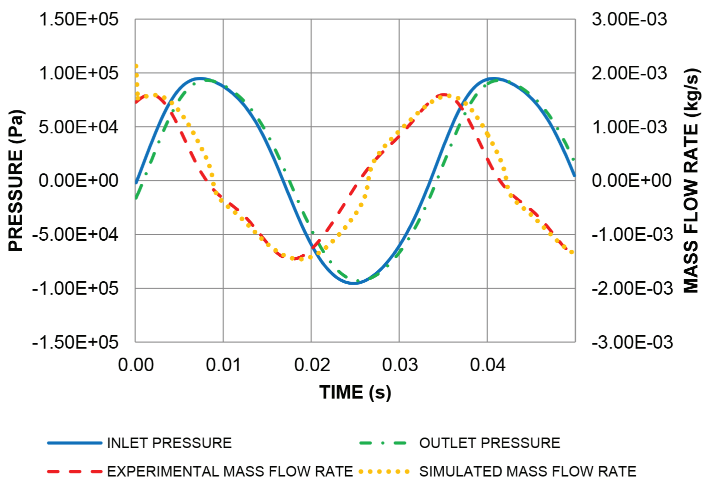

Figure 8. Simulated mass flow rate at 75K and 30VAC PWG input (ReK=2.71)

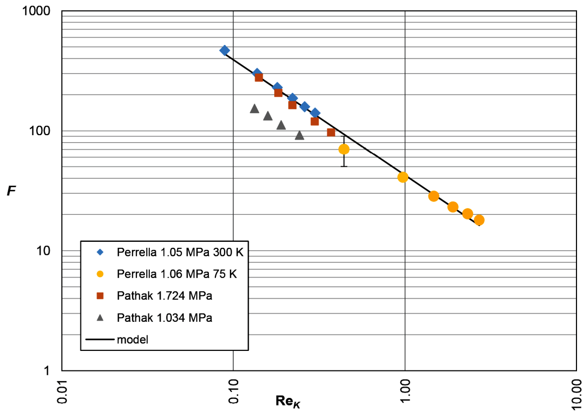

Figure 9. Variation of parameter F vs. pore-based Reynolds number, ReK

the magnitude of parameter F, the agreement between the two data sets is within a factor of two. The main reason for the disagreement could be the fact that in Pathak's experiments the Er0.5Pr0.5 particles were larger (63 to 75µm in diameter, compared with ≈50µm particle mean diameter) than in this study. More importantly, the results of this investigation show that the Darcy permeability and Forchheimer coefficient have similar values at 300K and 75K temperatures, suggesting that the coefficients may indeed be independent of temperature, as previously conjectured. The results also agree with the logarithmic relationship for Forchheimer coefficient derived by Pathak. The parameter F in this study was found to obey a power law relationship with pore scale Reynolds number, yielding the following correlation:

$$F = 42.48\,\mathrm{Re}_K^{\;-0.96} \tag{9}$$

A potential source of discrepancy between room temperature parameters and those measured at cryogenic temperatures is the possibility of subtle configurational changes in a regenerator, which is often assembled at room temperature, when its temperature is reduced to cryogenic levels. The apparent insensitivity of the solid-fluid momentum transport parameters in this investigation suggest that the effect of these configurational variations are negligible at least for packed powders.

While these findings are instructive, more work should be done to test the effects of varying operating frequencies, mean pressures, and even lower operating temperatures. More experiments addressing lower temperatures at as various frequencies and with different regenerator fillers are currently underway at Georgia Tech Cryo Lab.

CONCLUSION

In this study, a prototypical regenerator filled with spherical Er50Pr50 powder was assembled and tested under periodic helium flow at room (300K) and a cryogenic (75K) temperatures. The measured mass flow and pressure drop data were correlated with a porous media CFD model to determine the Darcy permeability and Forchheimer coefficients. The Darcy permeability, which depends only on the geometry of the porous medium, was taken from previous investigations using similar ErPr powder with a value of $K = 1.93\times10^{-11}\,\mathrm{m}^2$. The Forchheimer coefficients was found to decrease logarithmically with increasing pore-based Reynolds number according to Equation (9). The results of this analysis agree well with the results of previous investigations conducted at ambient conditions with slight discrepancies most likely due to the difference in ErPr particle size between the two studies. The values for Forchheimer coefficient in this analysis for temperatures of 300K and 75K at roughly the same mean pressure follow the same logarithmic trend and are well predicted by the same empirical correlation. Together, these findings suggest that the calculation of the porous media hydrodynamic resistance parameters is indeed independent of temperature as previously believed. Work is underway to strengthen this conclusion including experiments at lower temperatures with additional filler materials, mean pressures, and frequencies.

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